SHORT CIRCUIT

Newsletter of the Canberra Mathematical Association INC

VOLUME 11 NUMBER 9 DECEMBER 2020

NEWS AND COMMENT

At the 2020 CMA Annual General Meeting, the audited financial records were presented, and a new council for 2021 was appointed. The councillors are listed on page 4. It is possible that further councillors will be co-opted from the membership as the need arises.

Due to the cancellation of the conference, CMA received no income from that source but also had no related expenditure. The financial position remains sound and council anticipates that the 2021 conference will be exceptional and well worth the wait.

Several other professional development events are in the planning stages.

In this edition, we continue the sequence of articles by Heather Wardrop with the theme of literacy in the mathematics classroom. (See page 5.) There is also a further instalment from Andy Wardrop on mathematical black holes. (See page 3.)

As a slight departure, we have opted to hold back the solutions to the puzzle section until the next edition. Readers are thus encouraged to look for their own solutions and, if they feel so inclined, to submit their discoveries and comments for possible inclusion next time.

Congratulations to the education students from University of Canberra and Australian Catholic University who received awards from CMA for being highly successful in their courses in 2020. They are: Meagan de Puit and Merin Cruikshank from UC; and Jessica Lever, Catherine Langron and Mary Biboudis from ACU. We wish them and their fellow graduates well in their teaching careers.

The award winners receive a year's free membership of CMA, including the AAMT journals.

CANBERRA MATHEMATICAL ASSOCIATION

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MEMBERSHIP

Memberships run from 1 Jan to 31 Dec. each year. Membership forms can be accessed from the CMA website: http://www.canberramaths.org.au

Membership of CMA includes affiliation with the Australian Association of Mathematics Teachers and a subscription to one of two AAMT journals.

As a member, you are entitled to attractive rates for the CMA annual conference and CMA professional development events.

CMA members may attend conferences of the AAMT affiliates in other states, MAV, MANSW, etc. at member rates.

PUZZLES

This edition of Short Circuit has no back-page solutions to the puzzles. Instead, they will be delayed until the next edition so that readers can work out their own, free from the back-of-the-book temptation. If you have a comment or a nice answer to any of the puzzles, please send it in. You could be published!

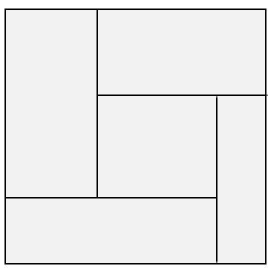
1 Slices of 3D

(a) Great circles are the largest circles that can be drawn on a sphere. (The equator and the lines of longitude on the roughly spherical earth are great circles.) Suppose you have in your hand a cube. One can imagine a sphere that just encloses the cube so that all eight of its vertices lie on the sphere's surface. How many of the vertices, at most, can belong to a single great circle?

(b) A regular tetrahedron has four vertices and there is a sphere that contains all of them. How many great circles, at least, would you need to draw on the sphere so that each vertex lies on one of the circles?

2 If but not only if

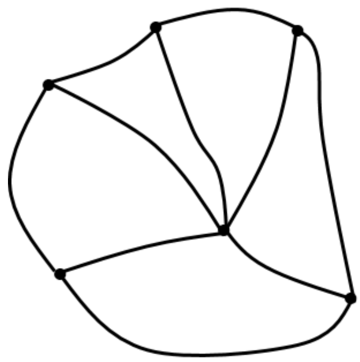
Five rectangles are arranged to form a square as in the following diagram.



If the rectangle in the centre also happens to be a square, it is clear that it could be moved sideways and vertically while preserving a similar arrangement with the four outer rectangles (which would take on various sizes). However, suppose the outer rectangles all have the same area. Show that under this condition the central rectangle has to be a square.

3 Pentagon colouring

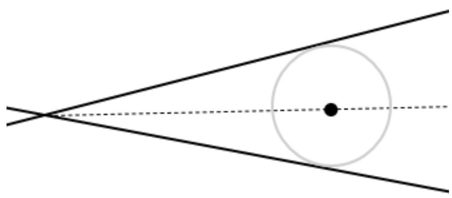
The graph depicted below is essentially a pentagon with triangles added internally. It has five vertices connected sequentially by five edges, and a central vertex connected to each of the other vertices making five three-sided regions.



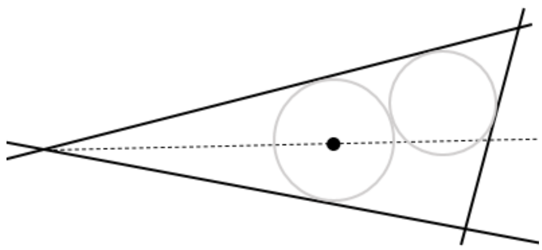
Each region is to be coloured differently. In how many ways can this be done? (Rotations and reversals of a colouring scheme are not considered to be different.)

4 Constructions

(a) Two lines intersect and a point is chosen somewhere on an angle bisector formed by the intersection. Using a straight edge and compass, how can you draw a circle that has the chosen point as its centre and is tangent to the two lines?



(b) Having constructed a circle that is tangent to two intersecting lines, let a third line, not too far away from the circle, intersect the lines as in the diagram below. How might you construct another circle tangent to the existing circle and to one of the first two lines and to the third line?



EVEN MORE MATHEMATICAL BLACK HOLES

By Andy Wardrop

The Collatz Conjecture

This mathematical process is credited to the German mathematician Lothar Collatz (1910-1990) who was working on the conjecture in 1937. Does the Collatz sequence eventually reach 1 for all positive integer initial values? This question is unanswered even though many great mathematicians have tried to solve it. It is also known as the 3n+1 problem, the 3n+1 conjecture, the Ulam conjecture, Kakutani's problem, the Thwaites conjecture, Hasse's algorithm and the Syracuse problem.

Start with any positive whole number. If it is even, divide it by 2. If it is odd, multiply the number by 3 and then add 1. Repeat the process until you discover the black hole. Note that the black hole is a loop 1-4-2-1-4-…

Ecker (1999) suggests a slightly different mathematical process that results in a mathematical black hole of 4. Start with any positive whole number and find the largest odd factor. Multiply this factor by 3 then add 1. Repeat the process until you get to the black hole.

Example
I choose 168. The highest odd factor of 168 is 29 (since $168 = 1 \times 2 \times 2 \times 29$).
Multiply 29 by 3 and add 1 to get 88
The highest odd factor of 88 is 11
Multiply 11 by 3 and add 1 to get 34
The highest odd factor of 34 is 17.
Multiply 17 by 3 and add 1 to get 52
The highest odd factor of 52 is 13
Multiply 13 by 3 and add1 to get 40
The highest odd factor of 40 is 5
Multiply 5 by 3 and add 1 to get 16
The highest odd factor of 16 is 1
Multiply 1 by 3 and add 1 to get 4
The highest odd factor of 4 is 1
Multiply 1 by 3 and add 1 to get 4… a black hole

Sum of Cubes

Pick a number that is a multiple of 3. (Any number whose digits add up to a multiple of 3 is divisible by 3.) Find the sum of the cube of each digit. Repeat until you find the black hole.

Example
I choose 126

$$1^3 + 2^3 + 6^3 = 1 + 8 + 216 = 225$$
$$2^3 + 2^3 + 5^3 = 8 + 8 + 125 = 141$$
$$1^3 + 4^3 + 1^3 = 1 + 64 + 1 = 66$$
$$6^3 + 6^3 = 216 + 216 = 432$$
$$4^3 + 3^3 + 2^3 = 64 + 27 + 8 = 99$$
$$9^3 + 9^3 = 729 + 729 = 1458$$
$$1^3 + 4^3 + 5^3 + 8^3 = 1 + 64 + 125 + 512 = 702$$
$$7^3 + 0^3 + 2^3 = 343 + 0 + 8 = 351$$
$$3^3 + 5^3 + 1^3 = 27 + 125 + 1 = 153$$
$$1^3 + 5^3 + 3^3 = 1 + 125 + 27 = 153...$$

There are four three-digit numbers that equal the sum of the cubes of each digit. They are 153, 370, 371 and 407. They are *narcissistic numbers* or *Armstrong numbers*. Only 153 has the black hole property.

Some Final Thoughts

In the first instalment of the three articles on Mathematical Black Holes, I attempted to define the concept as a repetitious process ending in a loop or single number. Some writers also include numbers that are limits of a converging sum or a repetitious process that approaches a limit. For example,

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$$

has the limiting sum of 2 but, in my opinion, 2 is not a black hole. The sum gets closer to 2 every time we add in a term but that is fundamentally different from the seemingly erratic behaviour of the examples I have written about. When you get to a mathematical black hole you get no further or you are in an endless loop. It is not a limit that you are approaching in the usual sense.

Reference

Ecker, M. W. (1999) "Number Play, Calculators and Card Tricks: Mathemagical Black Holes" in *The Mathemagician and Pied Puzzler: A Collection in Tribute to Martin Gardner*. E. Berlekamp and T. Rodgers (Editors) page 41

ABOUT THE CMA

The Canberra Mathematical Association (Inc.) is the representative body of professional educators of mathematics in Canberra, Australia.

It was established by, among others, the late Professor Bernhard Neumann in 1963. It continues to run - as it began - purely on a volunteer basis.

Its aims include

* the promotion of mathematical education to government through lobbying,
* the development, application and dissemination of mathematical knowledge within Canberra through in-service opportunities, and
* facilitating effective cooperation and collaboration between mathematics teachers and their colleagues in Canberra.

NEWSLETTER OF THE CANBERRA MATHEMATICAL ASSOCIATION INC

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http://www.facebook.com/pages/Canberra-Mathematical-Association/110629419011275

STRATEGY 5

The use of learning journals in mathematics
By Heather Wardrop

I have used journals frequently with mathematics classes at high school and college level. These are the advantages of having students write about their mathematics:

Students learn to progressively summarise their work.

Students reflect on their progress over a week and identify their own strengths and weaknesses. In the process of reflection, they often solve their own problems. They are often glad to see the problem they experienced explained to the whole class the next lesson and are then more inclined to identify areas of concern next time

Students identify what is important and learn how to express it in their own words as well as using mathematical language to construct their own meaning

Teachers find errors in approach, notation or expression and can rectify these before a test

Teachers can note what is NOT there. Students do not provide examples of work they do not understand

Communication is aided. The quiet student can tell you that they do not understand something. It is the best form of early feedback for the student and the teacher. It informs the revision plan for the week. It is useful for parent interviews and reports as it reflects effort.

Students can't cheat and get someone else to write it every week – they need to have been in class. I usually had the students do the first few journal entries in the classroom in front of me. I marked these but the mark didn't count and I handed out an example of a good journal entry for the week's work.

There are very rarely surprise test results and if there is one then you need to talk to the student about the discrepancy. Both the teacher and the student know how the student is progressing.

INSTRUCTIONS GIVEN TO STUDENTS

A journal is a summary of the work you have learned this week

All new terms and symbols must be explained in your own words

New mathematical language must be used by you

Relevant examples must be given and they cannot be ones I used on the board or just copied from your text book. It's great if you make up your own example. It must not be too easy! Include at least one worded problem.

The journal entry must include a comment on how you view your progress for the week. Are there things you can't do? Let me know exactly what they are.

Your journal should reflect that you have read your text book, looked at new terminology and studied the information given.

Criteria for assessment

The summary is complete. .../3
Terms and symbols are explained. .../3
Relevant examples are given. .../3
Mathematical language and symbols are used correctly. .../3
Progress is reflected on. .../3
Problems are identified OR interesting issues are raised. .../3
(This may show extra reading or further questions. For example "What if the triangle is not right angled?").

Whilst at Narrabundah College, I allocated some discretionary marks for the journals. One of the boys really complained but later, when I wanted some examples for some PD I was presenting, would not hand it over because,

"It has everything I need to know in it. All the hard questions I couldn't do are just sitting there corrected by you".

Another successful student said it was the best learning strategy she had ever used. She knew what she had learned, what she missed out, had examples to look at before the test, and had been forced to evaluate her own progress.

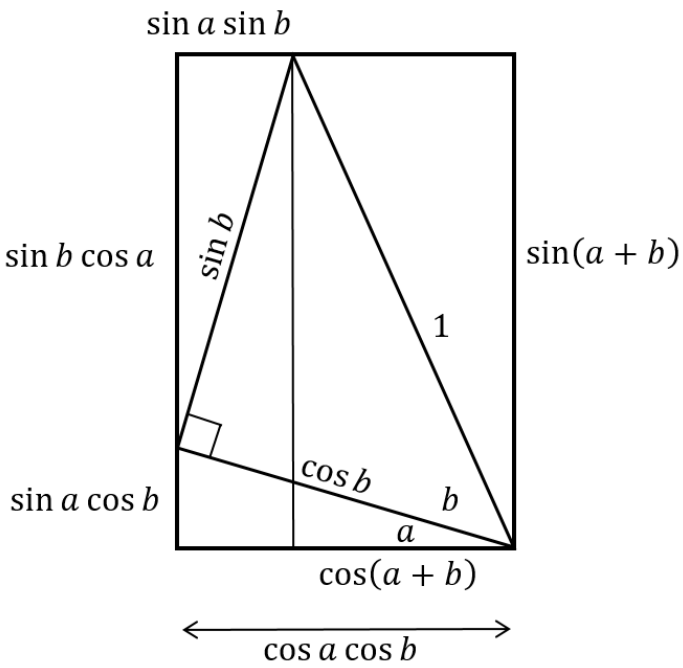
This technique can be adapted for younger students by the use of sentence starters:

Three things I learned this week are ……..

Something I found really difficult was……..

I would like more help with……….

A PROOF BY DIAGRAM



ACER—MORE ON MATHS FOR MASTERY

Geoff Masters AO, CEO of the Australian Council *for* Educational Research, has written an occasional essay: [*Time for a paradigm shift in school education?*] In it, Masters argues that mastery rather than time could be made the principle that guides a student's progress through the curriculum.

The thesis of the essay is that the schooling paradigm is in need of review. Currently, the role of teachers is to deliver a year-level curriculum to all students in a given year level. However, many students are not ready for their year-level curriculum because they lack prerequisite knowledge, skills and understandings. The resulting mismatch has unfortunate consequences for both teaching and learning.

Masters explores these ideas through analogy with the notion of the paradigm shift expressed in the book *The Structure of Scientific Revolutions*, by Thomas Kuhn. The essay may be accessed by mouse-clicking on the link above followed by selecting the download button.