# SHORT CIRCUIT

Newsletter of the Canberra Mathematical Association INC

VOLUME 13 NUMBER 8 AUGUST 2022

## NEWS AND COMMENT

The CMA 2022 conference is on 13th August. Registration is through the [CMA website](). We hope to see you there.

The **medals** that CMA provides for colleges will be available for collection at the conference. It will be a help if teachers would collect these on the day.

**Inside:**



**Coming Events:**

2022 CMA conference ADFA August 13.

**Wednesday Workshops:**

Check for notices sent separately.

## MEMBERSHIP

Memberships run from 1 Jan to 31 Dec. each year. Membership forms can be accessed from the CMA website: http://www.canberramaths.org.au

Membership of CMA includes affiliation with the Australian Association of Mathematics Teachers and a subscription to one of two AAMT journals.

As a member, you are entitled to attractive rates for the CMA annual conference and CMA professional development events.

CMA members may attend conferences of the AAMT affiliates in other states, MAV, MANSW, etc. at member rates.

## PUZZLES

**1. Not the Monty Hall problem**

There are two envelopes. One has X dollars inside and the other has 2X. You open one of the envelopes and you are then offered the chance to switch. Is it better to switch or not switch, or does it not matter what you do?

**2. A potato rings a bell**

Ninety-nine percent of a potato's mass is water. If the potato is dried up so that it consists of 98% water, what is the new mass of the potato as a percentage of its original mass?

## AMT—PROBLEMO

*Problemo*, from The Australian Maths Trust, is the go-to maths problem-solving resource for teachers. Designed to make lesson planning easier, it holds an extensive library of high-quality maths problems, all aligned to the Australian Curriculum.

* Create online quizzes and track student performance over time.
* Curriculum aligned across mathematics and computation and algorithms
* Scaffolded support with enabling, extending prompts and more than 60 lesson cards.
* Over 800 high-quality searchable problems.

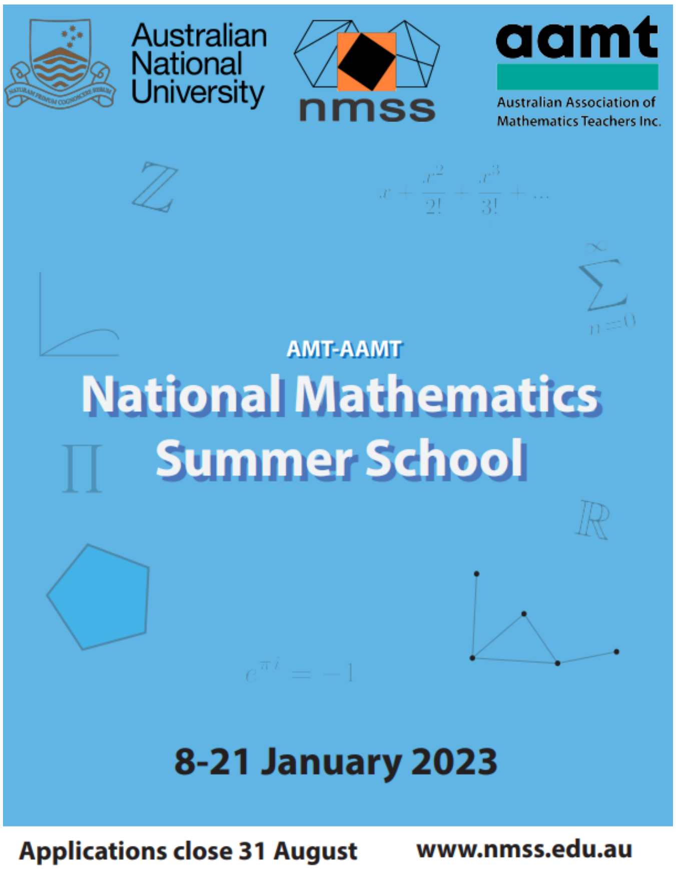
Find out more at [problemo.edu.au](problemo.edu.au).

## CMTQ

The Canberra Mathematics Talent Quest is the entrance point in the ACT for the National Mathematics Talent Quest. See the CMA web page for details.

For those teachers and schools who are able to take it on in these difficult times, the talent quest is expected to be of significant benefit.

## NMSS




The ANU-AAMT National Mathematics Summer School (NMSS) is a two-week residential mathematics school that takes place every year in January at the Australian National University. Since its foundation by Professor Larry Blakers AM in 1969, NMSS has enriched and engaged thousands of young mathematicians who are passionate and capable in mathematics. Academically sponsored by the Australian National University and the Australian Association of Mathematics Teachers, the National Mathematics Summer School is the oldest and most prestigious mathematics summer school in Australia.

The School is for students who are about to enter Year 12.

Further details and application forms are on the CMA website and on the NMSS website.

# CONFERENCE

## Change – one of life's constants?

### Canberra Mathematical Association
### Annual Conference 2022

## Australian Defence Force Academy

### Saturday 13th August 2022 9am-5pm

***Six sessions of talks/workshops for all levels Early Childhood - Yr 12***

***All food + President's drinks***

***Great prizes Trade stalls***

***Registration*: $100 member**

**$50 concession $150 non-member**

**Keynote speakers:**

**Prof Emma Sparks (Rector at UNSW Canberra)**

**Prof Jason Sharples (Bushfire Group UNSW Canberra)**

**(Neville de Mestre Memorial Lecture)**

**Details and registration at:**

***canberramaths.org.au***

**Contact: *p.mcintyre@adfa.edu.au***

**NEWSLETTER OF THE CANBERRA MATHEMATICAL ASSOCIATION INC**

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We're on the Web!
http://www.canberramaths.org.au/

# ABOUT THE CMA

The Canberra Mathematical Association (Inc.) is the representative body of professional educators of mathematics in Canberra, Australia.

It was established by, among others, the late Professor Bernhard Neumann in 1963. It continues to run - as it began - purely on a volunteer basis.

Its aims include

* the promotion of mathematical education to government through lobbying,
* the development, application and dissemination of mathematical knowledge within Canberra through in-service opportunities, and
* facilitating effective cooperation and collaboration between mathematics teachers and their colleagues in Canberra.

# THE 2022 CMA COMMITTEE

| | | |
|---|---|---|
| President | Aruna Williams | Erindale College |
| Vice Presidents | Bruce Ferrington | Radford College |
| | Paul Kruger | Marist College |
| Secretary | Valerie Barker | |
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| | Heather Wardrop | |
| | Andrew Wardrop | |
| | Sue Wilson | |
| | Yuka Saponaro | Amaroo School |
| | Jo McKenzie | ACT Education Directorate |
| | Joe Williams | |

Theresa Shellshear is CMA's COACTEA representative.

Sue Wilson is CMA's AAMT representative.

Joe Wilson is the website manager.

Short Circuit is edited by Paul Turner. **ISSN 2207-5755**

**Find us on Facebook**

http://www.facebook.com/pages/Canberra-Mathematical-Association/110629419011275

## CAREERS AND MATHEMATICS

From Frances Moore.

If you are investigating an aspect of mathematics or would like information about a person in that job, please contact me Frances Moore – I would be happy to hear from you.

Frances.Moore@onthejob.education
Mob 0410 540 608

We will explore a job and the mathematical activities involving this job from the website "On the Job".

Let's have a look at the Zookeeper.

**Context and relevance**: People are concerned with the reduction of biodiversity across the planet due to changes in habitats and climate change. Can zoos help? Zoos have changed their culture and focus to animal conservation. What is the mathematics behind keeping a range of animals in a zoo?

**Activities for the Classroom:**

**Activity 1: Design a suitable enclosure**

Primary Middle

[See the web page.] Students are to use the design of an enclosure from the Chester Zoo and their Painted Dog Enclosure Design to help them design an enclosure for the animal of their choice.

As an optional extra, students can undertake the Taronga Zoo – Design an Exhibit – activity.

**Activity 2: Zoo Mathematics** (Created by Taronga Zoo & NSW Education)

Primary Middle

Students have 8 different mathematical activities to undertake including the graphing of weights, looking at live TV and spotting 3D objects, animal patterns, animal diets, compare and contrast cubs, the 24 hour clock.

Additional work from Whipsnade Zoo includes calculations for vaccinations and weight, ratio and mean age.

**Activity 3**:

This is a range of educational websites, games and apps including another Taronga Zoo Learning Resource for students in Years 11 – 12 Business Studies – "It's Zoo Business".

**Careers & Mathematics can be found at**

https://onthejob.education/teachers_parents/Mathematics_Teachers/Careers_Mathematics_Index.htm

## TAX, SUPER + YOU COMPETITION

The ATO's 2022 Tax, Super + You competition is now open. The competition is a fun and engaging way for Australian high school students to learn about the value of tax and super. Not only does it help build financial literacy in students, but they also have a chance to win a share of the $7,600 prize pool for themselves and their school.

The competition invites high school students in years 7–12 to make a creative project about tax and super, with the topics based on the year the student is in at school.

You can watch one of the competition judges, ATO Deputy Commissioner Hoa Wood, talk about the competition and how students can get involved.

Entries will close at 5pm AEST on Friday 19 August 2022, and the winners will be announced in October. For more information about the competition, including how you can incorporate the Tax, Super + You teaching resource into your lesson plans, visit the Competition teacher's kit.

# SURROUNDED—THE GAME ANALYSED

The game *surrounded* was described in an article in last month's edition of Short Circuit. Its inventor Ed Staples explains why the game, despite appearances, is not fair.

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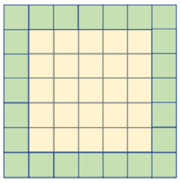
The electronic die in the game is loaded in such a way that the probability of any number $n$ being rolled is $k/n$ for some constant $k$. Since the sum of the probabilities is 1, we have $k(1 + 1/2 + 1/3 + 1/4 + 1/5 + 1/6) = 1$, and therefore $k = 20/49$.



If a 1 or a 5 is rolled, a green perimeter square lights up. To win the game, the player needs to light up all 24 of those cells.

The chance of rolling either a 1 or a 5 is given by $20/49 + 4/49 = 24/49$, and this exactly matches the ratio of the perimeter cells to the total cells in the square array of the game board. This means that there is a slightly less than even chance of rolling a number that will light up a perimeter cell (about 2% less chance than for a central cell).

From the Bank's perspective though, there is about a 2% higher chance that a central cell will light up on each roll. However, there are 25 cells to light up. In that sense the game seems balanced – either more cells to light with a slight edge in probability of lighting any one of them or else fewer cells to light with less chance of lighting any one of them. It might then seem that the game is fair.

Note also that at least 24 rolls are required to obtain a result, and at most 48. The largest stake that can accrue prior to a result is $47, occurring when 23 perimeter cells and 24 central cells are lit up, and then a win or loss of $48 by the player will occur on the 48th roll. The $50 minimum balance therefore ensures that the player will never run short of money before a result is known.

**Looks can deceive**

On any roll, the player has a probability of 24/49 of lighting a perimeter cell. We might as well think about the game as something akin to flipping a biased penny that might exactly model this dice game, with say $\Pr(H) = 24/49$ and $\Pr(T) = 25/49$. On each flip the Bank seeks Tails, and the player seeks Heads and whoever gets their quota first (25 for the bank or 24 for the player) wins all the accumulated staked money, however that much may be.

To understand why the game is biased against the player, let's imagine a simpler version of the game where the biased coin is flipped until either the player flips two heads or else the bank flips three tails, with $\Pr(H) = 2/5$ and $\Pr(T) = 3/5$. It seems fair – the bank has the bigger task but has the better chance of the coin landing tails on each flip.

Here are the possible outcomes, along with their probabilities and the stake won or lost shown in brackets. (The common denominator in the probabilities corresponds to imagined runs of 625 games.)

**Scenarios**

Player wins

| | | |
|---|---|---|
| HH | 100/625 | (+2) |
| HTH | 60/625 | (+3) |
| THH | 60/625 | (+3) |
| TTHH | 36/625 | (+4) |
| THTH | 36/625 | (+4) |
| HTTH | 36/625 | (+4) |

The sum of the winning probabilities for the player is 328/625.

Bank wins

| | | |
|---|---|---|
| TTT | 135/625 | (-3) |
| TTHT | 54/625 | (-4) |
| THTT | 54/625 | (-4) |
| HTTT | 54/625 | (-4) |

The sum of the winning probabilities for the bank is 297/625.

It looks at first glance that the player has the advantage, with the expectation of winning 328 of

625 games played. But how much will the player win given the payouts listed? For example, in 100 of those wins the player wins 2 dollars, and in 60 of them the player wins 3 dollars, etc. However, in 135 losses, the player loses 3 dollars, etc.

It is easy to work out the expected result over the 625 games played. The average payout $P$ is given by $P = \{100(2) + 60(3)(2) + 36(4)(3)\} - \{135(3) + 54(4)(3)\} = -61$. In other words, the player is expecting a payout of \$ $-61/625$ per game, or a loss of 9.76 cents per game.

This bias was first notified by Paul Abbott on a *LinkedIn* post, and subsequently followed up by a complete analysis of the game by Denis Entemeyer, 1st Assistant Professor, Université de Lorraine in France, via the same platform. The mathematics is made more difficult because there is a range of possible stakes involved each time the game is played. However, it is possible to determine that, in the game of *Surrounded*, the probabilities of the player and bank are approximately 50.077% and 49.922% and the expected loss for the player is about 3.7 cents per game.

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If you wish to receive a copy of Denis Entemeyer's analysis, please contact either the editor or myself on edsownmail@gmail.com.

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This might be a great game to model on a spreadsheet or similar. Estimates could be made on the average stake that a player is likely to need in playing a game, and an estimate of the probability of a player winning, based on relative frequencies over many trials, could also be developed. Other measures could be established as well, including measures around tracking 'lead sizes' as a game is played – how far ahead or behind a player might be at any intermediate stage of a game.

In typical gambling parlours, the bank almost always develops a way to create a profit margin on any game. Thought could be given to a way a commercial entity could vary it to build in a small bank margin.



## PUZZLE SOLUTIONS from Vol 13 No 7

### 1. Strange but true

Someone found these equivalences:
$\sqrt{(2\ 2/3)} = 2\sqrt{(2/3)}$, $\sqrt{(3\ 3/8)} = 3\sqrt{(3/8)}$ etc.

If the term on the left is $\sqrt{(a\ a/b)}$, it can be written $\sqrt{(a(b+1)/b)}$. We can set $b+1$ a square. Its square root comes outside the radical sign and we can put $a = b+1$. Thus, for example,
$\sqrt{(10\ 10/99)} = 10\sqrt{(10/99)}$. The pattern continues indefinitely.

### 2. Devilish exponentials

(a) $(8^x - 2^x)/(6^x - 3^x) = 2$.

By inspection, $x = 1$ is a solution. Plotting the function $(8^x - 2^x)/(6^x - 3^x) - 2$ on a graphing calculator gives the impression that $x = 0$ is another, but the function is undefined at that point.

(b) $(4 + \sqrt{15})^x + (4 - \sqrt{15})^x = 62$

Again, by inspection one might spot the solution $x = 2$. However, after dividing both sides of the equation by both the terms on the left, we have

$$(4 + \sqrt{15})^{-x} + (4 - \sqrt{15})^{-x} = 62$$

and so, $x = -2$ must also be a solution.

(c) $3^x - 54x + 135 = 0$

If this is rewritten in the form $3^{x-3} - 2x + 5 = 0$, it is relatively easy to see that both $x = 3$ and $x = 4$ are solutions. The expression on the left is positive for $x < 3$ and for $x > 4$. By differentiating the equivalent function $e^{(x-3)\ln 3} - 2x + 5$ it can be shown that there is a single minimum value between the zeros and so, these are the only solutions.

### 3. Target practice

Players $A$ and $B$ take turns shooting at a bullseye. Player $A$ goes first. The first player to hit the bullseye wins. Player $A$ hits the bullseye with probability $a$, and $B$ with probability $b$. What is $P(A)$, the probability that $A$ hits the bullseye first?

Player $A$ wins on the first shot with probability $a$, on the next try with probability $(1-a)(1-b)a$, and on the $n$th try with probability $(1-a)^n(1-b)^n a$. Adding infinitely many of these, gives the total probability that $A$ hits the bullseye first, $a/(a + b - ab)$.