SHORT CIRCUIT

Newsletter of the Canberra Mathematical Association INC

VOLUME 13 NUMBER 4 APRIL 2022

NEWS AND COMMENT

The Australian Government Department of Education, Skills and Employment has released the report of its Initial Teacher Education Review, conducted in 2021 by an Expert Panel chaired by Ms Lisa Paul AO PSM.

The panel made 17 recommendations, the first two of which concern raising the status of teaching and attracting high-quality candidates.

The full report titled *Next Steps: Report of the Quality Initial Teacher Education Review* is available on-line. There is also a 4-page condensed version.

For this edition of Short Circuit, we have again received a pleasing number of contributions from readers. If you have favourite lessons or well-tested teaching ideas, some interesting maths or puzzles or entertaining anecdotes, consider sending them in.

While it is difficult to come up with completely original ideas and your favourite lessons might already be well known to some techers, bear in mind that many readers are relatively young and may not yet have come across the gems you have accumulated.

Humour is also a possibility. For students at any level who are wrestling with difficult mathematical concepts, lessons are not usually occasions for levity, but the struggle might seem less dreadful if they were. (See Bruce Ferrington's piece on page 2.)

CANBERRA MATHEMATICAL ASSOCIATION

Inside:

Coming Events:

2022 CMA conference

Wednesday Workshops:

Check for notices sent separately.

MEMBERSHIP

Memberships run from 1 Jan to 31 Dec. each year. Membership forms can be accessed from the CMA website: http://www.canberramaths.org.au

Membership of CMA includes affiliation with the Australian Association of Mathematics Teachers and a subscription to one of two AAMT journals.

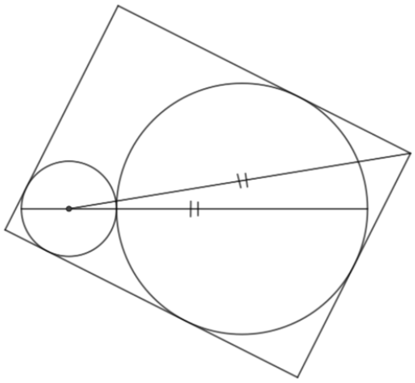
As a member, you are entitled to attractive rates for the CMA annual conference and CMA professional development events.

CMA members may attend conferences of the AAMT affiliates in other states, MAV, MANSW, etc. at member rates.

PUZZLES

1. Another constant

Last month we included a geometrical puzzle from Colin McAlister. Here is another one in a similar vein. Colin calls it his Chopsticks and Bowls construction.



Two mutually tangential circles are contained within a rectangle. The small circle is tangent to two sides of the rectangle, and the large circle is tangent to three of the sides. The distance from the centre of the small circle to the opposite corner of the rectangle is equal to the sum of the diameters of the two circles. Assume that the small circle has radius 1 unit.

Can you develop an equation that, when solved, provides the exact value of the radius of the large circle?

2. Another face of geometry

Sue Wilson found this one.

From a square piece of paper, there are various ways in which a net can be cut out that folds into a cube. How should this be done so as to maximise the volume of the cube? (The net must be in one continuous piece.)

What proportion of the square does the maximal net take up?

FAREY INTERESTING

Suppose we choose a number, a positive integer b, and form the sequence F_b of all the non-negative fractions (written in their lowest terms) that are less than or equal to 1 and list them in order of their size. For example, choosing $b = 5$, the eleven fractions of F_5 in ascending order are

0/1, 1/5, 1/4, 1/3, 2/5, 1/2, 3/5, 2/3, 3/4, 4/5, 1/1

Such an ordered list is known as a *Farey sequence*.

The British geologist John Farey Sr., in 1816, wrote about them in the *Philosophical Magazine*. He became intrigued by them because of a curious property they seemed to exhibit. Although he couldn't prove it, he found that by taking any three consecutive fractions in the sequence, the middle term's numerator and denominator seemed to always be the sum of the two neighbouring term's numerator and denominator. For example, $3/5 = (1+2)/(2+3)$. (Note that this is a very strange way to add fractions).

Alas, he was not the first to notice this. The French mathematician Charles Haros had written about this sequence fourteen years earlier in 1802 and had proved the property Farey had noticed. Unfortunately, the name Farey sequence stuck!

The fraction $(p+r)/(q+s)$ is termed the *mediant* of p/q and r/s and, provided the individual fractions are in their lowest terms, the mediant always has a value between them..

This follows from the two relations showing positive differences between the general terms. (We assume $r/s > p/q$.)

$(p+r)/(q+s) - p/q = s/(q+s).(r/s - p/q)$, and

$r/s - (p+r)/(q+s) = q/(q+s).(r/s - p/q)$.

Any two consecutive Farey fractions are called *Farey pairs*. The difference between the terms of a Farey pair is always the product of the reciprocals of the denominators. For example, the differences in F_5 are

1/5, 1/20, 1/12, 1/10, 1/10, 1/15, 1/12, 1/20, 1/5.

Note the symmetric pattern in these differences. This is just one of the many properties revealed by these strange sequences. If you're intrigued, why not do some digging yourself.?

ES

MY FAVOURITE LESSONS

Holding Hands to learn about straight lines

From Heather Wardrop

This practical activity has the whole class eventually holding hands, giggling and collecting data to learn about straight line graphs. It gives meaning to intercept and gradient. In these COVID times they could hold a ruler between them or you could provide hand sanitiser. They could also stand apart and pass a note or whisper a word.

I have used this from year 8 to year 12 but I think it could be done with upper primary school students to demonstrate graphing as it only uses positive whole numbers.

I learned this at a mathematics PD course but I cannot remember the presenter.

Choose a spot about 5m from the board. Get a student (student 1 or S1) to run to the board whilst a nominated timekeeper times it using their phone or a stopwatch.

Next have a student stand next to this student holding hands and, on the word, "Go" student 2 squeezes the hand of student 1 who runs to the board. (Alternatively a note could be passed or some other signal.)

Then student 3 joins. On "Go" S3 squeezes the hand of S2 who then squeezes the hand of S1 who runs to the board and the timekeeper records the time. This follows until the whole class is involved.

When this information is plotted (time taken from "Go" to S1 reaching the board against the number of signals passed) a straight line is formed where the gradient is the time taken for one squeeze to be passed to the next student and the intercept is the time taken for S1 to reach the board with no signal time. The data could be stored and graphed on a graphical calculator or in a spreadsheet. A table with a hand drawn graph is effective as well.

This is a fun lesson to teach.

MATHEMATICAL HUMOUR

From Bruce Ferrington

I was reading a book I picked up at Lifeline Bookfair last year. Here are a few short snippets.

The book is *Professor Stewart's Casebook of Mathematical Mysteries*, by Ian Stewart (2014) (Profile Books: London.)

On pages 80-81, with the heading *Master of All He Surveys*, we read,

> A farmer wanted to enclose as large an area of field as possible, using the shortest possible fence. Perhaps unwisely, he called the local university, who sent an engineer, a physicist and a mathematician.
>
> The engineer built a circular fence, saying it was the most efficient shape.
>
> The physicist built a straight line so long that you couldn't see the ends, and told the farmer that to all intents and purposes it went straight around the earth, so he had fenced half the planet.
>
> The mathematician built a tiny circular fence around himself and said, *I declare myself to be on the outside.*

On page 158, under *Isn't Statistics Wonderful?* the author says,

> Statistically, 42 million alligator eggs are laid every year. Of those, only half hatch. Of those that hatch, three quarters are eaten by predators in the first month. Of the rest, only 5% are alive after the first year, for one reason or another.
>
> If it wasn't for statistics, we'd all be eaten by alligators.

NEWSLETTER OF THE CANBERRA MATHEMATICAL ASSOCIATION INC

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We're on the Web!
http://www.canberramaths.org.au/

ABOUT THE CMA

The Canberra Mathematical Association (Inc.) is the representative body of professional educators of mathematics in Canberra, Australia.
It was established by, among others, the late Professor Bernhard Neumann in 1963. It continues to run - as it began - purely on a volunteer basis.

Its aims include

* the promotion of mathematical education to government through lobbying,
* the development, application and dissemination of mathematical knowledge within Canberra through in-service opportunities, and
* facilitating effective cooperation and collaboration between mathematics teachers and their colleagues in Canberra.

THE 2022 CMA COMMITTEE

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Theresa Shellshear is CMA's COACTEA representative.

Sue Wilson is CMA's AAMT representative.

Joe Wilson is the website manager.

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Find us on Facebook

http://www.facebook.com/pages/Canberra-Mathematical-Association/110629419011275

CAREERS AND MATHEMATICS

From Frances Moore
This is the second article in a series.

Mathematics is in every job - we all know that but do our students? We will explore a different job and the mathematical activities involving this job from the website On the Job.

Let's have a look at the Biometrician. Detailed information about the Biometrician can be found here: Biometrician - Research and Development - On The Job

Context and relevance: There are several *Types of Biometrics* and today our culture is using them more and more for security:

- DNA Matching
- Ear
- Eyes - Iris Recognition
- Eyes - Retina Recognition
- Face Recognition
- Fingerprint Recognition
- Finger Geometry Recognition
- Gait
- Hand Geometry Recognition
- Odour
- Signature Recognition
- Typing Recognition
- Vein Recognition
- Voice - Speaker Recognition
- Voice - Speaker Verification/Authentication

Activities for the Classroom:
https://onthejob.education/classhome_activities/research/biometrician.htm

Activity 1: Your Ear: A Medical & Technical Illustration: Intercepting with Biometrics!
Primary, Middle, Secondary

This activity looks at Biometric Verification using the ear. Students are to measure different parts of their partner's ear and photograph it. The students are to submit their measurements into a class Excel spreadsheet to see if any ear is the same. They are to then work out what other factors/characteristics could differentiate the ears.

Activity 2: 1984: A Comparison with today
An activity for **Secondary** students studying Literature.

Activity 3: Voiceprint & the ATO. This activity is linked to the Call Centre Operator. Secondary. An activity for students studying Technology.

Activity 4: Hand Biometrics Technology (from TryEngineering.org) **Primary, Middle, Secondary**

This lesson is on engineering applications of biometric technologies for identification or security applications. After exploring hand geometry biometrics, students work in teams of "engineers" to evaluate pros and cons of incorporating a hand recognition biometric technology into a new security system for a museum.

Careers & Mathematics can be found at
https://onthejob.education/teachers_parents/Mathematics_Teachers/Careers_Mathematics_Index.htm

Contact Information
If you are investigating a job or person in that job, please contact me Frances Moore – I would be happy to hear from you.
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Mob 0410 540 608

A LOOK INTO MY LIBRARY

From Valerie Barker

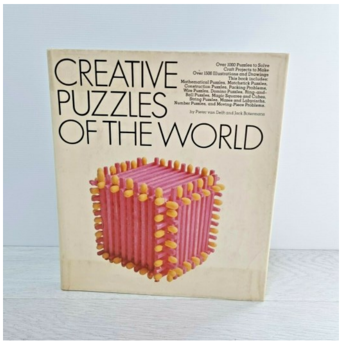
Like so many teachers, I am a collector…of books, puzzles, games, objects, photos and other eclectic memorabilia that has something – a little or a lot - to do with mathematics.

In this occasional series, I would like to share some of these with you, mostly books. I have tried to check whether some of the books are still in print but hard to source, or are more readily available. Other materials may no longer be available, but they may give you ideas of new sources or things to look out for.

So what is on my bookshelves? My April selection is as follows.

1. **Creative Puzzles of the World**,
   Jack Botermans and Pieter van Delft (1978)

This is now long out-of-print, but it seems to be available, although not cheaply, through second-



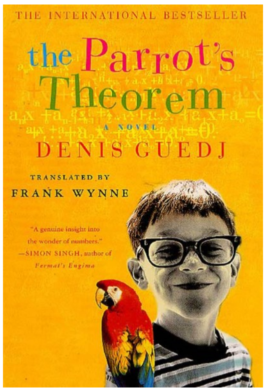
hand and antiquarian booksellers.

This book has over 1000 puzzles to solve, and offers instructions for making many of the classic puzzles: tangrams and other dissection puzzles, matchstick and dominoes problems, magic squares, string puzzles, mazes and labyrinths, and lots more. It offers hours of mental wandering and wondering; I have used many of its ideas to create, or have my students create and enjoy a range of puzzles, many of them classics of their kind.

2. **The Parrot's Theorem**
   a novel by Denis Guedj (2000), Weidenfeld & Nicolson, UK.

In this story Max, a deaf boy whose dysfunctional family live with Mr Ruche (a reclusive Parisan bookseller), finds a voluble parrot in a local fleamarket. The parrot turns out to be a bird who discusses maths with anyone who will listen. Meanwhile, Mr Ruche receives a letter from a long lost friend in the Amazon bequeathing him a vast library of mathematical books… The parrot and the bequest lead to a great exploration, both mental and physical.

Originally published in France in 1998, where it was a best-seller, it is a delightful novel which takes the reader on a journey through the history of mathematics in the guise of a first-rate mystery!

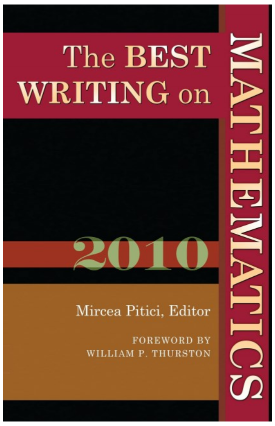


3. **The Best Writing on Mathematics 2021**
   Edited by Mercea Pitici, published by Princeton University Press, Princeton and Oxford.

This is actually still waiting to be placed on my shelves, alongside previous editions dating back to 2010, when I first discovered this annual publication. (In fact, 2010 was the first of the subsequent annual editions).

These anthologies bring together the year's finest mathematics writing from around the world. Featuring promising new voices alongside some of the foremost names in the field, *The Best Writing on Mathematics (2010 - 2021)* makes available to a wide audience many articles not easily found anywhere else - and you don't need to be a mathematician to enjoy them. These writings offer surprising insights into the nature, meaning, and practice of mathematics today. They also delve into the history, philosophy, teaching, and everyday occurrences of maths, and take readers behind the scenes of the hottest mathematical debates. And the structure of these anthologies means that they are easily dipped into and picked up for those brief few minutes of quiet in our busy lives.

Even the introduction of each edition, the editorial itself, is a fascinating overview of the most recent technical and non-technical writing.



DIVISIBILITY BY 11

Years ago, it was customary to discuss whole number divisibility rules in the classroom. One of these was divisibility by 11.

In seeking such a rule, a pattern begins to emerge when it is seen that single digits multiplied by 11 result in a pair of the same digit, and that two-digit numbers are easily multiplied by 11 by adding the digits and placing the result between them (adding a 'carry' to the left if necessary)

The divisibility rule that generalises this is that, for any number N, if the absolute difference between the sum of the *odd-positioned* digits and the sum of the *even-positioned* digits was either zero, or else a multiple of 11, then N is divisible by 11.

To see how it worked, consider the number 65 967. The *odd positioned* digits 6, 9, and 7 add up to 22 and the *even positioned* digits 5 and 6 add up to 11. In this case 22 – 11 = 11 and so 65 967 is divisible by 11.

What about 416 301? The odd positions sum is 10 and the even positions sum is 5, so their difference is 5 and thus 416 301 is not divisible by 11. How about a huge number like 342 576 159 431 000? The odds add to 25, the evens 25, so the answer is 'yes'.

So, why does it work?

The following proof is for a five-digit number, but it could easily be adapted to any whole number.

Suppose we think about the number $N = edcba$, as written in standard decimal notation. We could show it as $N = 10000e + 1000d + 100c + 10b + a$, and then split each section up so that
$N = (9999e + e) + (1001d - d) + (99c + c) + (11b - b) + a$.
Finally recombine the terms to
$N = [9999e + 1001d + 99c + 11b] + (e + c + a) - (d + b)$.

All that is required now is to show that the terms in the square brackets are each divisible by 11, and we can do this by applying modular arithmetic.

It is clear that $10 \equiv -1 \pmod{11}$ and therefore, even powers of 10 become $1 \pmod{11}$ and odd powers of 10 become $-1 \pmod{11}$.

This means that $9999 = (10^4 - 1) \equiv 0 \pmod{11}$ and similarly $99 = (10^2 - 1) \equiv 0 \pmod{11}$. Also, $1001 = (10^3 + 1) \equiv 0 \pmod{11}$ and similarly $11 = (10^1 + 1) \equiv 0 \pmod{11}$.

Therefore, the four terms in the square brackets are all divisible by 11. If it happens that the other part, $(e + c + a) - (d + b)$ is also divisible by 11, then the number of the form $N = edcba$ must be divisible by 11.

ES

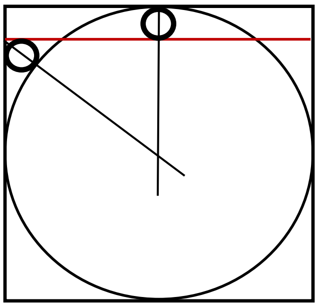
PI DAY SUDOKU SOLUTION

Every row, column and region contains the first 12 digits of pi.

3	2	5	1	5	4	6	3	1	8	9	5
4	1	5	2	3	8	5	9	5	1	3	6
6	1	4	5	9	3	5	8	3	1	2	5
5	3	3	1	8	5	9	2	5	6	4	1
8	9	2	6	5	1	1	5	4	3	3	5
5	8	1	5	2	9	4	3	3	5	6	1
1	5	3	8	1	6	2	4	9	5	5	3
9	4	5	3	5	1	5	6	8	2	1	3
2	3	6	5	1	5	3	1	5	4	8	9
3	6	8	9	4	5	1	5	1	3	5	2
1	5	1	3	6	3	8	5	2	9	5	4
5	5	9	4	3	2	3	1	6	5	1	8

PUZZLE SOLUTIONS from Vol 13 No 3

1. **A constant**



The configuration was explored by Colin McAllister using 'geogebra'. Colin saw that the ratio between the diameters of the circles was close to 9:1. With ordinary geometric calculations we find that the ratio is in fact

$$\frac{R}{r} = 3 + \sqrt{2} + \sqrt{10 + 8\sqrt{2}}$$

which is 9.03089776… .

The ratio is clearly an irrational number but there are good rational approximations. 9/1, 289/32, 587/65, 876/97, 2339/259, … .

2. **Coffee**

'You have a cup of black coffee and an equal quantity of milk in another cup. You take a spoon of the milk and stir it into the coffee. Then, you take a spoon of the coffee and milk mixture and stir it back into the cup of milk.'

At the end of this operation there is as much milk in the coffee as there is coffee in the milk. To see why, note that the coffee cup contains a little less than a spoon of milk. But since both cups still contain the original amounts of fluids, the same amount of coffee has been displaced from the coffee cup. It is now in the milk cup.