SHORT CIRCUIT

Newsletter of the Canberra Mathematical Association INC

VOLUME 12 NUMBER 3 MARCH 2021

NEWS AND COMMENT

Education is not managed solely by Education Departments. Families, community organisations, the arts, businesses, and many other formal and informal institutions can be educators. Sometimes, in order to be realised, a good idea in education has to become a commercial enterprise. This is not to say that all commercial educational enterprises spring from sound principles, but some do. On page 9 is outlined one such good idea, made real in the form of a touring theatre production.

Teachers try to take delays and interruptions calmly these days, hoping that the realisation of a good plan that has been set aside will yet occur. Recently, a flyer announced a year-long professional development program through Deakin University. This has been postponed but is likely to begin in semester 2. Foretastes by presenter Steve Thornton are expected before then and will be advertised.

CMA president, Bruce Ferrington, writes of the workshop held on Saturday 20th February at Mother Teresa School, Harrison:

> I would like to thank Anna, Valerie, Theresa and everyone who was involved in the great workshop this morning at Harrison. It was exciting to be meeting face to face with people again. We had 67 people register and there were a lot of very happy teachers by the end of the morning. Great energy and a very positive start to the year.

This edition of Short Circuit has run to nine pages! It includes work from three main contributors. Keep those wise words coming, folks!

CANBERRA MATHEMATICAL ASSOCIATION

Inside:

Coming Events:

CMA conference: 7 August at ADFA

Welcome Workshop for secondary teachers. TBA

AGM: 10 November.

Wednesday Workshop:

MEMBERSHIP

Memberships run from 1 Jan to 31 Dec. each year. Membership forms can be accessed from the CMA website: http://www.canberramaths.org.au

Membership of CMA includes affiliation with the Australian Association of Mathematics Teachers and a subscription to one of two AAMT journals.

As a member, you are entitled to attractive rates for the CMA annual conference and CMA professional development events.

CMA members may attend conferences of the AAMT affiliates in other states, MAV, MANSW, etc. at member rates.

PUZZLES

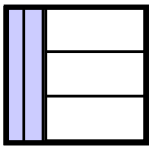
1 Some product

There is a collection of numbers whose sum is 10. How large could the *product* of those numbers be?

2 Risk

In a gambling game, three cards are dealt from a small but well shuffled deck of four, exactly one of which is red. If the red card appears, the game is lost and the player loses their stake of $17. If no red card appears the player wins $50. Who wins in the long run?

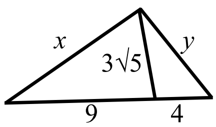
3 Algebra anybody?



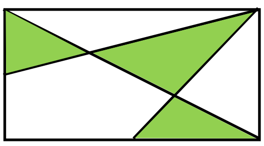
The shaded and unshaded rectangles inside the square have the same perimeter. What is the ratio of the area of a shaded rectangle to the area of an unshaded rectangle?

4 Maximum difficulty

The combined distance $x + y$ in the diagram is certainly greater than 13 but how much greater can it be?



5 Pieces of rectangle



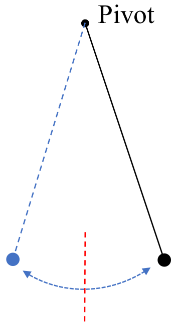
The sloping lines cut the rectangle at three vertices and at the mid-points of two sides. What fraction of the rectangle is shaded green?

PI TIME

By Ed Staples

(A π-day idea adressed to students…)

The word pendulum derives from the ancient Latin word *pendulus* meaning to hang down and that's exactly what a modern-day pendulum tends to do, unless of course it's pushed by some force whereupon it begins to swing back and forth for a while until various frictional forces stop it. A simple pendulum consists of a weighted *bob* at the end of a light *rod* suspended from a *pivot* so that it can swing freely.



Galileo Galilei in 1602 was (it is said) first to realise that a pendulum could be used as a timekeeper. He noticed that if the pendulum didn't swing too wildly, the angle of swing didn't seem to affect the time it took to go from one side to the other and back. The other thing he noticed was that the longer the rod, the slower the bob would swing back and forth.

We introduce some mathematical language to tighten this idea up. Suppose we held the bob out to the right from its central position (marked as a vertical red line in the diagram) about 10 to 15 degrees and then let it go. The bob would swing down through an arc of a circle, passing the centre and out to the other side where it would momentarily stop and then travel back through the centre again and up to where it began its motion (we'll ignore friction). That whole journey is called an *oscillation* and the time taken to complete one oscillation is called a *period.*

With a bit of experimentation, Galileo was able to show that the period could be determined from the length of the rod.

More precisely, the period was directly proportional to the *square root* of the length of the rod. Today we know that a rod length of almost one metre (0.9936 m) will swing with a period of 2 seconds. This is the length of pendulums in Grandfather pendulum clocks, so that each sweep from one side to the other is exactly a second.

Much later than Galileo, mathematical physicists derived the scientific law governing the period of the pendulum. The formula for the period P involves the length of the rod l, the gravitational acceleration constant $g = 9.8$ and the mathematical constant π.

$$P = 2\pi\sqrt{\frac{l}{g}}$$

(In fact g is not quite a constant. It varies slightly according to one's location on the earth, and careful measurements of the period of a pendulum are used to determine the value of g at particular places.)

Let's test the formula with a rod of length 0.9936 metres. Using a scientific calculator, we see that

$$P = 2\pi\sqrt{\frac{0.9936}{9.8}} \cong 2$$ seconds.

Pi day is on its way again, but this year it's on a Sunday. The date is the 14 March simply because 3.14 is a good approximation of the constant. Here's an idea! What if your class were to build a Pi-Pendulum, a pendulum with a period of π seconds! All we have to do is solve the equation

$$\pi = 2\pi\sqrt{\frac{l}{9.8}}$$

Square both sides so that

$$\pi^2 = 4\pi^2\left(\frac{l}{9.8}\right)$$

and then multiply both sides by 9.8 and divide both sides by $4\pi^2$ to reveal $l = 2.45$ metres. (Get your teacher to help you if you found this difficult.)

A taught string could be used for the rod and a heavy bolt or nut could be used for the bob. All we need is a suspension point higher than 2.45 metres. A normal house ceiling height of 2.4 metres doesn't quite cut the mustard, but most school building heights are higher than that. Dress it up as you like in your display, and make sure you test it – remember not to swing it too wildly. It should take just over 30 seconds to swing ten times.

Teacher notes

Exercises around this theme are suited to a range of student years from primary through to secondary. For primary students strings and bolts can be used, and periods can be determined quite accurately corresponding to strings of various lengths and the results can be graphed. To minimise timing errors, stop watches can be used to record times for, say, 10 oscillations and those times can be divided back to get each period. Combining and plotting results from students would be insightful. For lower secondary students, manipulating the key formula for rod length and period and using graphing software would be conceptually powerful as well. Upper secondary students meet the mathematics of the pendulum in more detail in course material but would also benefit from doing the heuristic investigations suggested.

POPULATION MODELLING part 2

By Peter McIntyre

(This is the remaining part of a piece addressed to students. See the previous edition for part 1.)

2 Population Problems

Mathematically, the problems here are about *iteration* and about *exponential processes*. Iteration is the process of carrying out the same operation over and over again. Let's take a simple example, that of multiplying by 2. Start with the number 1. Multiply it by 2 to give 2. Multiply the answer 2 by 2 again to give 4. Multiply 4 by 2 to give 8, and so on.

If you have a standard calculator, you may be able to do many of the calculations in the problems here just by pressing the = or × key. To do the calculation here, try this:

Press 1 × 2 = , then just press = or × (depending on your calculator) to multiply by 2 each time. You'll have to keep count of how many times you have multiplied by 2. If this quick method doesn't work on your calculator, experiment to see what does.

NEWSLETTER OF THE CANBERRA MATHEMATICAL ASSOCIATION INC

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We're on the Web!
http://www.canberramaths.org.au/

ABOUT THE CMA

The Canberra Mathematical Association (Inc.) is the representative body of professional educators of mathematics in Canberra, Australia.
It was established by, among others, the late Professor Bernhard Neumann in 1963. It continues to run - as it began - purely on a volunteer basis.

Its aims include

* the promotion of mathematical education to government through lobbying,
* the development, application and dissemination of mathematical knowledge within Canberra through in-service opportunities, and
* facilitating effective cooperation and collaboration between mathematics teachers and their colleagues in Canberra.

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Short Circuit is edited by Paul Turner. **ISSN 2207-5755**

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http://www.facebook.com/pages/Canberra-Mathematical-Association/110629419011275

POPULATION MODELLING part 2

continued from page 3

(On a TI-84/CE: press [1] [ENTER] × [2] [ENTER] [ENTER] ... On a Casio 9860, use [EXE] instead of [ENTER].)

Iterating by multiplying by a constant (2 here) is an example of an *exponential process*. You may have heard the term exponential growth, which many people interpret to mean 'grow quickly'. But exponential growth has a precise mathematical meaning, and some interesting properties which we shall explore shortly.

Exponential iteration models a number of processes such as radioactive decay, population growth and absorption of light. If the constant we multiply by is larger than 1, we get exponential growth; if it is less than 1 (but greater than 0), we get exponential decay.

The use of scientific notation makes writing down our calculations much easier. For example, if we start with 5 and multiply it by 2 three times, we get 5×2×2×2, written as $5\times2^3=40$. 2^3 means three 2s multiplied together. If we multiply 5 by 2 ten times, we have $5\times2^{10}=5120$. 2^{10} means ten 2s multiplied together. Some calculators have an exponentiation key, usually y^x or ^ so, to calculate 2^3, we would press [2] [y^x] [3] [=] or [2] [^] [3] [ENTER] .

2.1 Exponential Iteration

Write down the results of the first 10 iterations of multiplying by 2, starting with 1.

2.2 Lots and Lots of Bacteria

Bacteria multiply (increase in number) by dividing — into two. One type of bacterium, *Streptococcus exponentiae*, divides every minute. If we start with 1 bacterium, it divides into 2 bacteria after 1 minute. Each of these 2 bacteria divides after 1 more minute, and so on. *The number of bacteria grows exponentially.*

Make up a table with time in the first column and the number of bacteria in the second column. *How many bacteria are there after 10 minutes? after 20 minutes? after 1 hour? after* n *minutes? Why isn't the Earth covered metres deep in these bacteria?*

2.3 Malthus and Exponential Growth

Thomas Robert Malthus (1766–1834) made some worrying predictions for the world population, and his name is often associated with the idea of exponentially growing populations. Look up Malthus to find out the details of his ideas. Why was he worried about the world's population?

Malthus looked at the United States population to try to verify his ideas. He concluded the growth was exponential. *From the numbers in the table below, can you tell if he was correct for the years until he died?*

Hint: If the growth is exponential, each population should be a constant multiple of the previous value. Try a multiplier of 1.35, meaning the population increased by 35% every 10 years. The numbers you obtain only need to be close to the actual numbers, not exactly the same.

What about the population growth after about 1860?

Year	Population (millions)
1790	3.90
1800	5.30
1810	7.20
1820	9.60
1830	12.9
1840	17.1
1850	23.2
1860	31.4
1870	38.6
1880	50.2
1890	62.9

Year	Population (millions)
1900	76.0
1910	92.0
1920	106
1930	123
1940	132
1950	151
1960	179
1970	203
1980	227
1990	249

Why might the populations not continue to increase exponentially?

2.4 Cane Toads

The Hawaiian cane toad *Bufo marinus* was introduced into Australia to control sugar-cane beetles. From the original 101 toads released in north Queensland in June 1935, the population grew rapidly and spread across the countryside. The table below shows the total land area of Australia colonised by cane toads for the years 1939 to 1974.

Year	Area ($1000 km^2$)
1939	33.8
1944	55.8
1949	73.6
1954	138

Year	Area ($1000 km^2$)
1959	202
1964	257
1969	301
1974	584

Is exponential growth a good model here? You can get a

(POPULATION MODELLING)

rough idea by the process we used for the Malthus data—finding ratios of successive values—but a plot of the data together with an exponential fit (graphics calculator required) will provide a better answer. *What is the exponential equation of best fit?*

Given that the area of Queensland is 1728 thousand km² and the area of Australia is 7619 thousand km² when, according to the exponential model, will (did) the cane toads colonise all of Queensland? all of Australia?

The cane growers were warned by Walter Froggart, President of the New South Wales Naturalist Society, that the introduction of cane toads was not a good idea and that the toads would eat the native ground fauna. He was immediately denounced as an ignorant meddlesome crank. He was also dead right.

3 Other Exponential Problems

3.1 Piles of Paper

A ream of paper (500 sheets) is about 50mm thick, so that one sheet is about 0.1mm thick. Take a sheet of paper, cut it in half and put the two halves one on top of the other. Cut this pile of 2 pieces in half and make a pile of 4 pieces. Keep cutting the pile in half and stacking the pieces up.

Now suppose you could make 42 cuts altogether (you'd need big scissors!). *How high would your final pile be?* Try making up a table like the one below to keep track of your pile. *Where would your pile reach to?* Kilometres might be a good unit to use eventually. *Write down a formula that tells you the height after n cuts. What units will you use?* Be careful!

Cut number	Height of pile in sheets	in mm
1	2	0.2
2	4	0.4
3	8	0.8
4	16	1.6

This is an example of where maths lets you find an answer to something you can't actually do in real life.

3.2 Shoeing a Horse

A rich man sends his horse to the blacksmith to have 4 new horseshoes put on. Each shoe needs 5 nails. The blacksmith offers to charge either $100 per nail (they're gold!), or 1c for the first nail, 2c for the second nail, 4c for the third nail, and so on, the cost doubling each nail. *Which offer should the rich man take?*

Think first which offer *you* would take. Then do some calculations. Don't forget to add up the total cost at the end. A calculator might be useful. *Did you pick the better offer? Was there much difference?*

Perhaps you might like to write down a function for the cost of the second offer after n nails and graph it. Write down a new version of the problem if each shoe needed 6 nails. What could the first offer be in this case?

3.3 Interest Rates

Once you have some money in the bank, you start to think about interest, and you might want to answer a question like the one below to work out how much money you will have some time in the future.

If the annual interest rate on a bank account is 12% compounded monthly and you deposit $10, how much money will you have after 1 year? after 5 years? after 10 years?

What does this mean? In simpler terms, it means that every month the bank will pay you an amount of interest equal to 1% (an annual interest rate of 12% means a monthly interest rate of 12%/12=1%) of the amount you have in the account at the end of the month. So, after the first month, the bank will pay into your account 1% of $10 or 0.01×$10=$0.10=10c in interest, and you will then have

$$\$10 + \$0.10 = 1.01 \times \$10 = \$10.10$$

in your account.

After the second month, the interest will be 1% of $10.10 or 0.01×$10:10=$0.101=10.1c, and you will then have

$$\$10.10 + \$0.101 = 1{:}01 \times \$10.10 = 1.01 \times (1{:}01 \times \$10) = \$10.201$$

in your account. Although the bank won't pay you the 0.1c, they leave it in for future calculations.

Can you see a pattern? At the start of each month, the new amount in your account will be the amount you had last month times 1.01.

Now can you answer the question above? Can you write down a formula using exponential notation for the amount in your account after n months? How long before you have $15? $30?

The full text of this article includes further sections giving answers to the questions and information about calculator technology and other software that is useful in population modelling. This and other articles by Peter McIntyre are accessible as resources on the CMA website.

TEACHER MAGAZINE

The latest Teacher Magazine from ACER includes, among other things, an article by Geoff Masters titled The Equity Myth.

As a taster, we quote the final paragraph.

'Given community faith in the fairness of current curriculum and assessment processes, the common belief that educational outcomes are meritocratic and deserved, the far-reaching life consequences of those outcomes, and the fact that existing inequities are most likely to disadvantage those least likely to question or object, we have a responsibility as educators to redouble our efforts to make equity more than a myth.'

The rest interrogates insightfully this list of factors and is definitely worth reading.

THE MATHS SHOW

By Brad Felstead

The Maths Show is a live performance that aims to entertain and engage students with the wonder and power of mathematics.

The genesis of the show lies in the idea that live performance and storytelling, combined with maths history and a little bit of maths magic would enthral students and create a sense of amazement around maths and numbers.

Here is some magical maths mind reading that we use to start the show. Rumour has it that this little known mathematical gem was first discovered by ancient Babylonian mathematicians nearly 4000 years ago and has been passed down through the millennia.

Think of a single digit number, not zero, (because the Babylonians never discovered zero). Now type that number three times into your calculator or tablet. (The Babylonians invented the first mathematical tablet device, although it was, of course, a clay tablet.) For example, if you thought of the number seven, type 777 into your calculator. Add the three digits together in your head, then divide the number on your screen by the number in your head, and hit enter.

Imagine 100 students in the auditorium doing this at the same time–each beginning with whatever random single digit number they chose. Imagine them simultaneously calling out the number that they see on their screens. They, like you, would all be yelling out '37' - and feeling a mathematical *aha* moment. That is a taste of *The Maths Show.*

The show was first performed in 2017 and has since been seen in hundreds of schools with performers based in New South Wales, Victoria, Queensland and Western Australia.

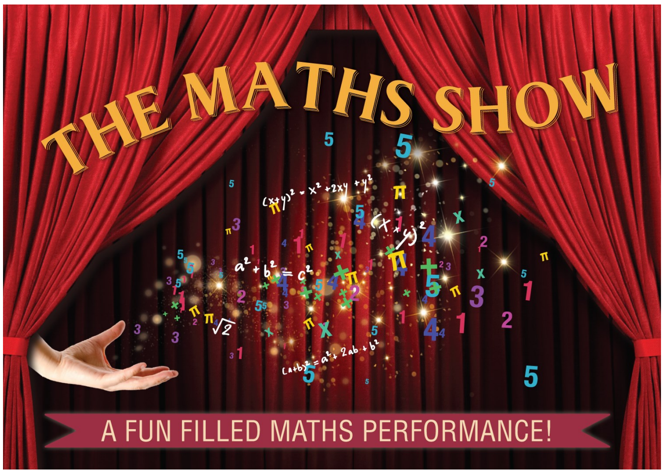
One of the most delightful things about the show is the sense of wonder that it creates in students.

When one sees the look of amazement on a student's face when their mind has been read using maths, or when they have been wowed by a card trick that is based on mathematical principles, or when they have been intrigued by a visual maths puzzle that they so badly want to solve but just can't quite–so that they stay back at lunchtime and beg the performer for just a few extra clues, one has stepped into the joy and charm of The Maths Show.

The Maths Show is designed for students in Years K to 8 with tailored versions for different age levels. The show will be touring the ACT in term two, 2021. For further details:

Visit: www.felstead.com.au.
Email: info@felstead.com.au.
Phone 0402 223406.

And, by the way, if you're still wondering about the first magic trick–it's just a little bit of algebra: $111x$ divided by $3x$ will always be 37.



PUZZLE SOLUTIONS

Solutions to puzzles from the February edition.

1 A coin game for two

Players take turns to choose a coin from either end of a row of 50, attempting to maximise their sum.

Player one computes and compares the sums of the odd and even positioned coins. If the odd sum is the greater, player one chooses the leftmost coin which means player two only has access to even numbered coins. Otherwise, player one chooses the rightmost coin since it has even parity. In either case, player one continues to pick coins consistently with the same parity and player two has no choice but to take coins with the opposite parity.

This strategy guarantees that player one does not lose, but it may not be optimal. It is possible that player one can do better. See You Tube channel stable sort, for example.

2 Eyes: blue, brown and green

Suppose you are one of the 200. On the first day you scan the assembled people and see no one with blue eyes, yet the seer says 'I see someone with blue eyes'. You conclude that yours must be the only blue eyes. Accordingly, you leave the island on the first evening.

It may happen that on the first and second days when you scan the assembly, you see one person with blue eyes. If on the first day that person had seen no one with blue eyes they would have concluded that they were the only one and they would have left on the first evening. Since they are still there on the second day, that person must have seen one other person with blue eyes, which can only have been you. You each conclude that you are the only two blue-eyed people and you both leave on the second evening.

Continuing in this way, it follows that if on day 100 the seer sees a blue-eyed person (and you can see 99 others with blue eyes who would have left the previous evening had they been the only ones), 100 will leave on the 100th evening. The remaining people have brown eyes because the seer can no longer see a blue-eyed person. They will leave on the next evening.

3 Amazing hinged triangles

Observe that triangles ABE and CBD are congruent and deduce that the smaller angle at the intersection of CD and AE is always $\pi/3$.

PUZZLE SOLUTIONS continued

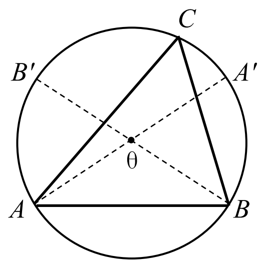
4 Missing middle

Call the bases of the left and right rectangles a and b respectively, and the height of the rectangles h. Then, $ah = 38$ and $bh = 34$. Thus, $(a + b)h = 72$. But, $a + b = 16 - 7 = 9$. So, $h = 8$ and the coloured region has area 56 cm^2.

5 Probability but no counting

Three randomly selected points on a circle make a triangle that contains its circumcentre with probability 1/4. The following explanation is essentially one from Ross Pure, with some details expanded.

A triangle contains its circumcentre when none of its angles is greater than $\pi/2$. Suppose two points A and B have already been selected. We may consider, without loss, just the subset of triangles that have a side parallel to AB. Side AB subtends an angle θ at the centre and we take this angle as the important random variable.



Define point A' diametrically opposite point A, and B' similarly. The triangle ABC contains the centre when the third point C is on the arc $A'B'$ and this occurs with probability $\theta/2\pi$.

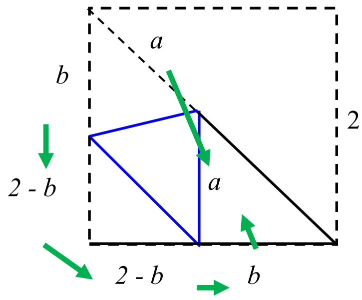
As AB moves vertically, the angle θ varies from 0 to 2π. However, by symmetry we can consider just the range 0 to π and double the result.

Since θ is uniformly distributed on $[0, 2\pi]$, the probability that θ is in an interval of length $\delta\theta$ is $\delta\theta/2\pi$, so that the probability that a triangle contains the centre *and* θ is in an interval $\delta\theta$ is given by $\theta/2\pi \times \delta\theta/2\pi$.

Putting all of this together yields the result

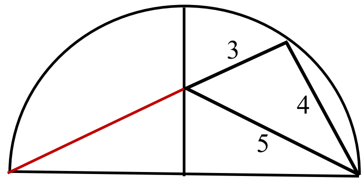
$$2\int_0^\pi \frac{\theta}{2\pi}\frac{1}{2\pi}d\theta = \frac{1}{4}$$

6 Paper fold



Follow the green arrows to see that $b = a$. Then, considering the main diagonal, $\sqrt{8} = a + a\sqrt{2}$, so that $a = 4 - 2\sqrt{2}$. The largest right triangle has area 2 while the two smaller ones have areas respectively $4(3 - 2\sqrt{2})$ and $2(3 - 2\sqrt{2})$. Thus, the parallelogram has area $2 - 6(3 - 2\sqrt{2})$ and the blue triangle is half this: $6\sqrt{2} - 8$.

7 Half the picture is never enough



Only the right-hand half of the diagram was given. The black triangle has a right angle, so we may add the red line and make a semicircle to complete the picture. The red segment has length 5. Then the diameter is $\sqrt{80}$ and the quadrant has area 5π.

8 Convenient shopping

The number 711 has prime factors $3^2 \times 79$. One of the items must have cost p times \$0.79 and the sum of the others must be $q \times$ \$0.79 such that $p + q = 9$. These are clues, but experiment is needed get the result: \$3.16, \$1.50, \$1.25, \$1.20.

9 Real sums

In coordinate geometry the sum of the three squares represents a sphere, and the sum of the three linear terms a plane. The surfaces intersect in a circle. Points like (1,2,2) are on the circle, as are uncountably many others. For example, (1.1, 1.646, 2.254).