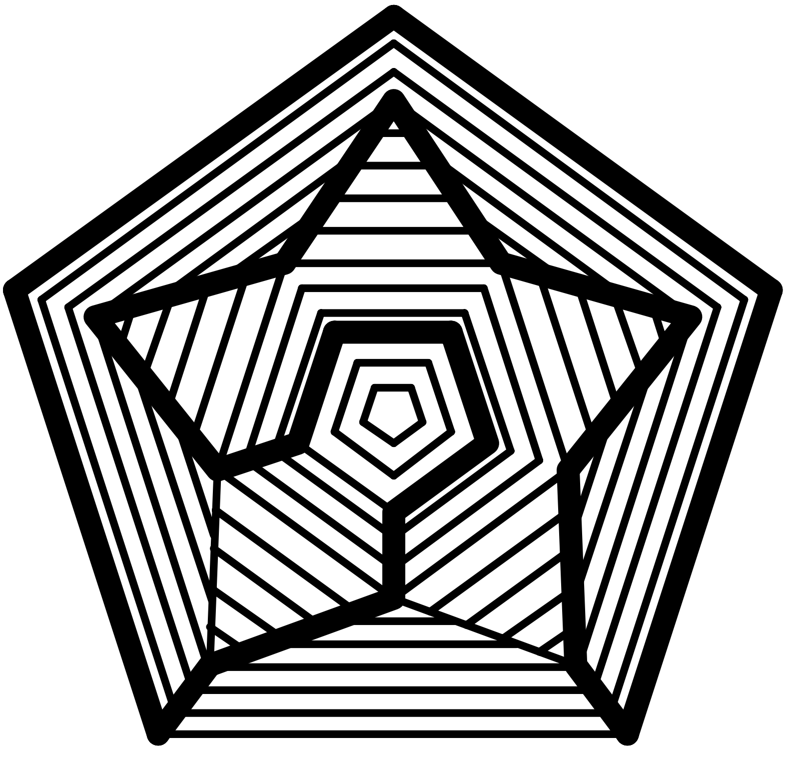
CIRCUIT

Published by the
Canberra Mathematical
Association

June 1998



CANBERRA MATHEMATICAL ASSOCIATION

OFFICE BEARERS 1998

EDITORIAL AND PUBLISHING TEAM FOR *CIRCUIT* 1998

Peter Enge
Canberra Institute of Technology
Phone 6207 4882
e-mail peter.enge@cit.act.edu.au

Margaret Rowlands
Lake Tuggeranong College
Phone 6205 6222
e-mail margr@oaf.opana.org.au

Kevin Taylor
Canberra Grammar School
Phone 6295 5759

CMA INTERNET HOME PAGE

http://education.canberra.au/projects/cma/home/html

CONTENTS

Page

The objects of the Canberra Mathematical Association are to promote interest in mathematics, to encourage improvements in the teaching of mathematics and its applications, to provide means of communication among teachers and students and to advance the views of the Association on any question affecting the study or teaching of mathematics and its applications.

The Canberra Mathematical Association Logo depicts a Hamiltonian Circuit on a dodecahedron.

CONTRIBUTIONS

The Circuit publishing team is always keen to receive articles, notes, problems, letters and information of interest to members of the CMA. Please contact a member of the publishing team if you wish to contribute to Circuit.

FROM THE PRESIDENT

Hello again.

First, a word about the AAMT National Mathematics Conference which we will host in Canberra in January 2001. The CMA executive has already started the planning. We have booked facilities at ANU for the conference and have developed a very broad outline of the program. We are planning sessions to appeal to teachers of mathematics at all levels preschool to tertiary. The title and theme for the conference is "Mathematics: Shaping Australia". We will be looking to build on the past and future of mathematics teaching and learning to plan for the future. We will be considering three major groups: learners, teachers and the community.

The conference theme will underpin all sessions. We will provide for full conference registration together with day registrations so that people may choose to come for one day and so that a school may choose to send a different teacher each day if it wishes. To ensure each day's program is cohesive, there will be four major sub themes underpinning the conference. In addition, one sub theme will be the focus for one day of the conference. The four sub themes are:

- The classroom
- Mathematics
- Technology
- Numeracy

We seek to make the conference a practical PD opportunity for all teachers of mathematics, especially teachers from Canberra and the surrounding regions.

Small planning teams are preparing for January 2001, but we are seeking teachers and CMA members who would like to play a role, however small or short term, in the planning. In particular, if you would be interested in contributing to a couple of brain storming sessions to flesh out aspects of the program, please let us know by calling Steve Thornton on 201 2017 (W) or Beth Lee on 282 5962 (BH). We would appreciate your ideas so that the ACT can host a conference as challenging and interesting, and friendly as we did in 1980.

Numeracy: forgotten and ignored yet again

Our primary and high school members may be encouraged by the news of a new public school literacy strategy. It appears that the money will be centralised with little flexibility for a school to address its special student population. The strategy is a short term 'solution'. The Reading Recovery and Early Literacy in Childhood (ELIC) programs supported all teachers in long term, practical ways. Why were these discontinued or cut right back? How will a central 'literacy team' support teachers at the school base? How will primary and high schools be supported to select and sustain their 'literacy coordinators'? What is the Department's long term strategic plan? How will this be funded long term?

These questions are all relevant to a consideration of numeracy education from preschool to Year 12? The key issue for CMA and its members is that once again, numeracy has been ignored! Below is a copy of a letter related to this issues sent to Kate Carnell by your CMA Executive on behalf of its members.

Beth Lee, 23 June 1998

23 June 1998

Chief Minister
ACT Legislative Assembly
Civic Square
London Circuit
Canberra ACT 2601

Dear Ms Carnell

On behalf of the Canberra Mathematical Association (CMA) and its members, I am writing to express our deep concern about what appears to be the ACT Department of Education's lack of a strategic plan and ongoing funding for the development of numeracy in ACT schools. Our primary and high school members may be encouraged by the news of a new public school literacy strategy. However, the $400 000 allocated to serve all primary and high school students and teachers is insulting in comparison with the million dollar allocations to sporting bodies in the ACT. Yet again numeracy is pushed to one side.

The CMA is concerned that many key questions do not appear to be being addressed or even considered by the ACT Department of Education, Training and Youth Affairs. When will students for whom mathematics is a disappointing and frightening learning experience get their chance to develop numeracy skills that will serve them lifelong? When will teachers of mathematics get practical ongoing support to upgrade and extend their knowledge and skills to support ALL learners to become numerate? How can it be in the interest of individuals, the ACT and Australia, to fail to empower ALL members of the community to contribute fully to their community?

The Department's lack of concern for numeracy is clearly reflected in the fact that the CMA was not invited to join the Department's numeracy working party, even after this request was made informally. It was left to chance, for individual schools to nominate a teacher with an understanding of mathematics and training in mathematics, to represent the ACT's professional mathematics teachers' association. What outcomes have come from this group? When will their discussion be published? How will these outcomes be applied to future numeracy education planning and funding in the ACT?

Two members of the CMA Executive welcomed the opportunity to contribute to a publication on numeracy for parents. What has become of this publication? When will it be available to parents, teachers and the wider community? The national numeracy conference in Perth in 1997 produced a definition of 'numeracy' and documented key parts of the discussion. When, and how, is this to be built upon in the ACT?

Little has been done to support teachers of mathematics preschool to Year 12 since the successful and comprehensive Mathematics Inservice Network Course (MINC) ended once external funding finished in 1989. The FAMPA program supported teachers and parents for more than ten years. This initiative was run by ACT primary and secondary teachers who gave up their weekends for no remuneration.

On reflection, over the period from 1975 it is difficult to see what the ACT Department of Education has done for mathematics education. Nor is it clear what it intends to do.

In January 2001 the CMA will host the national conference of the Australian Association of Mathematics Teachers. We have already begun planning and seek to make this a major professional development activity for ACT teachers and a highlight of the 2001 celebrations in the ACT.

The CMA seeks assurance that the Department of Education has well established strategic plans for numeracy and mathematics education in the ACT, that there is a published time line for these initiatives, and that there is appropriate and ongoing funding for implementation, teacher training, and public awareness campaigns to parallel Department initiatives.

Yours sincerely,

Beth Lee
President
on behalf of the CMA Executive and CMA members

cc Minister for Education

cc Chief Executive, ACT Dept of Education, Training and Youth Affairs

NOTICE BOARD

Missing Copies of The Australian Mathematics Teacher

A small number of members of CMA apparently had their first copy of The Australian Mathematics Teacher for the year (Volume 54 Number 1, March 1988) go astray. Our Treasurer Jan MacDonald asks members who do not receive copies of AAMT material they have paid subscriptions for to make contact with her so that any problems may be rectified. Her phone number at Stromlo High School is 6205 6166, or she may be contacted by mail through CMA at PO Box 3572, Weston, ACT 2611.

Problem Solving Task Centre Network

For twelve years the P*roblem Solving Task Centre Network* has been supporting primary, secondary and tertiary maths teachers in the establishment of Maths Task Centres.

Hundreds of teachers and school communities have come to see the value of hands-on mathematical problem solving with an emphasis on cooperation, teamwork and the use of concrete materials. Mathematics teachers are able to establish a Task Centre in their school for comparatively little cost. Further information regarding the Network can be found on the web at

http://www.srl.rmit.edu.au/mav/PSTC/index.html

For support in the establishment of a Maths Task Centre at your school, contact:

PROBLEM SOLVING TASK CENTRE NETWORK
c/- Michael Richards
Mordialloc-Chelsea SC
1 Station Street, Mordialloc
VICTORIA 3195

Phone: 03 580 1184
Fax: 03 587 5443
email: jami@zx.ent.au

FROM THE PUBLISHING TEAM

Winter is once again upon us and to warm the cockles of your heart this *Circuit* contains two major pieces, one local and the other from Victoria, plus the usual sections. Steve Thornton's observations on and analysis of the current situation in Australian school mathematics raise a host of issues for consideration and discussion. Equally, Gail FitzSimons' overview of vocational mathematics for schools, presented last year at the AMT Conference in Melbourne, outlines some exciting possibilities for mathematics curriculum change but also sounds several warning bells. As always, we would be delighted to print responses from individual *Circuit* readers or groups of teachers to either of these pieces.

Remember *Circuit* is your publication and that your contributions, feedback and suggestions are vital to ensuring that *Circuit* stays in touch with grass roots CMA members. Why not put together a joint contribution with colleagues in your workplace?

Mathematics: Transition, Choice and Learning

Text of an address to parents and teachers at a Canberra Grammar School Education Forum on Tuesday 10 March 1998. The views expressed in this address are those of the author, not necessarily those of either Canberra Grammar School or the Australian Mathematics Trust.

Transitions All the Way

Transition from one level of education to another has been the subject of widespread debate in recent years. Increased emphasis has been given to early childhood education in an effort to ease the transition from home to school, the middle school movement has focused attention on primary to high school transition, and senior school courses have been redeveloped with school to work and school to tertiary education transitions in mind. The discussion below focuses particularly on the transition from primary to high school, but I believe that the principles are similar at all levels.

Transition seems to be a particularly significant issue in mathematics, where students are expected to carry skills and concepts from one year to the next. There has been a strong movement towards more explicit specification of prior knowledge and better reporting of each student's mastery of that knowledge. Most state education systems have adopted, in one form or another, recommendations from *Mathematics: A Curriculum Profile for Australian Schools* as a basis for making judgments about students' development. In theory primary schools should be in a better position to plan a curriculum that enables students to meet the expectations of high schools, while high schools should be better informed about their incoming students' level of achievement and hence better able to match their curriculum to student needs.

Outcome Statements

Yet the reality is very different. Rather than being a vehicle for improved communi-cation and more effective teaching and learning, student outcome statements have become a noose around teachers' necks. One effect of closer specification of required prior knowledge is to place greater pressure on primary school teachers who feel a deep responsibility to help their students reach the level required for high school. Gone is the enjoyment of learning and the lateral enrichment. The other is to heighten high school teachers' awareness of the inadequacies of the incoming students. How often do teachers at all levels complain that the students did not appear to learn anything in the previous year?

From Primary to High School:

Three Critical Factors

So what really matters in moving from primary to high school? First and foremost I believe that there is no substitute for familiarity and confidence with number. Students do need to know basic addition, subtraction, multiplication and division facts; they need to feel comfortable with mental arithmetic calculations and to have devised some strategies to help them; they need to see fractions and decimals as friends rather than foes. This emphasis is not meant to diminish the value and importance of students being familiar with measurement, geometry, or chance and data, but mental computation is the glue that strengthens students' achievement in mathematics. Those who have a fear of numbers are unlikely to feel comfortable about high school mathematics.

The strongest predictor of student success in mathematics is confidence. The recently conducted Third International Mathematics and Science Study (TIMSS) (see Lokan et al 1996) examined a range of factors influencing student achievement in mathematics and science. Student confidence was clearly the variable most strongly correlated with success. Students who responded positively to the statement 'I usually do well in mathematics' did, indeed, achieve much higher results than those who felt less confident. So students entering high school need to have positive attitudes towards mathematics; they need to be willing to take a risk; they need to be unafraid of wrong answers; they need to be prepared to contribute their ideas and value those of others.

The third essential factor in students' readiness for high school is that they possess mathematical common sense. Students need to see mathematics not as symbols written on a page but as something meaningful. They need to recognise when an answer makes sense; they need to be able to identify when mathematics is likely to be useful and to use their knowledge to make sense of situations with which they are confronted.

Meaningful Mathematics

Facility with number, confidence and mathematical common sense are, I believe, far more important than any amount of algorithmic skill or content knowledge. If that is the case, how are these attitudes and dispositions developed? It seems to me that the old adage 'Practice Makes Perfect' is a myth that is responsible for many of the prevailing community attitudes toward mathematics. There is nothing wrong with practice, but too often we judge success by quantity rather than quality. Students who are quick to grasp new ideas and complete the work expected of them very quickly are given more of the same, only with harder numbers. Students who struggle to grasp new ideas are given more of the same as remediation.

It was fashionable a few years ago to talk about the 'spiral curriculum'. The idea was that topics would be revisited from year to year to help students to revise their skills and consolidate their understanding. This model of learning is evident in almost all school mathematics texts. However it usually seems to mean a chapter of revision at the start of the year in which students are asked the same questions as before in the same context. The Year 7 text contains a chapter on fractions because we know that the students will not have learnt them very well in primary school. The Year 8 text commences with a chapter on fractions because we know that they will not have learnt them very well in Year 7, and students cannot succeed in Year 8 algebra without knowing fractions. The Year 9 text commences with a chapter on fractions because we know that they will not have learnt them very well in Year 8, and students cannot do compound interest calculations without them. And so it goes. The net result is that some students are very good at fractions by the end of Year 9. However these are almost always the same students who were very good at fractions in primary school. The only difference is that they are now bored and uninterested in mathematics. A lot of students still cannot 'do' fractions at the end of Year 9; in fact many of them may know less than they did at primary school. Revisiting the same topics in the same contexts because they are seen as essential pre-requisite knowledge for something else does not work - it never has done!

Meaningful remediation and enrichment must be set in new contexts and provide challenges that require students to make sense of their mathematical knowledge in different situations. The motto should be “deeper, not lower or higher”. Rather than being given more and simpler fraction additions, students who have always struggled with fractions should be asked, for example, to make double quantities of recipes requiring one third of a cup of flour. Rather than being given more and harder fraction additions, students who have been quick to understand and complete the 'core' requirements should be asked, for example, to partition one unit fraction as the sum of two different unit fractions and to prove that their solution method always works. All students should be asked to apply their knowledge of fractions in both practical and mathematical settings, and to demonstrate understanding not just knowledge.

Is there knowledge without understanding?

Has Mathematics Teaching Lost Its Way?

One of the most significant decisions in every student's education is the choice of subjects, particularly in senior secondary school or college. The choice of which mathematics course to study, in particular, causes much consternation for many students, parents and their teachers. Factors such as maximising marks for tertiary entrance, future career pathways, interest and past success all influence the choice of whether to study 2, 3 or 4 Unit mathematics courses, or, indeed, no mathematics at all.

There has been considerable recent publicity surrounding falling enrolments in tertiary mathematics, science and engineering courses and the lack of adequate mathematical preparation of many students for their tertiary study. Enrolment numbers in academic mathematics courses in New South Wales have fallen dramatically in recent years, from, for example, 10603 3-Unit students in 1991 (19.7% of the total HSC population) to only 7688 (14.1%) in 1997. At the same time the number of 4-Unit enrolments has halved to only 2200. In the United States, of four million students entering high school in 1977, only ten thousand graduated with Doctorates in mathematics in 1992. There is no doubt that Australia is confronted with a similar shortage of people qualified in the mathematical sciences.

But encouraging students to continue their study of high levels of mathematics is not as simple as saying 'maths multiplies your choices'. Falling enrolment numbers are not simply the product of ignorance of the career choices open to mathematics graduates, nor of new scales of HECS fees. The perception of mathematics as a career which is poorly paid and impractical is a significant deterrent to students. Widely held beliefs about mathematics are reinforced by media stereotypes that portray mathematics as hard and mathematics teachers as uncaring and conservative. Students who excel in, for example, the Australian Mathematics Competition, and visiting academic mathematicians, are invariably photographed in front of a blackboard filled with formulas. Teaching is a particular victim of these perceptions, even among teachers themselves. The TIMSS data collected in Australia showed that 63% of teachers teaching Year 8 mathematics chose teaching as their first choice of career, but that 52% would now change careers given the opportunity. This contrasts with, for example, Portugal, where only 41% chose teaching as a career, but only 27% would now change. Even more worrying is the finding that a mere 26% of Australian mathematics teachers felt valued by society. There is a clear message to parents to thank their children's mathematics teachers and to let them know how much they are appreciated! A severe shortage of qualified mathematics teachers already exists in Australia; the shortage can only worsen in the near future.

These perceptions impact upon enrolments in academic mathematics courses and need to be addressed. But are they only perceptions? The reality of many school mathematics courses is that they are irrelevant and tedious, and, for most students, hard. The school mathematics curriculum seems to be an inexorable progression through smaller and smaller hoops that squeeze out more and more students along the way. Tests of algorithmic proficiency become increasingly stringent and increasingly effective in conveying the message that mathematics is only for the minority. Instead of being liberating, and opening up new opportunities, mathematics is all too often confining and limiting.

Reaffirming the Place of Mathematics

The contribution that mathematics makes to critical thinking seems to have been ignored in the current debate. Mathematics is not seen as an integral part of a rounded education, or as a vehicle for developing an appreciation of logic, structure and proof. Rather it is portrayed as a set of skills which must be mastered if students are to keep open their future career options. By stressing the skill-based, utilitarian aspect of mathematics perhaps we have done the subject a grave disservice, and devalued its true worth. I sometimes wish that there were no requirements for students to reach a particular skill level to progress to tertiary mathematics courses, that there were no prerequisites at all. Perhaps if mathematics competed for students 'on the open market' with other subjects, attracting students by virtue of its inherent interest, challenge and worth, we would be in a much healthier state, and students would make much wiser choices. But I doubt that that will ever happen!

Making Wise Choices

In the meantime we do need to help students to choose wisely, and we need to face the reality that school mathematics is, for many, a process of jumping through hoops. And there is little point in trying to force students through hoops if that is not what they want. Student readiness for a particular course is primarily a question of motivation and confidence. By all means encourage students to pursue as high a level of mathematics as possible. But students who have not succeeded in previous years are highly unlikely to suddenly excel in academically demanding mathematics courses. Mathematics only multiplies your choices if you are successful. We would do better to encourage students to see the wider picture and to study subjects they are likely to enjoy and succeed with.

At the same time, our primary and high school courses need to help students to enjoy mathematics for its own sake and to begin to appreciate the value of mathematics as encouraging logical thinking. Students who excel should want to continue to study high levels of mathematics because they enjoy the challenge rather than because of a perceived need. Others will choose appropriate levels of mathematics because they recognise its value and want to broaden and extend their education. We need to emphasise the inherent worth of studying mathematics at the highest level possible. The nature of much of the current school mathematics curriculum, coupled with negative perceptions of mathematics as a career, make knowledge required for tertiary courses an extremely problematic motivator in encouraging students to undertake academically demanding senior secondary mathematics courses.

Helping at Home

Dos and Don’ts for Parents

Parents have a vital role to play in their children's mathematical development, but many are unsure of how or how much to help. Some children are obstinate and refuse to admit to needing help or to accept it. Some parents are 'pushy' and insist on children completing a fixed number of hours of homework regardless of whether or not it is necessary, often finding extra work to help their children progress as quickly as possible through skills they have not yet encountered at school.

As a mathematics teacher I usually encounter one of three reactions when I talk with non-teachers. Sometimes the revelation that I am a mathematics teacher is met with complete indifference as no one wants to talk about mathematics. Sometimes it is met with the statement 'I was never any good at maths.' It is remarkable that it is socially acceptable, or even desirable, to admit to not being able to 'do' mathematics, yet illiteracy makes one a social outcast. The third reaction is 'I loved maths at school', followed by a sermon on the deficiencies of modern teaching methods. Each of these is harmful when conveyed to students. As parents and teachers we need to be positive, encouraging and interested.

The Mathematics Teaching and Learning Centre at the Australian Catholic University has recently prepared a guide for parents entitled *Helping Your Children With Mathematics* (ACU 1997). It contains several common sense ideas and strategies, recommending that parents

- build on success rather than finding fault;
- encourage their children to work things out for themselves rather than providing answers;
- give children time to think and time to answer;
- encourage children to talk, and especially to verbalise their problems;
- use mathematical words as a part of everyday language;
- buy books with mathematical themes or ideas;
- allow children to progress at their own rate rather than trying to force rapid skill development;
- above all, support, encourage and work with their child's mathematics teacher.

Encouragement, time and communication are the keys to success. Parents can be encouraged by the findings of the TIMSS. Students spending up to one hour per night on mathematics homework performed significantly better on the achievement tests than those doing no mathematics homework. However too long spent doing mathematics was also counterproductive. Up to two hours of television watching had no effect on student achievement, but students watching more than that achieved poorer results. Of course, these relationships are not necessarily causal, but they do give some pointers to the importance of a constructive home environment that values learning.

Tutoring in Its Place

One of the big questions faced by parents is whether or not their child needs tutoring. I believe that the likely success of a course of tuition depends more on the student than the tutor. Success relies on the student's willingness to accept help and motivation to succeed. I worry about the burgeoning tutoring industry; it seems that some will do anything to get ahead. The main benefit of private tutoring seems to me to be increased confidence, which may translate to improved performance at school. Effective tutoring programs build on success, encourage students to work things out for themselves, allow them to progress at an appropriate rate and give them time to internalise new ideas. It seems to me that this is precisely what every parent should be aiming for as they try to support their children's learning. Tutoring can be a valuable way to help build students' confidence but it does not replace the support of school and home.

The Essence of Mathematics Learning

The prevailing myth of student learning is one of inexorable progress along a continuum of skills. Each student's store of knowledge builds up bit by bit like a scaffold reaching to a higher plane. This model of learning is the reason that textbooks are best sellers. They perpetuate a myth of content built upon content, and you must buy the whole series and follow it religiously in order to construct a successful mathematics program.

Yet learning mathematics is far from linear. Mathematical knowledge did not evolve in a linear fashion; neither does student learning progress linearly. Learning is more like a 'Choose Your Own Adventure' story, or a surf on the Internet.

Learning is full of dead ends and incomplete Web pages. Just when we think we are getting somewhere we find that it makes no sense at all. Learning sometimes leads us down paths we do not expect. Often we search for one thing on the Web, but discover something else entirely. In the same way, as teachers we think that our students are learning what we are teaching them, but the reality is often quite different. Learning is also characterised by sudden leaps of intuition when all becomes clear - a Web page that exceeds all expectations. As teachers and parents we need to create an environment which challenges students to take the next step, to fit their learning into a coherent structure and to transform the anarchy of the Web into mathematical meaning.

Mathematics is, in part, about content knowledge. Almost every curriculum document in the world is built around five areas of mathematics: number, space, measurement, chance and data, and algebra. Mathematics is also about processes and thinking skills. Across Australia increasing emphasis is being given to the development of attitudes and appreciations, to the skills of mathematical inquiry, and to the need for students to be able to choose and use mathematics effectively.

But I am convinced that, above all, mathematics is about making connections. It is about making links between different areas of mathematics, and between mathematics and other fields of human endeavour. Students need a good supply of mathematical knowledge; they need a stock of problem-solving skills and strategies. But they also need to be able to access that knowledge and those skills when they need them. For this they need to have made the connections and created the links. As teachers and parents it is our job to promote connection-making by constructing situations in which similar ideas are encountered in different contexts, and by encouraging students to use visual as well as symbolic reasoning to solve problems. Student learning proceeds when they are able to say 'Aha!'.

Mathematics is about content knowledge, it is about processes, but above all it is about the development of insight.

References

Australian Catholic University. (1997). *A Guide for Parents: Helping Your Children With Mathematics*.

Curriculum Corporation. (1994). *Mathematics: A Curriculum Profile for Australian Schools*. Carlton, Victoria, Curriculum Corporation.

Lokan, J., Ford, P., & Greenwood, L. (1996). Maths and Science On the Line: Australian Junior *Secondary Students' Performance in the Third International Mathematics and Science Study*. Melbourne: Australian Council for Educational Research Ltd.

Steve Thornton
Director of Teacher Development,
Australian Mathematics Trust.
e-mail: SteveT@amt.canberra.edu.au

Vocational Mathematics for Schools

This paper briefly traces the history of vocationalism in education and differentiates between competency-based education and training (CBT) and the Key Competencies (KCs). Recent developments in mathematics for the vocational education and training (VET) sector are highlighted, and contrasted with the KCs. The concept of vocationalism in schools and some issues associated with the implementation of VET mathematics curricula are addressed. Finally the paper outlines possible developments and warnings for schools offering VET mathematics programs.

Introduction

Historical context: On the 29th November 1996, Dr David Kemp, Minister for Schools, Vocational Education and Training announced a package of some $20 million allocated to school authorities for the development and delivery of programmes which contribute to the expansion of vocational education in schools. In Australia, since the mid-1980s, there has been a concerted effort by federal governments to link education and employment in response to the widely-held perception (among many English-speaking countries) that economic problems were caused, not by any fault of industry, but by the education system. The last decade has seen major changes to policies and operations in the VET sector, which included the introduction of an open training market, so that TAFE providers no longer had a monopoly but were to compete with other government as well as private and industry-based providers. Another such change was the nationalisation of curricula. A third major shift was the introduction of CBT as a mandatory requirement of all nationally accredited courses.

Competency-based education and training

CBT entails a prescriptive system of curriculum and assessment procedures which must be rigorously adhered to by module writers. According to Porter (1993) it is intended to emphasise

> what a person can actually do in the workplace as a result of education and training. . . . It specifies the expected learning outcomes, the assessment criteria for those learning outcomes, the knowledge, skills and behaviours to be developed, and the methods and strategies for assessment of the learners' achievements. (p. iii)

Jackson (1993) has asserted that there have been two decades of research to indicate that CBT has not and probably will not improve learning in most areas where it has been applied. Jackson argued further that CBT be viewed not as an instrument of curricular reform but as a an administrative means of keeping programs accountable to ministerial policies.

Key Competencies

The Mayer Committee was set up in order to further develop the employment-related key competency (KC) concept — originally recommended by the 1991 Finn Report which addressed the participation of young people in post-compulsory education and training. Seven KCs evolved through a process of consultation with industry and other stakeholders, and provide a curriculum framework rather than a prescription. The KCs, which are described as being essential for effective participation work and in other social settings (Mayer, 1992), are as follows: (a) collecting, analysing and organising information, (b) communicating ideas and information, (c) planning and organising activities, (d) working with others and in teams, (e) using mathematical ideas and techniques, (f) solving problems, and (g) using technology. An eighth competency, (using) cultural under-standings, was proposed but not officially endorsed. According to Down and Lilly (1996) it could not be assumed that the KCs are automatically integrated into VET curricula, and they raised a number of issues which included the following:

1. The issue of transfer is central. The KCs will need to be explicitly written into course design and module outcome statements, and where appropriate, into learning outcomes and assessment criteria. This also implies a shift to develop the necessary cognitive skills which underpin transfer. "The development of competency needs to shift from a content and specific outcome orientation to one which is strongly grounded in process and approach" (Down & Lilly, 1996, p.4).

2. The written curriculum needs to be translated into an experienced curriculum so that the teaching and practicum reflect a "whole of work" competence.

3. The specified performance levels have not received widespread industrial support. Rather, they need to viewed in a developmental context, and accommodate the complexity of that context.

4. "Professional development is a vital component in all aspects of the process of integrating KCs" (p. 5).

These issues are of major import for the VET sector, and will take time and financial support to implement — they have certainly not been addressed from the point of view of mathematics. People responsible for the delivery of VET courses in any other sector, especially schools, need to be aware of these issues and where possible work to overcome existing problems.

Vocational Mathematics

The National Vocational Mathematics Curriculum Project (NVMCP)

With a seemingly endless range of mathematics units being offered across the TAFE sector throughout Australia, and diverse regulations in each state, the NVMCP was set up in 1992. It conducted an audit of mathematical skills included implicitly and explicitly in existing TAFE courses. To assist further curriculum development it recommended a set of topic packages be developed and mapped onto an associated topic network; topic packages could be assembled into modules or included in other mainstream modules as required. The NVMCP was required to conform to CBT arrangements, and its stated intention was to develop a mathematics curriculum which was nationally consistent across all vocational areas. The final product is an extensive atomised topic network composed of 94 topic packages, ranging from fractions and decimals to numerical methods (for example), arranged in a more or less linear progression of pre- and co-requisites.

Although the KCs are mentioned in the rhetoric of the NVMCP, the actual product is virtually entirely content based. For example, mathematics modules frequently contain learning outcomes of the form: "Solve vocational mathematics problems using ... [techniques]," and proceed with heavily content-oriented material that differs very little from the traditional school mathematics. Problem solving and modelling, in the rich and complex form of recent years' senior school assessment tasks (e.g., VCE) are non-existent. In practice there is evidence that some teachers are continuing to hand out worksheets with repetitive low-level skill questions, followed by so-called applications, which may or may not be of vocational relevance to the student group. On the other hand, the KCs provide a framework so that relevant content may be selected by the teacher, school, or accrediting authority. As mentioned above, the KCs evolved through a process of consultation weighted towards the needs and interests of industry, but they offer possibilities for making a positive contribution to the education of students in school and in the VET sector. The reality of current CBT mathematics curricula in the VET sector is that, within mathematics modules, meaningful interpretation of the Key Competency *Using Mathematical Ideas and Techniques* is not being realised, and there is only a small likelihood of other KCs being addressed to any substantial degree. Similarly, there almost no chance of the mathematical KC being realistically addressed in other non-mathematics modules — consistent with the findings of Down and Lilly (1996).

It is now accepted by many in mathematics education that Mathematics is a socially constructed phenomenon with its own values, institutions and relationships with society at large, which is both an object of study (a logical, formal system, which is nevertheless fallible) and a means of conceptualising the world (understanding mechanisms of social and scientific life — although this is not a definitive categorisation). In our technological society people are more able to understand and critically evaluate social and scientific developments when they have sufficient mathematical knowledge to do so. However, just as school mathematics is a subset of all mathematics known, mathematical facts and skills are a subset of all mathematical thinking. This includes knowing which mathematics to use, how, and when; recognising when one's skills are not enough and to seek further help; evaluating the results of one's calculations or judgements and follow up this evaluation in a constructive manner by iterating or abandoning the strategy adopted — together with reflection on this process. This is the thrust of the *Rich Interpretation of Using Mathematical Ideas and Techniques*, a DEET-funded project managed by the Australian Association of Mathematics Teachers (Hogan, in preparation). This KC, however, does not include a critical reflection on the uses to which mathematics is put, as suggested by writers such as Skovsmose (1994).

It should also be noted that the learning outcomes and assessment criteria, as expressed in the CBT mathematics modules, are very different from the interpretation of outcome-based education (OBE) made by Willis and Kissane (1995), although there are similarities in terminology. Unlike CBT, OBE views teaching as a professional rather than a technical activity, requiring a high level of professional judgement and autonomy. It also utilises a sophisticated model of standards-based rather than criterion referenced assessment. In contrast to CBT which requires checklists of skills and knowledge, aggregated into a final score, OBE has a fixed set of standards by which to judge a student's competence, providing learners with feedback and an indication of levels they are expected to attain, according to Willis and Kissane.

Vocationalism in schools

Sedunary (1996), in her critique of the trend towards vocationalism in schools, appears to form a dichotomy between discipline-based education and a narrowly constructed view of *instrumental* education. She also appears to trivialise the concept of *contextualisation*. There is a large body of research which indicates that learning is enhanced by the use of contextualisation which also addresses the question of transfer (see, e.g., Billett, 1996). My experience as a TAFE teacher of mathematics suggests that most adults and many young people have left school believing that they "cannot do" mathematics, although they lead otherwise successful lives. In social settings people are ambivalent towards mathematics, seeing it as publicly important but denying its use in their private lives; under the impression that they are incompetent in this domain (Benn, in press; Coben, in press). I would argue that this is precisely because mathematics has been taught, in the past, as a set of decontextualised skills, or, where a context appeared to be provided, as in the case of word problems, that this was a pseudo context — words wrapped around the topic of the day — stripped of any real world meaning and providing no opportunity for transfer.

Sedunary asserts that the new vocationalism is harnessing the intellectual training of discipline-based academic education for instrumental purposes. Although Skemp's (1978) distinction between instrumental and relational understanding of mathematics is widely known and appreciated, Mezirow (1996), referring to adult education, makes the point that instrumental and communicative learning are complementary and interactive, serving different purposes — as with Skemp's categories of understanding. According to Mezirow, the former refers to control and manipulation of the environment or other people, and the latter refers to "learning what others mean when they communicate with you" (p. 163). It could be argued that in the past the emphasis in mathematics education has been on the former, whereas the latter is an integral part of the KCs and important for personal empowerment of students in life generally as well as the workplace.

What are the Possibilities?

The adoption of VET mathematics curricula in schools offers some exciting possibilities. Although under CBT the actual learning outcomes and assessment packages look to be very prescriptive, there are spaces for experienced mathematics teachers to adapt and refine modules, to incorporate the whole range of KCs. (For industry trainers with no experience of mathematics pedagogy and limited mathematical qualifications, the story is very different!) Existing curricula have been based on extensive lists of skills which are heavily content-oriented at the expense of mathematical processes. But, by starting with the work which is actually relevant to an industry, school mathematics teachers may be able to reverse the process.

Industrial experience

There are opportunities for mathematics teachers to work with the teachers of vocational subjects and to observe and discuss what are the actual uses of mathematics in other school subjects and on the job. This was shown to be a very powerful experience for teachers in the AAMT project (Hogan, in preparation). In TAFE it is possible for teachers to team-teach, although it is not a common occurrence. Sefton, Waterhouse, and Deakin (1994) provide an interesting model for integrating curricula in ways that are meaningful to students and useful for employers.

Professional development

Under the ministerial School to Work Program Administrative Guidelines (19/2/97) there will be one year of professional development for school teachers to be able to deliver VET subjects, and ongoing funding for projects such as professional networks of VET teachers in schools, teacher-industry exchanges, and projects which combine VET, enterprise and KCs. In most, if not all states, there is currently no professional development for TAFE mathematics teachers, and voluntary networks have collapsed under increased workloads, so school teachers should make the most of any opportunities presented. Industrial release is seldom related directly to mathematics education, but can be enlightening for mathematics teachers who are able to appreciate that the accountant is not the only person in a company who uses mathematical ideas and techniques!

What are the functions of school?

It could be argued one of the functions of school is to prepare children in some way for their lives outside, both currently and in the future. Accepting this argument would require that schools prepare their students for adapting to a variety of situations and work practices: full-time, part-time, casual, short-term contract, ongoing tenure, self-employment, chronic unemployment, to name a few. (How can one arrange a home loan or hire purchase agreement when prospects of continuous employment are so uncertain?) Mathematics classes should not be exempt from this function. Students also need to be aware of the reality of *credential creep*, where there is a distinct possibility that employment gained will be below their level of qualification (and expectations). They need to be aware of the changing situations with respect to unionism and enterprise bargaining; of unscrupulous work practices by employers or fellow staff, and to critically evaluate just whose interests are being served in the employment situation. Are mathematics teachers in a position to assist? Yes, according to Skovsmose (1994), but these topics are not to be found in current VET mathematics curricula.

What are the Pitfalls?

Funding

Under VET funding guidelines, each module is allocated a nominated duration, arising from decisions made about *typical* students. The validity of these hours may be open to question as there appears to be no solid research base for them, and the question of who is a typical student has not been seriously addressed. More alarming is the fact that if a student does not reach competence in the required time there is no money for additional tuition. The student is then required to re-enrol, but the problem here is that providers, in some states at least, are monitored for their course completion rates, and students' failure to complete can carry a penalty for the provider.

Assessment

The NVMCP materials contain suggested assessment material. Experienced teachers will quickly discern that the tasks are very traditional, and even assignments and projects are reminiscent of traditional textbook material. As with the curriculum it is difficult to see the theoretical underpinnings. There are only general guidelines, and it is difficult to see the educational rationale behind much of the "exemplary" material. Teachers are exhorted to move away from the traditional reliance on timed tests and examinations; they are instructed to ensure that each of the many assessment criteria have been addressed (although not necessarily individually), and that the results are valid and reliable. Such a task is daunting, especially when, for example, in one 50 hour module there are * assessment criteria, while the time devoted to formal assessment can not exceed 10% of the module time. Professional development, apart from a one day general introduction in 1995, is not available for VET mathematics practitioners. Teachers who are concerned with assessment issues and interested to improve their knowledge are recommended to refer to Galbraith (1995a, 1995b).

Flexible delivery

In the VET sector there is a trend towards flexible delivery. Although it has been suggested as a means of accommodating the needs of working people for varying times and places of tuition, it is also gaining popularity as a means of institutions saving money. Misko (1994) warned that it is only suitable for people who are self-motivated, good at time management, and who have adequate English language reading skills. This is not my experience of most TAFE mathematics students — just the opposite. In fact, many students are in TAFE precisely because of their failure to cope with the traditional mathematics presented in school classrooms. Personal experience of past didactic materials presented under the banner of flexible delivery shows, not surprisingly, strong adherence to the transmission paradigm, limited pedagogic knowledge of the writers, and a failure to anticipate the depth of student learning difficulties. Once more, students are likely to have their self-confidence destroyed and to be convinced that it is somehow their own fault. Schools are warned to be very wary of treading this alluring path towards self-paced instruction, no matter how brilliant the instructional design techniques or technologies (including the Net).

In a similar vein, teachers are warned to beware of *Trainer's Guides*. Although they have the advantage of being produced for particular industries, and so are contextually relevant, the actual pedagogies employed are, once again, generally transmission-based. "Explain to the student that . . ." "Run through a few simple examples." They are very much geared to rote learning with plenty of rules, recipe-style, with "helpful hints" supplied by the trainer (or worksheet) to make things easier — even to the extent of filling in the spaces of catechisms for multiplication by powers of 10 through adding noughts or shifting the decimal point, for example. To the unsuspecting learner the rules of mathematics would appear to have fallen from on high; all the mathematising has already been done by others and the learner remains powerless and dependent.

Conclusion

In this paper I have given a broad overview of the history, possibilities, and perils associated with the implementation of VET mathematics curricula in schools. Curriculum and assessment practices continue to be dominated by a traditional school-based model and further restricted by the imposition of a CBT regulatory framework. The concept of KCs has been given little more than lipservice and they have yet to be integrated into any VET curricula in a meaningful way although they have exciting educational possibilities. At the same time teachers have to beware of narrowing school curricula to vocational interests at the exclusion of more general needs of students as critical citizens. As with all changes there are opportunities presented, and school teachers are encouraged to utilise any industrial release, professional development, and/or curricu-lum development programs available, as well as to take part in informal discussions with vocational teachers and other contacts to improve on existing VET curriculum and assessment practices. This paper has focused on general issues; for more in-depth analyses, see FitzSimons (1996, 1997a, 1997b).

References

Benn, R. (in press). Silent murmurings. In G. E. FitzSimons (Ed.), *Adults (returning to) study mathematics: Papers from the 8th International Congress on Mathematics Education (ICME 8) Working Group 18*. Adelaide, SA: Australian Association of Mathematics Teachers.

Billett, S. (1996). The transfer problem: Distinguishing between sociocultural and community of practice. In *Learning and work: The challenges: 4th Annual International Conference on Post-Compulsory Education and Training (Vol. 4)* (pp. 21-36). Brisbane, Qld.: Centre for Learning and Work Research, Griffith University.

Coben, D. (in press). Mathematics or common sense? Some reflections on research into adults' mathematics life histories. In G. E. FitzSimons (Ed.), *Adults (returning to) study mathematics: Papers from the 8th International Congress on Mathematics Education (ICME 8) Working Group 18*. Adelaide, SA: Australian Association of Mathematics Teachers.

Down, C., & Lilly, M. (1996, October). *The identification and further integration of Key Competencies into the curriculum, delivery, assessment and reporting of apprenticeships (Automotive, Building and Construction, Electrical/Electronics, Engineering, Hospitality).* Paper presented at the Social Organisation of Education Practice seminar, Melbourne.

FitzSimons, G. E. (1996). Does it add up? Adult education, mathematics and economic rationalism. *Education Links, 53,* 20-24.

FitzSimons, G. E. (1997a). *Gender and Adult Education*. Manuscript submitted to Mathematics Education Research Journal.

FitzSimons, G. E. (1997b). *Mathematics in the Vocational Education and Training sector: A case study*. Paper submitted to 20th conference of

Mathematics Education Research Group of Australasia.

Galbraith, P. L. (1995a). Assessment in mathematics: Developments, innovations and challenges. In L. Grimison & J. Pegg (Eds.), *Teaching secondary school mathematics: Theory into practice* (pp. 289-314). Sydney, NSW: Harcourt Brace.

Galbraith, P. L. (1995b). Assessment in mathematics: Purposes and traditions. In L. Grimison & J. Pegg (Eds.), *Teaching secondary school mathematics: Theory into practice* (pp. 271-288). Sydney, NSW: Harcourt Brace.

Hogan, J. (in preparation). *Final report of the Rich Interpretation of Using Mathematical Ideas and Techniques Key Competency Project.*

Jackson, N. (1993). Reforming vocational learning? Contradictions of competence. In W. C. Hall (Ed.), *What future for technical & vocational education & training?*, (pp. 105-122). Adelaide: National Centre for Vocational Education Research.

Mayer, E. (Chair). (1992). *Report of the Committee to advise the Australian Education Council and Ministers of Vocational Education, Employment and Training on employment-related Key Competencies for post-compulsory education and training*. Melbourne: Australian Education Council and Ministers of Vocational Education, Employment and Training.

Mezirow, J. (1996). Contemporary paradigms of learning. *Adult Education Quarterly, 46*(3), 158-173.

Misko, J. (1994). *Flexible delivery: Will a client focus system mean better learning?* Leabrook, SA: National Centre for Vocational Education Research.

Porter, B. (1993). *Developing competency based curriculum modules: A guidebook for TAFE teachers and curriculum writers.* Sydney: NSW TAFE Commission.

Sedunary, E. (1996). 'Neither new nor alien to progressive thinking': Interpreting the convergence of radical education and the new vocationalism in Australia. *Journal of Curriculum Studies*, *28*(4), 369-396.

Sefton, R., Waterhouse, P., & Deakin, R. (Eds.). (1994*). Breathing life into training: A model of integrated training*. Doncaster, Vic.: National Automotive Industry Training Board.

Skemp, R. R. (1978). Relational understanding and instrumental understanding. *Arithmetic Teacher, 26*(3), 9-15.

Skovsmose, O. (1994). *Towards a philosophy of critical mathematics education*. Dordrecht: Kluwer.

Willis, S., & Kissane, B. (1995). *Outcome-based education: A review of the literature*. Perth, WA: Education Department of Western Australia

Gail FitzSimons
Swinburne University of Technology
e-mail: gfitzsimons@swin.edu.au

QUOTABLE NOTES AND NOTABLE QUOTES

Priorities

A lawyer, an artist and a mathematician are arguing over whether it is better to have a spouse or a paramour. The lawyer argues for a spouse, stressing the advantages of legality and security. The artist argues for a paramour, emphasising the joys of freedom. The mathematician says: "You should have both, then when each of them thinks you are with the other you can get on with mathematics."

It's a Dangerous Existence

Risk assessment is a serious business for governments, experts and the public. The table below opens up possibilities for exploration, discussion and calculation in the classroom. Does anyone have the corresponding figures for Australia?

Living Dangerously: Annual Risk of Death in Britain	
Smoking 10 cigarettes per day	1:200
All natural causes, age 40 years	1:850
All kinds of violence and poisoning	1:3300
Influenza	1:5000
Road accident	1:8000
Playing football	1:25 000
Accident at home	1:26 000
Accident at work	1:43 000
Homicide	1:100 000
Railway accident	1:500 000
Struck by lightning	1:10 000 000
Radiation release by nuclear power station	1:10 000 000

Source: British Department of Health, 1995 via **New Scientist**, 21 February 1998.

Geometrical Terms Through Puns = Fun

Most of these riddles will be familiar to one or other of our readers. If you can add to the list, please make contact so we can share on your material with readers in a future issue.

- What do you call a man who spent all summer at the beach?
  Tangent
- What do you say when you see an empty parrot cage?
  Polygon
- What do you call a crushed angle?
  A rectangle
- What did the Italian say when the witch doctor removed the curse?
  Hexagon
- What did the little acorn say when it grew up?
  Geometry
- What do you call an angle which is adorable?
  Acute angle
- What do you use to tie up a package?
  A chord
- What do you call a fierce beast?
  A line
- What do you call more than one L?
  A parallel
- What do you call people who are in favour of tractors?
  Protractors
- What should you do when it rains?
  Coincide

A Mathematical Web?

HTML, the standard mark-up language on the Web, cannot cope with mathematical expressions and symbols. Currently, to include mathematics on a Web page, the material must be saved as a GIF or JPEG image file, then embedded into the HTML. Loading images is much slower than loading text, and mathematical text images can't be cut and pasted.

Now along comes MathML, an application of XML (eXtensible Mark-up Language), the successor to HTML. MathML handles mathematical symbols and expressions and fits seamlessly into the software that mathematicians and scientists already use. After two years of negotiations the World Wide Web Consortium, the Web's governing body, has given MathML a "recommendation". This gives software writers the go-ahead to develop applications for MathML. Already there are prototype plug-ins which adapt Web browsers for MathML, and in the long run there should be a wholesale migration out of HTML to XML and much easier mathematics on the Web.

Adapted from *Problem Solved* by Mary Parlange, **New Scientist**, 4 April 1998.

Learn from the Greats:

Leonhard Euler, 1707-1783

The contribution Euler made to mathematics is underlined by the numerous terms, formulae, equations and theorems that bear his name. The November 1983 issue of *Mathematics Magazine*, published as a tribute to Euler, contained a glossary of 44 such items. Euler was a rare genius, but his enthusiasm, his insatiable probing curiosity, and his determination to procure deeper and deeper understanding still have the power to inspire.

In the words of Andre Weil (1984):
"Perhaps (Eulers) most salient feature is

the extraordinary promptness with which he always reacted even to casual suggestions or stimuli ... Every occasion was promptly grasped, each one supplied grist to his mill, often giving rise to a long series of impressive investigations. Hardly less striking is the fact that Euler never abandoned a problem after it had once aroused his insatiable curiosity .... All his life, even after the loss of his eyesight, he seems to have carried in his head the whole of the mathematics of his day, both pure and applied. Once he had taken up a question, not only did he come back to it again and again, little caring if at times he was merely repeating himself, but also he loved to cast his net wider and wider with never failing enthusiasm, always expecting to uncover more and more mysteries, more and more "herrliche proprietates" lurking just around the next corner. Nor did it greatly matter to him whether he or another made the discovery."

More from the AAMT Internet Mailing List Community

The plaintive but frequently asked question from mathematics students **"When am I ever going to use this?"** became the catalyst for a recent interesting exchange of views via the Mailing List. Here are some extracts.

First is John Mason (Open University, UK)

"Even if mathematics were taught solely for its utilitarian role in the lives of citizens, would you expect that every step in the encounter would have application?

Are students really driven by the utilitarian theme. For example, if whenever the question "When will I use this?" were answered immediately with "it is absolutely vital in retail and in the service sector", would the person asking the question be satisfied? Might they not actually be asserting through their question, "I don't understand; I am not coping; I can't do this"?"

Then Patricia Cretchley (University of Southern Queensland)

"I think John hits it on the head:

many students are indeed expressing their lack of confidence when they query the usefulness of the mathematics at the time, and their lack of taste for the task - discomfort robs them of enjoyment. The usefulness of the content will not fix all that!

But obviously we hope to persuade them to engage sufficiently deeply to reach some sense of satisfaction and achieve some logical goals - the ongoing challenge ...

I don't think we are alone in facing this kind of question: many students find the same way of voicing their discomfort with the study of Shakespeare, Latin, etc; others love it and will rationalise madly, to establish the value they clearly feel. Utilitarian, the Classics? Not to many!! But valuable nevertheless, to those who accept breadth in the challenge of learning.

It is not easy to verbalise to students the conviction we hold on the value of logic and exposure to modelling and problem solving generally. Will we ever regain the courage to focus more on that and assert the wider vision of the teacher, and will that ever be acceptable to a student population so sure of its rights, and so quick to defend them? ...

But for most groups of students, their expectation and life experience of what will indeed prove useful, is likely to be so varied that it is limited as a motivator!

Thank goodness some students (and not always the highest achievers) still express a liking for the "game" of mathematics, when they feel the tasks are within their reach. We can build on that!

Let's not underrate the value of logical toys and puzzles with some mathematical content, while we strive to establish good standards of mathematical literacy and numeracy."

Next is Lindy McKeown (Queensland University of Technology)

"Hi everyone from a lurker, parent, user of mathematics daily (like everyone else).

One aspect that doesn't seem evident in the discussion here is that of THINKING and SPEAKING mathematically. A children's story book that I purchased recently was about a child who woke up one day and could only see the world through the view of a maths problem - time to get ready, number on the bus, patterns in the rows of desks in class, etc, etc. She was thinking mathematically. As Patricia Cretchley said, there are wonderful qualities in literature, Latin and maths - in any topic of study for that matter. Part of that is thinking and being able to articulate that thinking with enthusiasm and confidence. Somewhere in kids' journey through school, they stop doing that or teachers stop doing it around them and they lose the joy of learning mathematics that was so powerful when they were younger. Sustaining that joy of learning is one of the challenges we face as teachers and it is through many strategies not one that we achieve it. Personal enthusiasm for the subject matter and articulation and demonstrations of personal passion for the content, form, process and challenge of maths (or any subject) is one way of helping students have that same experience. When was the last time YOU were passionate about maths and said so? Passionate teachers help create passionate classrooms filled with the energy of enthusiasm!!

Stepping off her high horse ..."

Now comes John Gough (Deakin University)

" ...Here is the larger picture. For many school-linked subjects, not just mathematics, we can see that something happens between pre-school and end-of-school that damages natural enthusiasm and curiosity. Yet the real point of school is that it should develop sufficient skills, knowledge and curiosity, that each student is let loose like a revved up learning machine, able and willing to continue life-long learning independently of formal instruction. I should stress that this does not mean independent of information resources (people, books, media, CD-ROMs, computers, Internet and so on) but not needing to be enrolled and interacting with a professional teacher in order to be able to learn. Self-motivated.

The abandonment of mathematics is typical of the abandonment of many things which young children seem to do almost spontaneously. Think of drawing, singing, making up songs, telling crazy stories, and asking "Why?" When was the last time you drew a picture, sang a song, made up a song, told a crazy story, or asked "Why?" If the answer is a long time ago, then "Why?" ...

Then some reflections from Walter Spunde (University of Southern Queensland)

" ...There is a difference though between being passionate about established old ways of thinking (that look stale from the outside) and passionate about opening our minds to new ideas (that look threatening to old heads). Children revel in the joy of the latter. Math teachers unfortunately seem too often wrapped in the former."

Finally, a slightly pessimistic wrap up from Lindy McKeown

"Funny you should bring up this point as a group of teachers and parents were discussing this with me on Friday afternoon. The (secondary) teachers were saying it was so sad that when they tried to get kids to "think" and "solve problems" and engage in design activities, they kept asking them to tell them exactly what the teacher wanted them to do, tell them what the teacher wanted them to do next. They had been trained to be "good" students ... not to think for themselves but to do what they are told, when they are told to do it and how they were told to do it. Things like thinking for themselves, creating the learning experience and showing initiative were unthinkable. This is a sad situation where kids see their role in the classroom in this way. They are not cognitively lazy, just trained in "school survival."

All in all an interesting interchange of views on matters at the heart of mathematics learning and teaching. Any AAMT member with access to the net can join this Internet Mailing List Community.

PROBLEMS AND ACTIVITIES

Remember that we include a coding system which attempts to indicate in terms of Year levels the suitability range for each item. Thus 6 - 8 suggests an item accessible to students from Year 6 to Year 8.

(1) A PROBLEM Problem

6 - 10

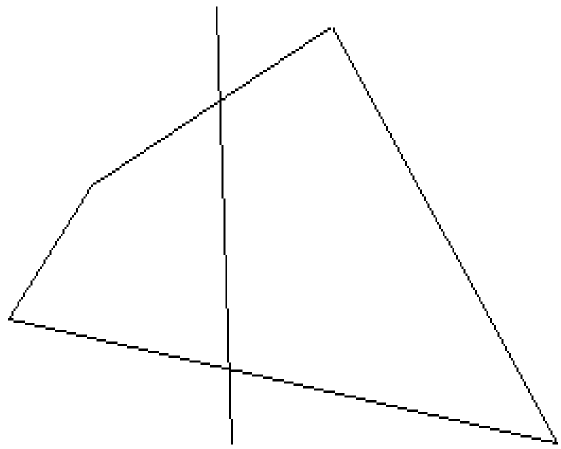
In how many ways can the word 'PROBLEM' be traced out in this diagram if you are allowed to move only one step at a time horizontally or vertically, up or down, backwards or forwards?

```
            M
          M E M
        M E L E M
      M E L B L E M
    M E L B O B L E M
  M E L B O R O B L E M
M E L B O R P R O B L E M
  M E L B O R O B L E M
    M E L B O B L E M
      M E L B L E M
        M E L E M
          M E M
            M
```

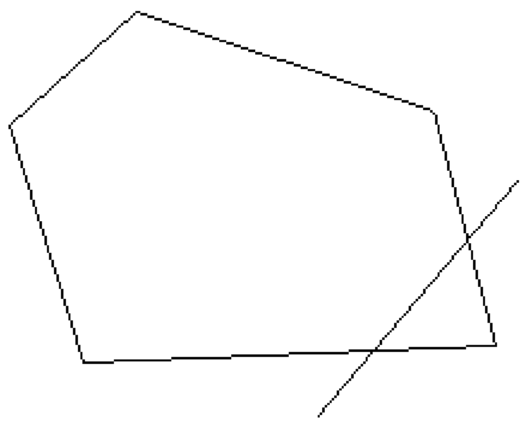
(2)Angle Sums

7 - 10

(a) How does the figure below suggest that **if** the angle sum of any quadrilateral is a fixed quantity, it must be 360°?



(b) How does the next figure suggest that **if** the angle sum of a polygon is fixed by the number of sides, then adding a side increases the angle sum by 180°

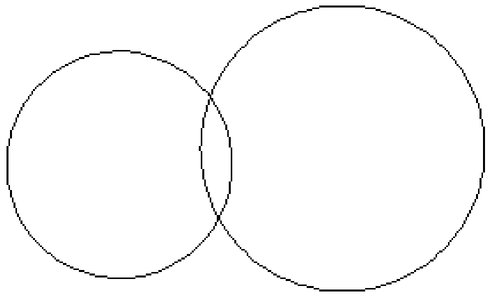


---

(3)Overlapping Circles

8 - 10

A circle of radius r intersects another circle of radius R, with R>r, as shown.



radius r radius R

Find an expression for the difference in the areas of the non overlapping parts of the circles.

---

(4) A Prize Problem

10 - 12

This is a problem from the 1997 M L Urquhart Prize Problem Solving Competition for senior secondary students, named in honour of one of the founders of the Mathematical Association of Tasmania.

Show that from any five positive integers which are not greater than 120 it is possible to choose two integers x,y satisfying the inequalities $1 \le \frac{x}{y} \le 3$.

---

(5) A Variety of Approaches to a Proof of Uniqueness

9 - 12

Prove that the only pair of positive integers (p,q) whose sum p+q equals their product pq is (2,2).

---

(6) A Knotty Regular Pentagon

4 - 12

(a) Take a thin strip of paper with parallel edges which is about ten times as long as it is wide. Now without crumpling it tie a simple overhand knot in it and then carefully flatten out the knot using sharp creases.

The result should be a perfect regular pentagon knotted in to the bent paper strip.

(b) Prove that the figure obtained in (a) is a regular pentagon.

THUMBNAIL (and longer) REVIEWS

Readers are welcome to contribute to this section. Reviews can cover books, periodicals, videos, software, CD ROMs, calculators, mathematical models and equipment, posters, etc.

aMath
Freeware from the World Wide Web,
by Marc Guillemot, Saint Gilles, France.
email address: mguillem@infonie.fr

aMath is a free and user friendly collection of macros for Microsoft Word 6 or later which simplifies the typing up of mathematical material. The mathematical text of the Problems and Activities and Solutions to Problems sections in this Circuit were produced using aMath. A full range of mathematical special symbols, plus angles, vectors, grouping symbols, limits, roots, algebraic fractions, vectors, matrices, sub- and superscripts, sums, products and integrals are provided via Auto text entries and dialogue boxes.

The Web address for downloading a Math is
http://perso.infonie.fr/a-guillemot/amath/

aMath is a boon to anyone who uses Microsoft Word for typing mathematical text material and in my opinion is much simpler to use than MicroSoft Equation Editor

Peter Enge

Mathematics
From the Birth of Numbers
by Jan Gullberg
Published by W. W. Norton, 1997
Hardcover, $69.23

This is a unique compendium of mathematics which runs to nearly eleven hundred pages. The author, a Swedish surgeon, was inspired to write the book during his son's college years in Indiana USA, when mathematics became the common subject of father and son conversations. In Sweden Gullberg is known as a writer on scientific and medical topics. This book is the result of a ten year effort to bring together the history of mathematics, a broad treatment of mathematical content extending as far as calculus and differential equations, and material on the foundations of mathematics.

The really striking thing about Gullberg's achievement is the thoroughness and clarity with which he lays out a comprehensive account of the history of mathematics from the birth of numbers and their associated language, side by side with his comprehensive survey and self-contained summary presentation of mathematical content. Having been a teacher of mathematics for 30 years and a reader of books about mathematics and mathematics education for pleasure, I was repeatedly struck by the freshness and thoroughness of Gullberg's treatment of hackneyed historical topics and his sustained attention to mathematical and pedagogical details.

This armchair reader's grand tour of the world of mathematics includes more than

1000 technical illustrations, a multitude of reproductions from mathematical classics and other works, and a generous ration of humorous asides and quotes, ranging from limericks and tall stories to cartoon and other humorous and decorative drawings. The book's usefulness as a reference work is enhanced by extensive cross references and detailed indexes.

From numbers, the tour proceeds through the primary realms of arithmetic, algebra, geometry, trigonometry and calculus to the final destination of differential equations, with excursions into symbolic logic, set theory, topology, fractals, probability and other topics. The treatment of mathematical content is systematic, dense and sustained along textbook lines. It includes hundreds of worked examples. As he confesses in the preface, Gullberg mainly wrote for himself - "as a means of exploring mathematics and its history more completely than I ever had before." All users of this book, bent on improving their own mathematical background for whatever reason, are the beneficiaries of his magnificent obsession.

The foreword by Peter Hilton, subtitled Mathematics in Our Culture, makes a significant contribution to the book. It begins with the assertions that mathematics is both worth doing for its own sake and an essential component in any education process aimed at producing citizens who are able to grow and adapt in accord with changing life needs. As all mathematics teachers soon appreciate, the usefulness of mathematics is both a blessing and a curse. But as Hilton indicates, this does not trouble Gullberg. His text really does succeed in conveying the content and spirit of an extraordinary amount of mathematics, past and present.

Where else, apart from in a book by a compulsive Swedish medical man writing about mathematics, would "hunting and gathering peoples - such as the aborigines of Australia, Tasmania and Papua New Guinea" have their approaches to number naming and counting mentioned on the first page? Later the Maori approach to counting is described, and Gullberg points out that the name Maori was only adopted after European settlement of New Zealand. Having lived in isolation for a long time, the New Zealand Polynesians had no need to name themselves until they wished to distinguish themselves from Europeans. Hence there adoption of the name Maori, meaning "normal". After all, isn't mathematics about making distinctions?

A couple more snippets to whet the appetite. First, Sheepish Division, an Australian mathematical tall story in the chapter titled "Cornerstones of Mathematics". A newcomer visited a sheep station in Australia. Asked to estimate the number of sheep in the milling flock, she responded 10 461 almost immediately. Stunned by the prompt and exact answer, the grazier asked how the visitor did it. "That's easy," was the reply, "I just counted the feet and divided by four." And an example of a limerick:

Many things have more than direction,
The magnitude is also a question.
With acceleration or force,
And many more things of course,
It's vectors that make the connection.

From impossible objects à la Escher and Penrose, to the multiline derivatives of assorted trigonometric functions, this book will be a standard reference for students and teachers in years to come. If you are interested to learn more about mathematics and purchase one mathematics book this year, consider making this the one. Then persuade your librarian to add a copy to the collection. (S)he and other library users will be grateful to have such a stimulating and comprehensive mathematical work available.

Peter Enge

Preparing a Profession
Report of the National Standards and Guidelines for Initial Teacher Education Project
Chair: Professor Kym Adey
Published by Australian Council of Deans of Education, Feb 1998, Canberra.

The major outcome of this project has been the drafting of 'Standards and Guidelines' to be used as the basis of a national, professionally driven framework for the initial education of school teachers. It is the result of wide consultation and collaborative discussions with the stakeholders in initial school teacher education across the country, held throughout 1997.

The body of the Report consists of three parts:

- National Standards and Guidelines;
- Development and Implementation;
- Appendices listing organisations and individuals involved in the consultation processes.

Apparently the AAMT made a written submission and the ACT Department of Education and the AEU ACT Branch participated in Advisory Committee hearings for the project.

The first part, National Standards and Guidelines, is intended to specify external review criteria for initial school teacher education programs seeking approval or accreditation. The hope is that the criteria will be suitable as a public accountability mechanism ensuring that graduates from an institution which meets the guidelines will be effective beginning school teachers with the potential to build a successful teaching career.

This National Standards and Guidelines section is subdivided into Graduate Standards and Guidelines, Program Standards and Guidelines and Organisational Standards and Guidelines. Literacy and numeracy appear as two of the fourteen subheadings under Graduate Standards and Guidelines. Anyone who doubts the complexity of the modern school teaching situation should browse this section of the report and its impossibly long list of requirements for graduates. Teachers really are expected to be superhumans.

The subsection on Program Standards and Guidelines attempts to specify conditions which will guarantee high quality outcomes for initial school teacher education students. The Organisation Standards and Guidelines subsection seeks to specify those general features of institutional, faculty and school field experience operation necessary for quality initial school teacher education.

The second part of the Report, Development and Implementation, deals with the substantive historical, social, political and structural issues which must be addressed in order to achieve the recommended National Standards and Guidelines. Six implementation principles are presented and discussed in this section. These principles are a pragmatic attempt to find sound operating procedures for implementation of the Report in the incredibly complex context of the Australian Federal system.

The Report makes important and timely recommendations. Unfortunately it completely dodges issues of standards and guidelines for funding of initial school teacher education. In the current climate of diminishing funding for all things public educational, one cannot be optimistic about the implementation of this Report by the Commonwealth Government under the current Minister for Education. More challenges for AAMT, CMA and other professional organisations for teachers!

Peter Enge

SOLUTIONS TO PROBLEMS AND ACTIVITIES

(1) Consider this portion of the original diagram.

```
M
E M
L E M
B L E M
O B L E M
R O B L E M
P R O B L E M
```

There is only one way of starting at the P and ending at the top M and only one way of starting at the P and ending at the M on the extreme right, in accord with the given conditions. To perform a count, we replace each letter in the arrangement by the number of different ways of proceeding from the P to that letter through letters in the correct sequence for PROBLEM.

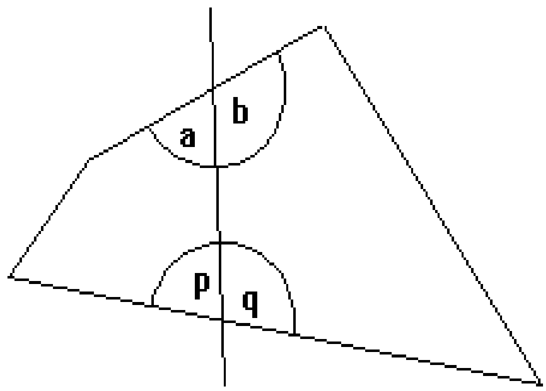
This gives the number triangle shown, built from the initial 1s along its horizontal and vertical edges.

```
1
1 6
1 5 15
1 4 10 20
1 3  6 10 15
1 2  3  4  5 6
1 1  1  1  1 1 1
```

(Notice that it is Pascal's triangle.) So, using the fact that separate quarters of the arrangement remain disjoint during the count, the number of ways of tracing out the word PROBLEM in the original diagram is 4 + 4 (6 + 15 + 20 + 15 + 6) = 252

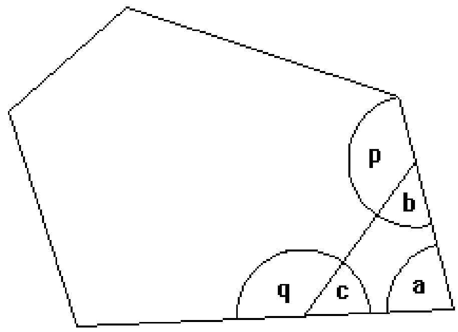
---

(2) (a)



If each of the three quadrilaterals in the figure has angle sum A, then the total angle sum of the two smaller quadrilaterals is A more than the angle sum of the outer quadrilateral. But the extra angles are a, b, p and q with a + b = p + q = 180.
Hence A = a + b + p + q = 360.

(b)



When the original pentagon in the diagram is converted into a hexagon by lopping off its corner, the angle sum goes up by

p + q - a = (p + b) + (q + c) - (a + b + c)
= 180 + 180 - 180 = 180

Thus, by adding one side, the angle sum has been increased by 180 and this will be true however many sides the original polygon has.

---

(3) If S represents the area common to the two circles, then the areas of the non-overlapping parts of the smaller and larger circles are πr^2 -S and πR^2 - S respectively.

The required difference

$= (\pi R^2 - S) - (\pi r^2 - S) = \pi (R^2 - r^2)$

Note that this expression applies even when the circles do not intersect.

---

(4) Divide the positive integers from 1 to 120 inclusive into the four disjoint sets

$S_1 = \{1,2,3\}$,

$S_2 = \{4,5,6, ..., 12\}$,

$S_3 = \{13,14,15, ..., 39\}$ and

$S_4 = \{40,41,42, ... 120\}$,

constructed so that the largest number in each set is no more than 3 times any other member, and each of the 120 integers is a member of just one set.

Now given five arbitrary positive integers no greater than 120, by the pigeonhole principle at least two must belong to one of these sets. Denote these two integers by x and y, x being the larger, and we are finished.

---

(5) Our discussion is based on that in Morsel 13 of *More Mathematical Morsels* by Ross Honsberger. Three clever solutions are topped off by a fourth gloriously simple and direct one due to Samuel Greizer.

(i) Let q denote the bigger number so that $p \leq q$.

Since p + q = pq, then

$p = \frac{pq}{q} = \frac{p+q}{q} = \frac{p}{q} + 1$

which is not an integer for $p < q$.

Hence p = q, giving $2p = p^2$ and 2 = p = q.

(ii) Clearly p/(pq - p).

But pq - p = p + q - p = q, so p/q

Similarly, q/p, giving p = q, etc.

(iii) As in (i), arrange the labels so that $p \leq q$.

Since we want p = 2, let's look at the consequences of p > 2, i.e. $p \geq 3$.

In this case,

$pq \geq 3q > 2q = q + q \geq p + q = pq$

and we have the contradiction pq>pq.

Thus $p \leq 2$.

But p = 1, leads to 1 + q = q, which is impossible. Hence p = 2, etc.

(iv) Now for Greizer's solution.

If pq = p + q, then

pq - p - q = 0,

pq - p - q + 1 = 1,

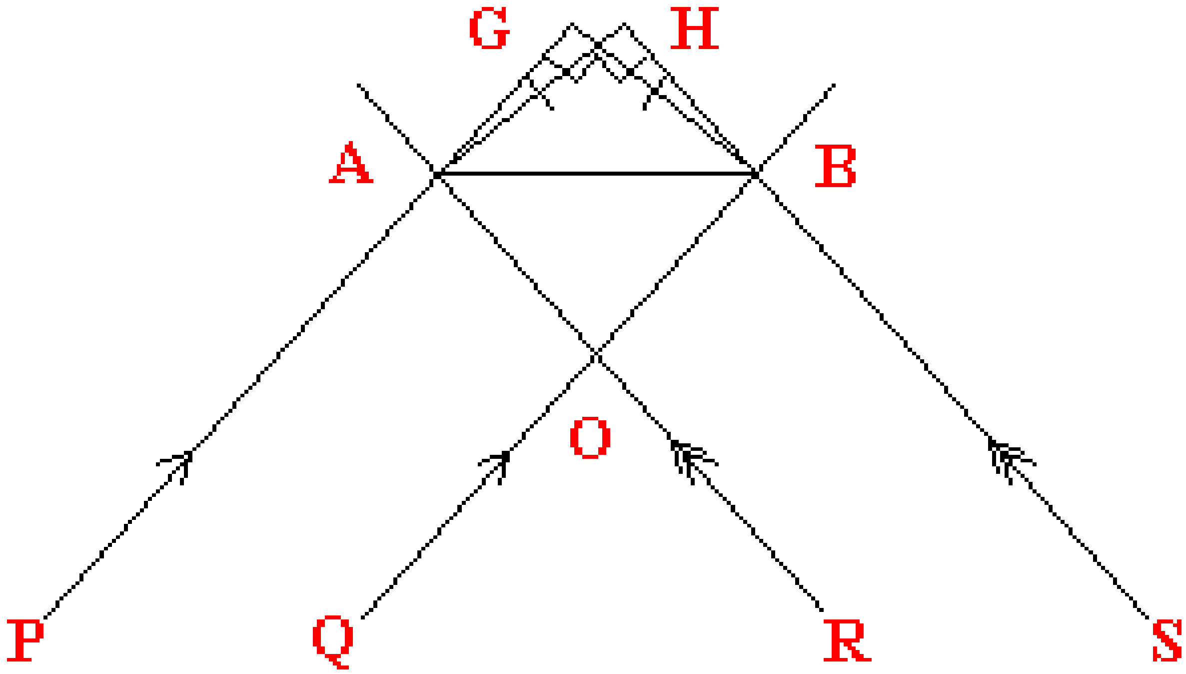
(p - 1) (q - 1) = 1,

making p - 1 = q - 1 = 1 and p = q = 2.

---

(6) We organize the proof so that two lemmas and various corollaries lead us to the desired theorem. The proof is essentially that from Alexander Bogomolny's website http://www.cut-the-knot.com/logo.html.

Does any reader know of a simpler approach?



Lemma 1

Consider (see the diagram) two pairs of parallel lines PA, QB and RA, SB such that the distances between the lines PA and QB equals the distance between the lines RA and SB (as when two parts of the strip of paper cross when the pentagon is folded). Let QB and RA cross at O.

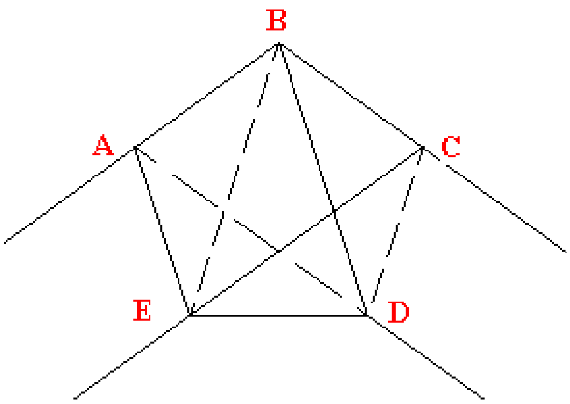
Then <PAB = <SBA and OA = OB.

Proof
Let H and G be the feet of the perpendiculars from A to SB and B to PA, respectively. Then the two right triangles ABH and ABG are congruent (why?),
and <GAB = <HBA.

But <PAB = supplement of <GAB and
<SBA = supplement of <HBA,
so <PAB = <SBA.

Next, <PAR = <SBQ since their arms are parallel.
Subtracting <PAR from <PAB and
<SBQ from <SBA gives <OAB = <OBA, so triangle OAB is isosceles and OA = OB.

The next figure represents the pentagon knotted from the paper strip, with hidden edges dotted.



The rest of our discussion refers to this diagram.

Corollary A

EA = AB = BC = CD

Indeed the above segments are each hypotenuses of right angles triangles congruent to triangle ABH in the first diagram.

Corollary B

AD = BE,

since, by Lemma 1, AD = BD and again BD = BE.

Lemma 2

The quadrilaterals (trapezia) ABCD and BCDE are congruent.

Proof

By Lemma 1, <ABC = <BCD

By Corollary A, AB = BC

By Corollary A, BC = CD

By Corollary B, AD = BE

The equality of these four corresponding elements of the two trapezia guarantees their congruence.

Corollary C

CD= DE and <BCD = <CDE

Using Corollaries A, B and C shows that the pentagon has sides of equal length and this in turn can be used to show that all five of its angles are equal (how?).

Finally then:

Theorem

ABCDE is a regular pentagon.