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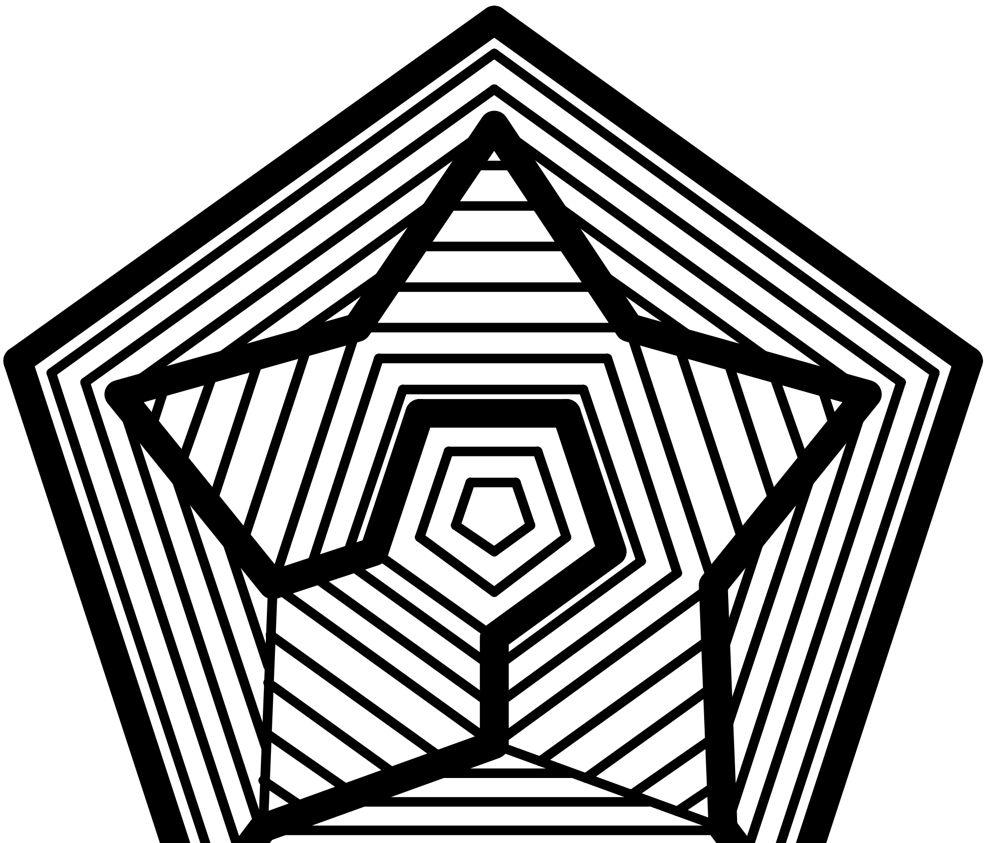

FROM THE PRESIDENT

Congratulations and Thanks

The CMA congratulates Mike Newman, a life member of the Canberra Mathematical Association and an active member since the inception of the CMA, on his appointment as Professor of Mathematics at the Australian National University.

Mike and Laci Kovacs founded the Friday enrichment evenings for senior seconday students nearly 30 years ago. Mike continues to play an active role as codirector of the Canberra Mathematics Enrichment Program.

The CMA wishes to formally thank Mike and Laci on behalf of the many teachers and students who have benefited from their extensive leadership and practical support for mathematics education in ACT schools. The CMA wishes Mike success and enjoyment in his new role.

The University of Canberra Maths Day, Friday 30 May 1997

The University of Canberra (UCAN) Maths Day was the sixteenth such "fun day" for Year 12 students from the ACT and surrounding regions. Forty schools, including a couple from Sydney, participated at the Sports Centre at the University.

Many thanks to the staff from the UCAN Information Science and Engineering faculty, who with administration staff from the Australian Mathematics Trust, planned and coordinated this very enjoyable day. Their hospitality was appreciated by teachers responsible for school teams. The new Head of the Faculty, Professor Robert Bartnick, took the opportunity to see the Maths Day in action and to speak with teachers and students.

The final event, the relay, remains a favourite with students and teachers.

Prizes were presented to:

University of Canberra Maths Day Trophy
Sydney Tech High School

UCAN Country Schools Shield
Shoalhaven High School

UCAN Capital Cities School Shield
Merici College

Certificates for the Poster Competition
Moruya High School
Wellington High School
Canberra College, Weston

FROM THE PUBLISHING TEAM

Third term is well under way and here we are with issue number three. The publishing team is pleased with the diversity of contributions so far this year in terms of topics covered and the educational sectors spanned by the writers. Remember that *Circuit* is your journal and that we welcome all contributions, including letters to the editor. Contributions for our final issue for the year should be received by mid October.

Numeracy - A Hot Topic on the Political Agenda

Why the debate about 'numeracy'?

The various costly national, state and territory initiatives in mathematics education since the Hobart declaration in 1988 and the national mappings in mathematics in 1989 do not appear to have come to grips with describing 'numeracy' in a way that supports the practical work of teachers and students' understanding and learning of mathematics.

The mathematics statements and profiles, and the national benchmarks and standards currently being developed, provide extensive and detailed information about aspects of students' mathematical learning. However, the information is not sufficiently focused and succinct to facilitate students' mathematical learning, teachers' instruction and assessment planning, and parental and community understanding of what mathematical competence and numeracy look like in practice. It is difficult to envisage how teachers, students, parents and the wider community will cope with the amount of information available.

How has numeracy been described?

Sue Willis argues that the Crowther Report in the UK (1959) originally introduced the concept of numeracy, and the word itself. It defined numeracy as "a word to represent the mirror image of literacy". On the one hand, it is necessary to have an understanding of the scientific approach to the study of phenomena - observation, hypothesis, experiment, verification. On the other hand there is the need in the modern world to think quantitatively, to realise how far our problems are problems of degree even when they appear to be problems of kind.

The USA 'back-to-basic' movement (1960s) policy documentation on numeracy had as the first in the list of mathematical requirements:

"The ability to perform with reasonable accuracy the computations of addition, subtraction, multiplication and division using natural numbers, fractions, decimals and integers".

The Education Department of WA (1977) declared that the term 'numerate' is understood to mean mathematical literacy. A person is considered to be literate and numerate when he has acquired the skills and concepts which enable him to function effectively in his group and community, and when his attainment in reading, writing and mathematics make it possible for him to continue to use these skills to further his own and his community's development.

The Cockcroft Report in the UK (1982), in considering the teaching of primary and secondary school mathematics in England and Wales, emphasised the importance of 'numeracy'as the possession of two attributes. The first was an 'at-homeness' with numbers and an ability to make use of mathematical skills which enables an individual to cope with the practical mathematical demands of everyday life. The second was an ability to have some appreciation and understanding of information presented in mathematical terms, using for instance, graphs, charts, tables, or percentages.

The Australian Mathematics Education Project (AMEP, 1982) stated that

"Basic skills involve more than arithmetic skills, and understanding of mathematical concepts and processes is more important than knowledge of isolated facts and skills."

The Beazley Committee of Inquiry into Education (1984) began with a broad definition of numeracy as "the mathematics for effective functioning in one's own group and community, and the capacity to continue to use these skills to further one's own development and that of one's community", but ended up with a subset of a traditional mathematics curriculum.

The US Standards (1989) emphasise 'mathematical literacy' as much more than familiarity with numbers and arithmetic. (NCTM; 1989, 287 - 288) Romberg gives a glimpse of what mathematical literacy looks like in practice when he suggests that in increasingly technologically demanding workplaces, workers will need mathematical understanding, the ability to formulate and solve complex problems, often with others, flexible problem solving approaches, the ability to explore and create new knowledge, and to read and interpret complex information. He suggests that students will need to value mathematics, to trust their own mathematical thinking, to become mathematical problem solvers, to learn to communicate mathematically and to reason mathematically.

Sue Willis (1990) suggests that "as we enter the last decade of this century schools and the broader community must together ask what 'being numerate' really means: what kinds of mathematical outcomes are most important for daily life now and what are likely to be important in the future, what ideas, attitudes and skills really count. It is essential that we rethink what is taught in Australian mathematics classrooms in the name of numeracy, how it is taught and how it is assessed. Finally, and importantly, we must address ourselves to the question of what kind of conditions are most likely to facilitate changes in school mathematics curriculum and pedagogy".

Malcolm Swan (1990) considers that in any discussion of numeracy, it is essential to pin down exactly what we mean by the word. Most people appear to equate the word 'numerate' with ability to perform basic calculations. Others maintain a much broader interpretation equated with the basic principles of mathematics and science.

Kaye Stacey (1990) suggests that numeracy, if conceived of as the accurate performance of algorithms alone, is of little value. "Mathematical skill developed without regard to problem solving and applications is frequently not useful and hence does not contribute to numeracy. Conversely, taking serious regard of real situations where mathematical ideas arise is important not only to learn how the ideas might be applied, but also for their acquisition".

Galbraith and Chant (1993) suggest that public perceptions of the role and purpose of school mathematics "indicate an overwhelming belief in the importance of arithmetic over all other branches of mathematics".

Yasukawa et al (1995) extended the definition of numeracy to entail a critical awareness (which) enables us to build bridges between mathematics and the real world, with all its diversity. They argue that being numerate requires one not only to have this critical awareness, but also to take the responsibility to reflect on that critical awareness in one's social practice.

The Australian mathematics benchmarks project (1996-97) describes 'numeracy' in a very general way as the effective use of mathematics to meet the general demands of life at school and at home, in paid work, and for participation in the community and civic life.

Why is the concept of 'numeracy' changing?

Willis suggests there are four major reasons why the mathematical ideas and skills needed by people to function effectively in all aspects of their lives are changing:

there is a need for new types of skills

new mathematical ideas are emerging

existing mathematical ideas are assuming a new importance

there is an increasing need for people to be able to question the assumptions upon which predictions and procedures are based.

“The need for certain mathematical skills, particularly arithmetic and algebraic computation is decreasing due to the ready availability and portability of calculators and computers. At the same time, other mathematical ideas, for example those associated with probability, statistics, orders of magnitude and estimation are assuming increasing importance, as is the need to understand the assumptions upon which a prediction or procedure is based. These are essential aspects of being numerate - permeating the media as they do - and are required for all levels of personal decision-making from relatively routine choices of car insurance to more complex decisions which may be necessary in court cases or in developing a viewpoint on the most appropriate balance between development and the environment”. (Willis; 1990)

How does Willis’ view of numeracy match what is taught in schools?

“It is disconcerting, therefore, to note that the core of mathematics taught in many Australian schools is surprisingly similar in content, and often also in pedagogy, to much of the mathematics taught early in this century. Changes brought about by such movements as ‘new maths’ in the 1960s and early 1970s and ‘problem solving’ in the 1970s and 1980s appear to have had only a superficial effect on mathematics curriculum in practice, as have the advent of inexpensive hand-held calculators and the extensive use of computing technology in the workplace. The school mathematics curriculum in many ways still reflects the demands and priorities of economies based on industry and agriculture, where a majority of people are prepared for jobs in factories or on farms and an elite minority are prepared to enter professional careers”. (Willis; 1990)

In conclusion

Willis suggests that the essence of informed numeracy is that students appreciate that with mathematical knowledge and understanding, they acquire desirable power and that an understanding that mathematics can help in the solution of their problems and their own decision making. (Willis; 1990)

The first in a series of articles on numeracy by Beth Lee

An Alternative Approach to Geometry In Years 11 and 12 for General Studies

Using computer technology as a fresh approach to teaching both the geometry of design and Euclid has given students the opportunity to develop underlying spatial concepts and to use these to create intricate geometric patterns as well as more formal geometric “proofs”. The package we have used extensively is *The Geometer’s Sketchpad* (GSP), other similar packages include *Cabri Geometer* and *The Geometry Supposers*.

Why use computers in mathematics ?

The major influence of technology on mathematics education is its potential to shift the focus of its instruction from an emphasis on manipulative skills to an emphasis on developing concepts, relationships, structures and problem solving skills. (Blane, 1986)

Michael Barraclough conducted a survey of years 7–10 in Victoria in 1990 and found that half of the respondents were using LOGO in year 7 for basic geometry concepts and shape design. This continued to a lesser degree in year 8. There was also considerable use of software to assist in problem solving in years 7 and 8. This continued to a lesser extent in years 9 and 10 using packages such as spreadsheets and *Green Globs*. However, the overall picture of the schools responding was that less than half made use of computer technology. In the ACT calculators would be used in all classrooms, K to 12, but computers are still under utilised.

Access may be part of the problem, however at present many mathematics courses place great emphasis on the development of competence in algebraic manipulation and the application of routine techniques and algorithms. This philosophy assumes that practising rote procedures is a prerequisite to understanding.

Heid (1988) encouraged students to use computers for conceptual work and then carry out routine manipulations at the end of the course. These students were reported as performing better at higher-level conceptual problems than students taught in the traditional method, but were not significantly different at routine manipulation. Other reports (Demana and Watts, 1990; Machie, 1992; Ransley, 1990; Tall, 1987; and Wild and Simmons, 1993) have also shown that computer technology can enhance pre-calculus courses, encourage students to become good problem solvers and to develop understanding of algebraic concepts and procedures. In fact performance in standard tests improved although the students may have had less "practice".

Why use computers in geometry?

Students need to be able to visualise and analyse before they can make formal and informal deductions. Students should create their own geometric constructions. Working in co-operative groups, or individually, they can discover properties by looking for patterns and using inductive reasoning. A computer screen allows this quickly and more effectively than pencil and paper. Most students do not need to present formal proofs, but they do need to be familiar with the concepts of space and design which underlie them. This is the place of GSP and other similar programs in mathematics curricula. By introducing the students in years 11 and 12 to the concepts via this computer package they have new insights into the basic realities of geometry as well as the language used.

One of the most powerful aspects of GSP is it's use of "unchangeables" or "invariants". Most importantly, it gives the ability and motivation, to reason rigorously by encouraging students to make conjectures about properties of figures and constructions. This avoids a rote rehash of traditional proofs.

Students have difficulty constructing the traditional Euclidean geometry proofs still expected in many mathematics courses. The proofs are often learnt by rote with little understanding or transference to new situations. Wertheimer (1990) states that students will not be able to create formal mathematical proofs in geometry until they have an intuitive understanding of geometry concepts. This can be generalised to other areas of mathematics as well. Proofs in geometry require an understanding of the language of geometry and spatial understanding of concepts as well as an ability and the motivation to reason rigorously.

Other important features which make the computer, with appropriate software, a powerful learning tool are that they remove tedious tasks from the learner, enabling the students to grapple with mathematical ideas which were previously masked by difficult, repetitive procedures. They engage the students in an interesting task, similar to a game, providing instant feedback; and which allow experimentation, in a non-threatening environment where correction, or debugging and refinement, of their work is normal and expected. When students make mistakes using pencil and paper, they often become discouraged, their work is ruined, they feel they have failed. With the computer the mistakes can easily be eradicated. The student's sense of achievement is greatly enhanced.

What sorts of things can be done?

We have used the package GSP, not as a proof developer as such, but as a way of demonstrating to the students the concepts. As a way of enabling them to produce quality work without the tedium of seemingly limitless, repetitive processes and colouring in. It is possible to conduct experiments without pen and paper. For example, each student in a traditional class may be asked to draw a circle, with angles at the circumference and at the centre, standing on the same arc. With each individual student using a pencil and paper, each student would have a different diagram and, hopefully the class would acquire the desired associations. However, each student has only one example on their paper. With the computer they have the capacity to accurately create many circles and angles, and to actually see the ratio remaining constant as they change the angles. They can make hypotheses and test their theory. The students own exploration is an important part of the learning process. For example:

Angle(EDF) = 58 °
Angle(EAF) = 116 °
Angle(EAF)/Angle(EDF) = 2.00

Select and drag any point E, D or F.
What happens to the ratio
Angle(EAF)/Angle(EDF) ?

Figure 1. The angle at the centre is twice the angle at the circumference standing on the same arc, as demonstrated by GSP.

Ask the students to construct a square, which cannot be changed into anything but a square. They are forced to focus on the essential features of a square and how they relate to each other. This exercise emphasises the importance of invariants- things which will remain constant. What is given is of primary importance and these relationships cannot be broken. In this example the square is a quadrilateral with adjacent sides equal and perpendicular. Ensuring this gives an invariant square.

These are two examples that give an insight into the value of using computers to develop student's knowledge and problem solving skills.

GSP has also allowed the students to appreciate a creative side to mathematics. They have designed complicated multi-coloured patterns, tessellations and animations after four hours. In this process they had grasped many line, circle, and transformation properties in order to develop these. The following is an example of what students can achieve in two hours. With a colour machine and printer they can achieve even more.

Figure 2. Tumbling blocks, using tessellations.

Conclusion

The staff found learning GSP painless and it was easy to teach the students. The students quickly grasped the fundamentals and were successfully working on worksheets constructing geometric figures and making conjectures within two, one hour lessons. Students and staff found it a positive and effective experience. The computer allows an exploration more complete than pen/paper/compass/ exercises, as well as bringing in a game like process incorporating trial and error and debugging with little or no retribution for mistakes.

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Software:
The Geometer's Sketchpad Produced by Key Curriculum Press. Distributed by Edsoft.

Cabri Geometre Distributed by AAMT.

The Geometry Supposer Series. Produced by Sunburst. Distributed by Edsoft.

Margaret Rowlands and Glenda Crick
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PROBLEMS AND ACTIVITIES

Remember that we include a coding system which attempts to indicate in terms of year levels the suitability range for each item. Thus **6 - 8** suggests an item accessible to students from year 6 to year 8.

(1) A Riddle

4 - 12

Nature requires five, Custom allows seven, Idleness takes nine, and Wickedness eleven.

What am I?

(2) The Missing Money

6 - 12

A fruit stall sells two varieties of apples, 30 of each variety every day. The different apples are sold initially at 3 for 60c and 2 for 60c, giving total takings of $15. After a few days, to simplify matters the stallholder decides instead to sell the 60 apples each day at 5 for $1.20, expecting the proceeds to stay the same. Surprise, surprise! The takings are 60c short.

Where did the money go?

(3) Intuition Beware

9 - 12

(a) Simple dice games often demand that a player throw a six. Calculate the number of throws of a single die required to give a greater than 99% probability of a six occuring.

(b) A classic gambling swindle tempts punters to bet on the chance of a double six within a certain number of throws of two dice. How many times must two dice be rolled to make the chance of double six better than evens?

---

(4) One Pile (aka Unipile)

3 - 12

In its original form, probably antedating noughts and crosses and other games of position, this game was played with pebbles or counters. For younger students it is still best played initially with pebbles or counters. Later it can be converted into a pure numbers game giving excellent practice in arithmetic and raising questions of who can win and how.

(a) Take the Last

A number of counters is placed in a pile. The two players draw alternately from the pile, the object being to gain the last counter. If the first player were allowed to seize the whole pile, the first player would win; if the draw were limited to one counter each turn, the result would depend on whether the original pile contained an odd or even number of counters. Therefore a minimum draw of one counter is set, with a maximum larger than one.

Suppose that the limits are 1 to 3 counters. Play this game a few times and explain how the player who can first leave the opponent facing a pile whose number is a multiple of 4 should be able to win.

Now investigate the game for different maximum and minimum draw numbers, and different size starting piles.

Can you find a formula for winning pile sizes w in terms of the least l and most m that may be drawn at each turn?

(b) Leave the Last

Now the object of the game is to force one's opponent to take the last counter. Investigate again and see if you can come up with a formula for the winning pile size w in terms of l and m.

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(5) An Intercept Problem

10 - 12

Three touching circles of equal radius R are drawn, all centres being on the line OE as shown.

From O, the outer intersection of the line of centres with the left hand circle, line OD is drawn tangent to the third circle. Calculate the length, in terms of R, of AB, the segment of this tangent which forms a chord in the middle circle.

(6) Digit Move

10 - 12

(a) Find the smallest counting number (with no leading zeroes) which is multiplied by four when its last (rightmost) digit is moved to the front.

(b) Find the smallest counting number (with no leading zeroes) which is doubled when its last digit is moved to the front.

QUOTABLE NOTES AND NOTABLE QUOTES

A Partnership Model for Learning and Teaching

The Student	The Teacher
• Chooses work which s/he identifies as worthwhile.	• Chooses work which s/he identifies as worthwhile.
• Selects items independently.	• Selects items independently.
• Is aware of criteria for choice and shares these with the teacher.	• Is aware of criteria for choice and shares these with pupil.
• Refers back to previous work if model needed.	• Selects new learning goals which build on previous learning.
• Evaluates progress by looking at own current and previous work.	• Evaluates progress by analysis of range of evidence.
• Can present peers, parents and teachers with evidence of learning success.	• Bases new teaching objectives on valid and reliable evidence.
• Extends awareness and vocabulary of personal and curricular assessment criteria by regular reflection.	• Develops independence in the learner.

Adapted from *Developing differentiation practices: meeting the needs of pupils and teachers* by Mary Simpson, **The Curriculum Journal**, Vol. 8 No. 1, Spring 1997.

"Maths is bouncing back in public esteem. How do I know? Well it's a bit of a guess really, but I'm impressed that a book called *Fermat's Last Theorem* is occupying magnificently the number one spot in the best-seller list in Britain this week. Not far back is another book with maths as a theme: *Longitude*. And I see Penguin have an Australian novel coming out soon called *The French Mathematician*."

Robyn Williams, Ockham's Razor, ABC Radio National, Sunday 29 June 1997.

If you are caught in a downpour, it is better (i.e you don't get as wet) to run for shelter than walk over a distance of 100 metres, researchers in the US advise. The research involved detailed mathematical modelling and experimental verification, plus reference to 1995 British work. Should you decide to take an umbrella, remember that opening it will spoil your aerodynamics.

New Scientist, 29 March 1997

"Statistical thinking will one day be as necessary for efficient citizenship as the ability to read and write."

H. G. Wells at the end of last century

"For some reason I am in everyone's scientifically selected sample. I suppose this is because I am a member of a relatively small class of female mathematicians. For the same reason questionnaires are almost impossible - I always fall between the boxes. Therefore I do not fill out questionnaires."

Julia Robinson, quoted in *JULIA - A Life in Mathematics* by Constance Reid.

The number you have dialled is imaginary. Please multiply by *i* and dial again.

–attributed to MIT Telephone Exchange.

SOLUTIONS TO PROBLEMS AND ACTIVITIES

(1) Hours spent in bed, according to Mother Goose. Are our modern lives more stressful or weren't they perhaps the good old days?

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(2) Selling all 60 apples at 5 for $1.20 certainly only raises $14.40. The average price of each apple sold this way is 24c, whereas it was 25c originally, and so we are 60 x 1c = 60c short. The simpleminded 'addition logic' of the stallholder is only correct if the starting numbers of less and more expensive apples are in the ratio 3:2, which for a total of 60 apples requires 36 cheaper and 24 more expensive apples, rather than 30 of each.

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(3) (a) The chance of not getting a six in x throws of a die is $(5/6)^x$. We want the smallest integer x for which $(5/6)^x < 0.01$. This requires

$$x > 2(\log 6 - \log 5)^{-1},$$

that is x > 25, so that 26 throws are required. The solution is easily confirmed using the power button on a calculator.

Thus on one occasion in a hundred we may still be waiting for a six after 25 throws of a die. Such a wait can be a sure recipe for childish tears, so game players need to use their discretion and perhaps modify the rules of games which demand that a six be thrown.

(b) The chance of not getting a double six in x throws of two dice is $(35/36)^x$. Solving

$$(35/36)^x < 0.5 \text{ gives } x > 24,$$

so that 25 throws are needed to give a better than even chance of double six.

Many punters fall prey to the fallacious reasoning that since the probability of obtaining double six on any particular throw is 1/36, therefore 18 throws should provide an even chance. (Where is the flaw in this reasoning?) Betting on this basis means losing more than 60% of the time (show this!).

Most people are surprised that both **(a)** and **(b)** have such large yet similar answers.

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(4) (a)

$$w = (l + m)n,$$

where n is a non-negative integer. This formula is quite general, being independent of the number of counters in the initial pile. If the initial number is a multiple of $l + m$, the first player should lose; if not the first player can win by reducing it to a multiple of $l + m$.

(b) A player can be forced to take the last counter only when there is a single counter left. Hence the formula is

$$w = (l + m)n + 1,$$

where n is a positive integer. If your opponent must draw from a pile of this size, you win by always ensuring that the sum of your opponent's draw and your reply is $l + m$.

---

(5) The length of AB is 8R/5. Our method for calculating it involves analytic geometry. We find the equation of OD and the equation of the middle circle, both in polar form, then use a result from the theory of quadratics.

By Pythagoras' Theorem,
length OD = $2R\sqrt{6}$.

From here on we use polar coordinates, taking O as the origin. The equation of OD in polar form is $\theta = \alpha$, where

$$\tan \alpha = R(2R\sqrt{6})^{-1} = \sqrt{6} / 12,$$

that is $\theta = \cos^{-1} (2\sqrt{6} / 5)$.

In Cartesian form, the equation of the middle circle is

$$(x - 3R)^2 + y^2 = R^2.$$

Transforming this into polar form by putting $x = r\cos\theta$ and $y = r\sin\theta$ gives

$$r^2 - 6Rr\cos\theta + 8R^2 = 0.$$

This circle intersects the line $\theta = \cos^{-1}(2\sqrt{6}/5)$ at A and B whose moduli r_1 and r_2 respectively are roots of the quadratic

$$q(r) = r^2 - (12\sqrt{6}/5)Rr + 8R^2.$$

In fact,

length AB $= |r_2 - r_1|$

$= |$difference of roots of $q(r)|$.

But

$|$difference of roots of $ax^2 + bx + c|$

$$= a^{-1}\sqrt{(b^2 - 4ac)}.$$

Hence, length AB

$$= \sqrt{(864R^2/25 - 32R^2)} = 8R/5.$$

If preferred, the calculation can be carried through using Cartesian rather than polar equations.

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(6) (a) Assume the number we are seeking has n digits and represent it as ab with b its rightmost digit and a the integer consisting of the remaining n–1 digits.

The condition stated in the problem requires that

$$4(10a + b) = 10^{n-1}b + a$$

with $1 \le b \le 9$ and a,b both integers.
So $39a = b(10^{n-1} - 4)$ ....(*)

Putting n = 2,3,4,5, ... successively gives the equations

39a = 6b, 39a = 96b, 39a = 996b, 39a = 9996b, ...

Cancelling the common factor of 3 on both sides of these equations, our smallest solution comes when b = 1 and 33…32 is first a multiple of 13.

Division gives

$$33332 = 13 \times 2564 = 1 \times 99996,$$

so a = 2564 and b = 1 appear to give the smallest solution to our equation.

But on checking this solution we find that $4 \times 25641 = 102564$. Our apparent solution does not work, since a leading zero must be included.

Other solutions of equation (*) for a in order of increasing size, are generated by calculating

2564 b for b = 2,3,…,9.

We have successively,

$2 \times 2564 = 5128$ and $4 \times 5128 = 205128$,

$3 \times 2564 = 7692$ and $4 \times 76923 = 307692$,

and

$4 \times 2564 = 10256$ and $4 \times 102564 = 410256$.

Thus the smallest solution with no leading zeroes is 102 564.

(b) The same approach works again, but this time the key equation is

$$19a = b(10^{n-1} - 2)$$

and the arithmetic is longer and more tedious.

The smallest solution is

105 263 157 894 736 842.

There is clearly room here for more investigations.

THUMBNAIL REVIEWS

Readers are welcome to contribute to this section. Reviews can cover books, periodicals, videos, software, CD ROMS, calculators, mathematical models and equipment, posters, etc.

Maths for Work: Intermediate
Published by Cambridge University Press, 1996, 107 pages, $12.95.

Produced as part of The School Mathematics Project by a team of writers, this is a workbook designed to be placed in student hands. It aims to teach and reinforce core mathematics skills across number, shape, space and measure, and data handling by presenting self contained case studies followed by a separate section on the skills themselves. Solutions to all questions and exercises are included. The work is pitched somewhere around Year 10 to Year 12 level in the Australian context.

The eleven case studies describe practical situations, present information and end with specific questions and mathematical activities. The topics are: Tin can labels, Food additives, Adventure park, Crisp boxes, Australia, The painter's problem, Landscaping, House prices, Relative business, Cycle computer, Planning permission. The core skill techniques section covers number work, probability, estimation and checking, formulas, units of measurement, perimeter, area, volume, plans and drawing, data collection, discrete and grouped data, mean, median, mode, range, and graphs. Explanations are clear and direct and the practice questions straightforward.

The back cover blurb talks about the case studies developing "core skills in a variety of vocational contexts". I take this to be new-speak for what in times past would have been described as practical applications of basic mathematics. The British already seem to be a fair way down the vocational road in schools, and this material gives some indication where we might be heading if our Federal Government has its way.

British contexts, vocabulary, scenes and money units probably prevent this book being used in class sets in Australia, but it would make a useful addition to high school and college library and mathematics staffroom collections.

Peter Enge

Board Games Round The World
A resource book for mathematical investigations
by Robbie Bell and Michael Cornelius
Published by Cambridge University Press, 1988, iv + 124 pages, $26.95

This useful book, reprinted in 1993, offers a selection of nearly sixty board games, chosen for their interest plus the possibilities they provide for motivation of students and investigation and analysis by them. The universal appeal of board games, perhaps even greater in this computer age, spans the proverbial age range from well before nine to beyond ninety. The snippets of historical and cultural background included here, plus occasional photographs of game boards and associated artefacts, give prominence to cultures and historical periods which do not often connect with mathematics classrooms.

The games are divided into five groups: Games of position; Mancala games; War games; Race games; Dice, calculation and other games. Of course, some games overlap the catgegories. A sample to indicate the range of games might include noughts and crosses, three men's morris, seega, mu torere (a Maori game), pong hau k'i (from China and Korea), solitaire, mancala (East and South Africa), draughts, tablut (from Lapland), vultures and crows (from India), tabula (played by the Romans), chasing the girls (Iceland), the snake game (dated from Egypt before 3000 B.C.), lu-lu (Hawaii), and rithmomachia (from the eleventh century). The games have been chosen to produce situations where mathematical investigation can take place. Of course most of the games in this collection can be played and investigated at different levels and ages.

Each chapter ends with game by game suggestions for investigations. The suggestions are little more than starting points and there is plenty of room for students and teachers to develop their own lines of thought. Notes on the investigations are included at the end of the book. Also included is a list of references and suggestions for further reading, as well as an index. Most of the material has been tried out with students in schools and the book includes a chapter dealing specifically with games and investigations in the classroom. It includes examples of worksheets, samples of student work, and teacher and student reactions, reflections and ideas.

This book shows how board games can be a wonderful source of fun and enjoyment in teaching and learning, and simultaneously increase student appreciation of the scope of mathematics. The one essential is that students must actually play each game before they analyse or investigate it. Certainly this is a book which should be workshopped, discussed and available in every primary school, high school and secondary college.

Peter Enge

At Home in the Universe
The Search for Laws of Complexity
by Stuart Kauffman
Published by Viking, UK, 1995.

Kauffman, who works at the Santa Fe Institute searching for patterns in complexity, believes that the grand architecture of nature expressed in the biological world arises through more than natural selection sifting through random mutations. He argues that while natural selection is important, it is underpinned by self organisation: in the living world, selection has always acted on those systems that exhibit spontaneous order. These principles of self organisation - laws of complexity only now beginning to be uncovered and understood - operate from the very beginnings of life. Molecules of all varieties join in a metabolic dance to make cells, cells interact with cells to form organisms, organisms interact with organisms to form ecosystems, economies and societies.

Over the past three centuries science has been predominantly reductionist, attempting to break complex systems into simple parts and then those parts, in turn, into simpler parts. The reductionist program has been spectacularly successful at using information gleaned about the parts to build a theory of the whole. The problem is that complex wholes may exhibit properties not readily explained in terms of features of the components. Here Kauffman outlines his search for laws of complexity that govern how life arises from a soup of molecules into today's biosphere. If he is correct, this underlying order, honed by selection, truly makes us at home in the universe.

Kauffman's discussion includes several order of magnitude calculations and arguments based on counting and probability, covering autocatalysis, self sustaining reactions and state spaces in various situations. Universal molecular toolboxes, random chemistry, fitness landscapes, learning curves, sandpiles and self-organised criticality, along with the edge of chaos are all considered. There is a brief bibliography, plus an index. Boolean networks are used to exemplify the emergence of spontaneous order. This is an original exploration of the dynamics of the living world and mathematics is Kauffman's major tool.

Peter Enge