SHORT CIRCUIT

Canberra Mathematical Association Inc.

VOLUME 15 NUMBER 5 MAY 2024

NEWS AND COMMENT

On April 10-11, delegates from every state and territory association of mathematics teachers attended a meeting in Adelaide organised by AAMT and ATSIMA, the Aboriginal & Torres Strait Islander Mathematics Alliance. There were three attendees from CMA.

Readers of this newsletter may have seen a news item on ABC television in which ATSIMA director Chris Matthews explained the purpose of the gathering. Professor Matthews said that the education system is not currently meeting the needs of Aboriginal students and that the AAMT affiliated associations like CMA, if suitably mobilised, could help teachers make a difference for this group.

The meeting produced a Commitment Statement which is reproduced on page 2 of this edition.

More on this initiative will appear in coming newsletters, including stories, history and items about culture, not always with an obvious and immediate connection to mathematics, but certainly with relevance to the formation of productive relationships with students.

MEMBERSHIP

Memberships run from 1 Jan to 31 Dec. each year. Membership forms may be downloaded from the CMA website: http://www.canberramaths.org.au

The several benefits of Membership of CMA may be found on the website.

NEWSLETTER

The CMA newsletter, Short Circuit, is distributed monthly to everyone on our mailing list, free of charge and regardless of membership status.

That you are receiving Short Circuit does not imply that you are a current CMA member but we do encourage you to join.

Short Circuit welcomes all readers.

Inside:

CANBERRA MATHEMATICAL ASSOCIATION

ATSIMA SUMMIT, APRIL 2024

STATEMENT

Aboriginal and Torres Strait Islander Mathematics Alliance

ATSIMA

Transforming Maths Education

aamt

Commitment Statement:

Our responsibility is to drive a cultural shift to make a systemic difference in mathematics education for Aboriginal and Torres Strait Islander learners.

We commit to :

- truth-telling which recognises the past and builds capacity for the future
- building relationships by listening to and learning from and with Aboriginal and Torres Strait Islander communities
- creating sustainable partnerships based on trust and respect
- leading and supporting culturally responsive practices and
- advocating for a shared understanding of success

In doing so, we agree to be unwavering and accountable in actioning this commitment to achieve positive outcomes for Aboriginal and Torres Strait Islander learners.

MATHS IN THE STREET

From Bruce Ferrington

On Saturday 23rd March, the CMA took maths to the streets. Tables were set up at Curtin shops and we spent an enjoyable few hours running maths puzzles and challenges for anyone walking past.

About a hundred adults and children stopped and played with the tasks we had set up. There was much positive conversation about how much fun and enjoyment could be had doing maths on a Saturday morning over a cup of coffee.

Several shoppers asked if we were going to do this on a regular basis. The short answer is yes – but we will be choosing different venues around Canberra to locate our Maths in the Street activities.

Stay tuned for venues in the next few months.

PUZZLES

1. Days and centuries

No century begins with a Tuesday, Thursday or Sunday. Can you prove it?

2. Missing magic

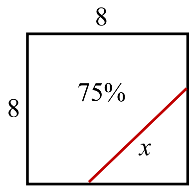
Find the missing numbers to complete this magic square. Is your solution unique?

4		
	7	
6	5	

3. Fish

One morning a fisherman catches 50 fish from a lake. He puts small tags on them and returns them to the lake. In the afternoon he catches 40 fish and 10 of them are tagged. Estimate the number of fish living in the lake.

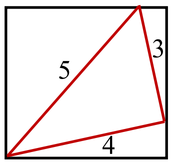
4. Cutting corners



We cut an isosceles triangle corner off a square piece of paper so that 75% of the area is left. What is the length of the cut?

5. Area

A square encloses a 3-4-5 triangle as in the following diagram. What is the area of the square?



PROBLEM SOLVING TASK CENTRE

From Sue Wilson -

Participants who attended my Problem Solving Task Centre workshop at the CMA conference will be interested to hear that another international school is teaching maths using the tasks - recently an eTask pack was sent to a school in Shenzhen, China.

For details of the eTask pack:

http://www.mathematicscentre.com/taskcentre/resource.htm#etasks

ABOUT THE CMA

NEWSLETTER OF THE CANBERRA MATHEMATICAL ASSOCIATION INC. INC

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We're on the Web!
http://www.canberramaths.org.au/

The Canberra Mathematical Association (Inc.) is the representative body of professional educators of mathematics in Canberra, Australia.
It was established by, among others, the late Professor Bernhard Neumann in 1963. It continues to run - as it began - purely on a volunteer basis.

Its aims include

* the promotion of mathematical education to government through lobbying,
* the development, application and dissemination of mathematical knowledge within Canberra through in-service opportunities, and
* facilitating effective cooperation and collaboration between mathematics teachers and their colleagues in Canberra.

THE 2024 CMA COMMITTEE

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Theresa Shellshear is CMA's COACTEA representative.

Bruce Ferrington is CMA's AAMT representative.

Joe Williams is the website manager.

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Find us on Facebook

http://www.facebook.com/pages/Canberra-Mathematical-Association/110629419011275

MULTIPLICATION

The classroom may still be one of the few special places on earth where multiplications are done without an electronic device.

As young people, many of us spent hours and months perfecting the difficult art of long multiplication, a skill that if mastered in a timely way, opened doors to future careers and opportunities. But perhaps there was a better way.

An old textbook describes, approximately as follows, how two positive whole numbers can be multiplied without using long multiplication. Instead, the algorithm uses divisions and multiplications by 2, followed by addition.

Example:
Multiply 27 by 73

Put the numbers at the head of two columns. (There will be fewer steps if the smaller number is placed in the first column.) Divide it by 2 again and again ignoring remainders where present, putting the result underneath each time. In the second column the larger number is to be doubled again and again, placing the result underneath each time. Strike out all numbers in the second column which come opposite even numbers in the first column. Add up the rest.

27	73
13	146
6	~~292~~
3	584
1	1168
	1971

Here is an explanation.
Clearly, when one number is halved n times and the other doubled n times, the product of the new numbers equals the original product. However, in the algorithm, dropping the remainders on division by 2 makes the numbers going down the left column too small and something extra would need to be added to get the correct result.

Suppose instead, the halving in the left-hand column is carried out without dropping the remainders, which are kept as unit fractions. The last entry in the column will be 1 plus a series of decreasing negative powers of 2. In the example we would have

$$
\begin{array}{l}
27 \\
13+\frac{1}{2} \\
6+\frac{1}{2}+\frac{1}{4} \\
3+\frac{1}{4}+\frac{1}{8} \\
1+\frac{1}{2}+\frac{1}{8}+\frac{1}{16}
\end{array}
$$

Observe that the results of multiplying each of the fractions in turn by the nth doubled term is none other than the list of numbers in the right-hand column, omitting the terms adjacent to the terms that have no remainder.

In the example this is

$$1168\times\left(1+\frac{1}{2}+\frac{1}{8}+\frac{1}{16}\right)=1168+584+146+73$$

Hence, we may as well drop the fractions and keep the second column.

In the traditional long multiplication algorithm, the user needs to know the product of every pair of digits from the set (0, 1, …, 9). But in the process just described, one only needs to know how to multiply and divide by 2.

The option of implementing the procedure in binary arithmetic on a machine is apparent. In binary, the given example might be represented as

1 1 0 1 1	0 0 0 0 1 0 0 1 0 0 1
0 1 1 0 1	0 0 0 1 0 0 1 0 0 1 0
0 0 1 1 0	0 0 1 0 0 1 0 0 1 0 0
0 0 0 1 1	0 1 0 0 1 0 0 1 0 0 0
0 0 0 0 1	1 0 0 1 0 0 1 0 0 0 0
	1 1 1 1 0 1 1 0 0 1 1

Thus, the calculation is accomplished by left- and right-shifts, and addition. Moreover, those anachronistic classroom lessons on multiplication, long or otherwise, can be recast as a response to students' curiosity about what happens when the 'times' button is pressed. There will always be a need for some people who understand the inner workings of calculators, and for very many more people who are willing to believe that there is a rational, though complicated, explanation for such things, and no magic.
PT

BEYOND THE QUESTION

From Peter Fox

In Short Circuit – Volume 15, Number 4 (April 2024), Paul asked us to consider the 'old question' that hailed back to Martin Gardner's problem where one coin is rotated around an identical stationary coin. The outer rotating coin, whilst staying in perfect contact with the inner coin, rotates twice, not once as most people initially assume.

To help understand why this occurs, take a moment to go back to Paul's article where he provides a lovely adaption involving squares followed by a wonderful extension combining squares and triangles. Just as intriguing as the rotation problem:

What is the path of a point on the circumference of the rotating coin?

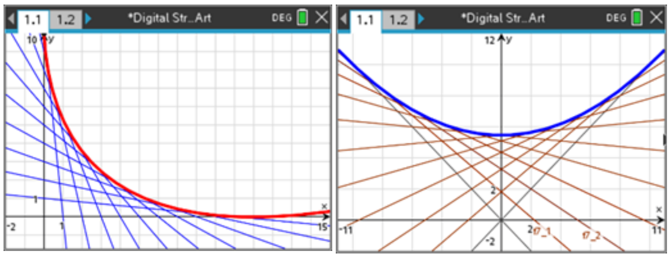
At the recent CMA conference, I presented a workshop on Digital String Art. The first section looked at the amazing images you can create by stitching the x- and y-axes together using mathematics typically covered in Years 8 to 10. Students determine the equation to a straight line given a point on the y-axis (intercept) and calculate gradient such that the line will pass through a specified location on the x-axis.

Students typically work through a series of lines (0, 10) connected to (1, 0); then (0, 9) connected to (2, 0) and so on. As the y-intercept and gradient change, a series of straight lines is produced resulting in something called an *envelope*.

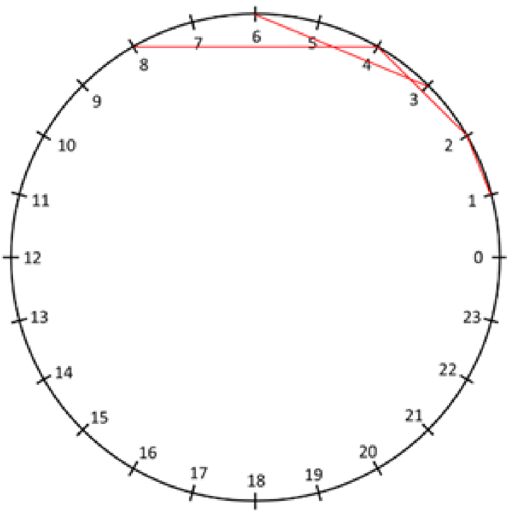
This and similar envelopes can be seen in the cables used in the construction of many bridges, including the Margaret Hunt Hill bridge in Dallas, Texas. The envelope or curve produced is a parabola. Indeed, if

you stitch the lines $y = x$ and $y = -x$ students will see a more typical parabolic curve which has a much easier equation to determine, a treasure trove of mathematics ensues.

Teachers wanting to take this mathematics ever further can use matrices to generate a rotation (and dilation) of this simple parabola and bring it into alignment with the original envelope formed by stitching the x- and y-axes.

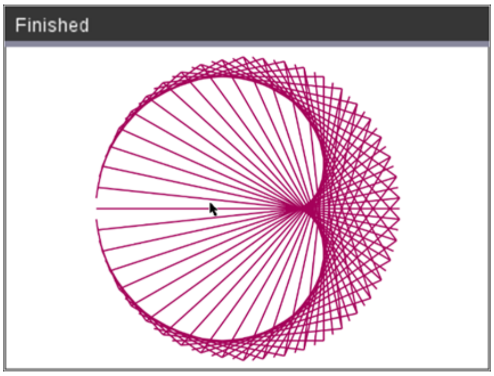


What has all this got to do with rotating coins? To help answer this question, we wrap the lines into a circle. The first patterns starts with stitching the 2 times tables, around the circle. [Burkhard Polster has a wonderful YouTube video on this.]



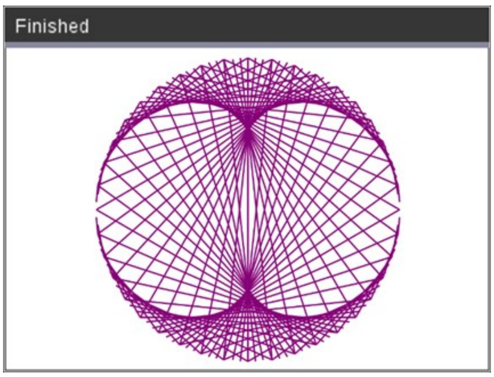
The circle diagram has the numbers from 1 to 23 around the outside. (24 aligns back to zero) The first straight line connects 1 to 2 since 2 × 1 = 2. The second straight line connects 2 to 4 since 2 × 2 = 4. The third straight line connects 3 to 6 since 2 × 3 = 6 and so on. Continue this pattern around the circle. As you go beyond 12, you will need to do some simple modular arithmetic. 2 × 12 = 24 but mod(24,12) = 0. [This is equivalent notation for 24 ≡ 0(mod 12). ed] Similarly, 2 × 13 = 26 but mod(26,12) = 2, (think remainder or clock arithmetic) so 13 is connected to 2. Continuing on, 2 × 14 = 28 but mod(28, 12) = 4 so 13 connects to 4, our pattern becomes symmetrical.

Our envelope is a little hard to see when these points are spaced apart. What happens if we put more and more points around the circle?



If we put 90 points around the circumference of the circle and keep connecting, the image will appear as shown. Take a look back at the path traced out by our rotating coin: is this envelope the same as the path?

If the 2 times tables create this envelope, what might the 3 times tables create?



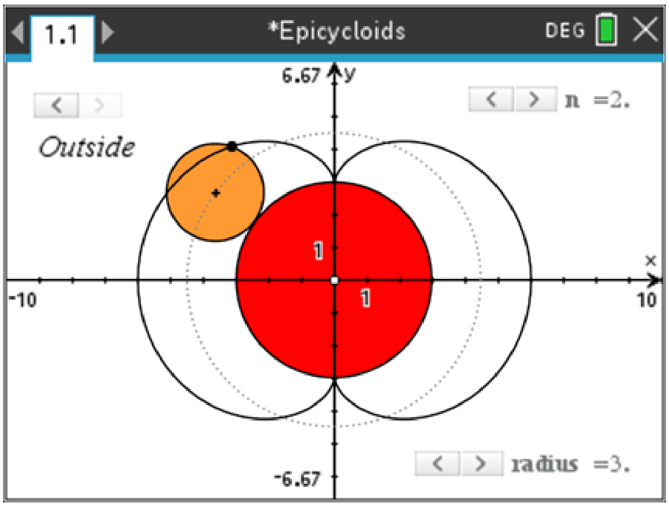
The 2 times table creates an envelope called a 'cardioid', which comes from Latin (cardio) meaning "pertaining to the heart".

The 3 times table creates an envelope called a 'nephroid' meaning kidney shaped.

Can we produce this kidney shaped path with two coins? The answer is "yes". If the outer coin is smaller than the inner stationary coin it stands to reason that it will rotate more times as it is rolled around. If the outer coin has a radius half that of the inner coin it will complete one more rotation.

Conjectures are now ripe for the picking. What would the 4 times tables look like? Would the envelope produced look similar to the path of the rotating coin that has a radius 1/3 the size of the stationary coin?

FYI: The Latin word for 3-leafed clover is: "trifolium".



The rotating coin provides a physical means of producing the path but also a conceptual crutch for the mathematical equations. Consider the movement of the centre of the rotating coin as it circumnavigates the stationary coin. The path is that of a circle with radius: $R + r$, where 'R' is the radius of the inner circle and r the radius of the outer circle. The equation to this path can be described parametrically:

$$x(t) = (R + r)\sin(t)$$
$$y(t) = (R + r)\cos(t)$$

Now imagine, in isolation, the outer coin rotating around its centre located at some point (h, k)

$$x(t) = r\sin(nt) + h$$
$$y(t) = r\cos(nt) + k$$

The locations of the centre (h, k) has already been described parametrically and the value of n depends on the relative sizes of r and R. For the graph in the TI-Nspire document the stationary circle has a radius of 3 units and the rotating circle a radius of 1.5 units. This would produce the parametric equations for the nephroid:

$$x(t) = 1.5\sin(3t) + 4.5\sin(t)$$
$$y(t) = 1.5\cos(3t) + 4.5\cos(t)$$

To align the graph precisely with the point shown on the outer circle a small translation will suffice:

$$x(t) = 1.5\sin(3t - \pi) + 4.5\sin(t)$$
$$y(t) = 1.5\cos(3t - \pi) + 4.5\cos(t)$$

OR

$$x(t) = -1.5\sin(3t) + 4.5\sin(t)$$
$$y(t) = -1.5\cos(3t) + 4.5\cos(t)$$

To run this activity even further, consider trying to find the cartesian equations (start with the cardioid). Also consider the gradient functions, are there any points where the gradient will be undefined or infinite?

All of these activities can be downloaded for free from the Texas Instruments Australia website.

ICME-15

The International Congress on Mathematical Education is the largest international conference on mathematics education in the world.

The 15th International Congress on Mathematical Education (ICME-15) will take place 7-14 July 2024 at International Convention Centre in Sydney, Australia. ICME-15 promises to be an innovative congress that builds on the well-established ICME program, showcasing established and emerging thought leaders from around the world.

RHYMING COUPLET

From Ed Staples—

A rational close to *pi* minus *e*?

Try sixty-nine over one-sixty-three!