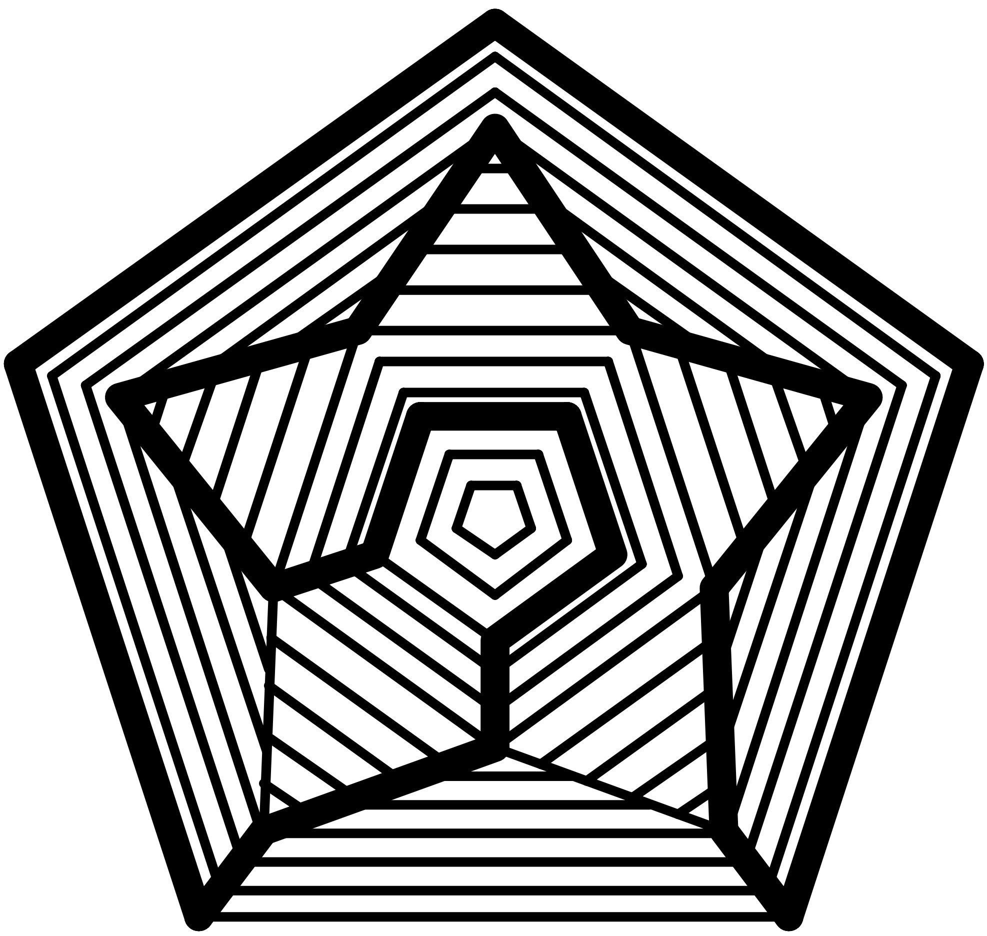
CIRCUIT

Published by the Canberra Mathematical Association

1997 Number 4



FROM THE PRESIDENT

1997 President's Annual Report

My thanks to all those who have contributed to making 1997 a successful year for CMA, in particular the 1997 Council members. The CMA relies on school and tertiary personnel to share their ideas and expertise with colleagues, as well as to help administer the ongoing business of CMA. Please let us know if you would like to play a part in CMA in 1998.

Our membership has increased across most sectors this year. A number of primary schools have taken up institutional membership this year and we hope more will do so. We seek to increase membership further next year so that CMA may better support mathematics teaching and learning in the ACT and achieve its goals:

- to enhance the mathematical confidence of teachers
- to act as a network and forum for discussion of issues related to the teaching of mathematics
- to provide professional stimulation and resources for mathematics teachers and educators
- to encourage the learning of mathematics by fostering a range of activities for students of all ages
- to act as a lobby for mathematics educators in the ACT
- to assist teacher networks across schools and sectors
- to foster the love of mathematics
- to foster better mathematics teaching and learning

1997 Council

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Beth Lee

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Heather Wardrop, Kambah High

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Kevin Taylor, Boys' Grammar School
Margaret McLaughlan, St Francis Xavier High
Andy Wardrop, Calwell High
Steve Thornton, Australian Maths Trust
Glenda Crick, Lake Tuggeranong

1997 Activities

Thank you to all those who have led workshops, those who have attended CMA activities or contributed to *Circuit* in 1997.

This year CMA has provided a range of opportunities for teachers to network together and gain some professional development. These have included a problem solving workshop, an open night at the University of Canberra, a language across the curriculum workshop, seminars on numeracy, a workshop on calculators for teachers of early childhood classes, a workshop on mathematical modelling for primary and secondary teachers, two workshops on Maple software, a workshop on integrating studies of Asia into mathematics, and a presentation on probability, Fibonnaci and topology.

Our special thanks go to the following people for the contribution they have made to these sessions:

Associate Professor Di Siemon, Professor Robert Bartnik, Malcolm Brooks, Peter Brown, Roger Curnow, Ian Lisle, Peter McIntyre, Steve Thornton, Heather Wardrop, Jan Macdonald, Naomi Hurst, Margaret Rowlands, Annabelle Cassells, Martin Ward, and Yvonne Williams.

A large number of primary and secondary teachers took the opportunity to attend the 1997 MANSW conference held in Canberra in September. It was great to see so many ACT primary and secondary teachers participating in, and leading, sessions.

Margaret Rowlands, Steve Thornton and Annabelle Cassells attended sessions at an OECD conference in September. Annabelle also attended a numeracy conference in Perth earlier this year.

Circuit

Very special thanks are due to Peter Enge, Kevin Taylor and Margaret Rowlands for their hard work this year in revitalising *Circuit*. They have encouraged contributions, edited, printed and collated articles. We encourage all teachers to contribute to the journal through articles and letters to the Editor.

CMA Home Page on the Web

A thank you to Warren Atkins for creating our home page and keeping it updated. If you have items appropriate for the Web, you could forward them to Warren at UCAN. His email address is:

atkins@education.canberra.edu.au

The CMA Home Page address is:

http://education.canberra.au/projects/cma/home.html

1998 - The Year Ahead

Continuing the partnerships in ACT mathematics education

In 1998, the CMA hopes to draw again upon the expertise of a range of ACT educators from all sectors of ACT education, as well as to link to existing mathematics initiatives and programs at national and local level.

In 1997 the Australian National University and the University of Canberra have continued to play a key role in supporting mathematics education in schools. Personnel from these tertiary bodies have led workshops for teachers and played a major role in student enrichment activities. We thank them for their ongoing support and look forward to working with them again in 1998. We hope to further our relationship with Signadou Catholic University and the Australian Defence Force Academy in 1998.

This year the CMA has been very fortunate to have Steve Thornton on Council. A special project officer with the Australian Mathematics Trust, and Secretary for the national Association of Mathematics Teachers, Steve has been very willing to add his experience and enthusiasm to the team. He is keen to work with schools and teachers in classrooms, and you can contact him on 6201 2017.

CMA professional development activities.

The Council has begun initial planning for a 1998 PD program. Our PD will seek to link to local, national and international mathematics initiatives and priorities. We plan to offer workshops to meet what teachers have identified as their areas of interest, and to use a variety of approaches: clusters (northside and southside), short afternoon sessions, mini conferences for special interest groups eg early childhood, primary, Y7 - 10, workshops on particular teaching approaches, resources and topics, sandwich courses with between unit activities, a forum with the ACT Dept of Education for two-way exchange of information and discussion on major issues such as assessment, and practical sessions such as resource sharing.

The PD emphases in 1998 will be to maintain and extend: tertiary links and input; student enrichment activities for students of varying mathematical abilities; teacher enrichment workshops on resources, new teaching ideas and topics; links to national initiatives eg numeracy, assessment; links between mathematical and language development; curriculum writing in mathematics to incorporate technology, extension materials, problem solving.

We plan to incorporate resources, teaching approaches and assessment as integral components of all workshops.

Teachers have identified technology as an area where they would like support. We have thus set up a draft program for 1998 as follows. Please watch for ads for these in *Circuit* or the Bulletin.

March	
Graphic Calculators	Years 9 to 12
May	
Early Childhood Calculators	K to 2
July	
Geometers' Sketch Pad	Years 6 to 12
August	
Developing teaching ideas for Maple	Years 11 to 12
October	
Spreadsheets	Years 7 to 12

Social activities.

The CMA Dinner and AGM in November each year is a great opportunity to meet colleagues and have a chat. Please look for notices about the dinner for 1998, also for other occasions such as another Casino Night (very successful in 1996). Council would welcome suggestions for other social activities, so please let us know if you have some ideas.

A tribute to Belle Gillies

Belle was a mathematics teacher for many years in Canberra. She was an active member of CMA after it was established in 1967, and her sound common sense and professional commitment were much valued. Belle continued to attend CMA dinners until a couple of years ago. She died in Canberra last week aged 76. CMA sends its condolences to her family.

Beth Lee
November 10, 1997

FROM THE PUBLISHING TEAM

Christmas is nearly upon us and here we are with our final issue for the year. General spring and early summer gardening activities and the increasing tempo of life in educational organisations in fourth term are our feeble excuse for this issue being a few weeks later than we meant it to be.

Once again we thank our contributors who have done us proud with their efforts. Members of the Association and others have submitted diverse and interesting pieces. The hopes expressed in this column in February in our first effort as Publishing Team have been vindicated. Undoubtedly, this year *Circuit* has benefited from Canberra's "wonderful concentration of mathematical and education expertise and institutions".

May we hope that the flow of contributions continues into 1998. Bear in mind that the authors of articles are always open for feedback, and that the publishing team welcomes comments and any contributions in printable form. (Address contributions to a member of the *Circuit* publishing team - see inside front cover for contact details).

Over the summer season, perhaps some of you will be able to turn ideas into *Circuit* articles, problems, activities, reviews, critiques or even letters to the editor. Should CMA consider awarding a prize for the best contribution to *Circuit* each year?

Peter Enge and Kevin Taylor would like to gratefully acknowledge the efforts of their wives, Rosemary and Gillian, in assisting with the input and formatting of *Circuit.* Without the generous help of these two CMA supporters *Circuit* would not have appeared this year in the form that it has. The publishing team also thanks all those CMA Council members who assisted with the actual printing and mailing out of *Circuit*.

Have a peaceful, safe and relaxing summer break.

Numeracy - A Hot Topic on the Political Agenda

My article on numeracy in the August Circuit outlined the various ways that numeracy has been defined and described, and the different emphases and foci reflected in these definitions. There was usually agreement that the content for numeracy was more than arithmetic, but not a lot of agreement on what numeracy content would include.

This second article summarises some of the common threads evident in various reports on numeracy since 1959.

Considered together, the various reports mentioned in my previous article (and others not referred to there) highlight the need for:

- agreement on why numeracy is important;
- a clear statement of what numeracy looks like in general;
- an understanding of the sorts of qualities that a numerate student will demonstrate;
- a statement of the more specific skills, processes and strategies that numerate students will demonstrate.

Numeracy is Important for People to:

- understand and interpret many aspects of their world;
- participate effectively in society;
- be able to serve the national interest;
- further their own and their community's growth and success.

An Articulated, Shared Understanding of Numeracy:

- provides a basis for discussion and debate on school mathematics;
- facilitates identification of changing educational demands and ways to address them;
- helps teachers make decisions about the incorporation of new topics and changing emphases in school mathematics;
- assists in establishing appropriate criteria for judging the quality of student achievement;
- recognises and values the increased use, relevance and importance of mathematics in our everyday lives.

Qualities a Numerate Student should Demonstrate in Dealing with Mathematics are:

- familiarity with mathematical knowledge, processes and skills;
- facility in using mathematics;
- flexibility in selecting and using mathematical ideas, knowledge and processes;
- mastery;

- confidence and a critical awareness of what they know and can do in mathematics;
- an at-homeness with mathematics.

Numerate Students should be able to:

- read, understand, interpret and write mathematical information;
- automatically recall;
- construct, check, condense and calculate;
- ask appropriate questions so that they can support or refute mathematical information;
- find and evaluate assumptions and constraints in mathematical arguments and proofs;
- collect, analyse and organise quantified information;
- extrapolate and interpolate from given mathematical information;
- explain and justify their results;
- develop, select and apply appropriate mathematical techniques;
- measure, estimate and approximate.

My next article will try to give an overview of functional numeracy at various levels.

The second in a series of articles on numeracy by Beth Lee

Why Would Anyone Want to be a Teacher?

The Third International Mathematics and Science Study (TIMSS) has raised more issues than those concerning student performances in Mathematics and Science. One section of the study involved a questionnaire given to the teachers asking about career choices they have made and also whether teachers feel they are appreciated by society and students. Of the countries questioned, Australia had the second lowest rate of appreciation levels. Do teachers in the ACT follow the Australian trend or do they feel they are appreciated by society and students

Although no official numbers are available, there are approximately 250 teachers of Mathematics in the ACT. These are people who teach in our high schools and colleges both private and public. They are people who instruct more than twenty-eight thousand students in the ACT. They are not merely people who teach our children how to add and subtract, but they are also the people responsible for moulding the attitudes of our future generation towards Mathematics and enlightening students to the beauty of Mathematics.

Unfortunately, the number of qualified Mathematics teachers appears to be dwindling in the ACT. This may be in part due to the comparatively low income of the teaching profession on the whole, the stress associated with the job, the increasing amount of paper work and administrative tasks required of the classroom teacher or it may be the fact that teachers do not feel appreciated by society.

In 1996, the Australian Council for Educational Research (ACER) published a book titled *Maths & Science On the Line* (Lokan, J., Ford, P., Greenwood, L.) which outlined the section of TIMSS which dealt with teaching as a career. This questionnaire contained several questions relating to teachers' opinions about teaching as a career choice. The questions were:

1. Was teaching your first choice as a career when university or teacher training?
2. Would you change to another career if you had the opportunity?
3. Do you think that society appreciates your work?
4. Do you think that students appreciate your work?

These questions were included in an optional section that could be omitted if desired. Fourteen countries declined to participate in this section. There were some teachers in Australia who refused to answer these questions.

The results from these questions show some interesting facts. Australia and New Zealand stand out as the two countries with the highest percentage of teachers who would rather change to another career. Teachers from Latvia, England and Australia had the lowest level of belief that society appreciated their work. However, the teachers believed that their students appreciated their efforts more than society. Australia and Latvia still had the lowest level of belief that students appreciate their work.

As a result of this questionnaire I decided to conduct a survey of my own, this time looking for why teachers did or did not think they were appreciated by society and students. I sent surveys to fifty Mathematics teachers in the ACT public and private schooling system. Of the fifty sent out forty-four were returned, an 88% response rate. Seventeen replies came from public colleges, fifteen from public high schools and the remaining twelve from private colleges. The teachers involved seemed to be quite happy to vent their opinions. While many teachers gave detailed answers, some respondents still answered simply Yes or No, but I was still able to get the overall feeling of the questionnaire. Although the numbers involved in this questionnaire were relatively small, I decided to convert all fractions to percentages so that easy comparisons may be made.

The first two questions related to the type of school the teacher was currently teaching in. This question was included to give me an idea of what type of educational institutions the questionnaires were coming from. The third question asked how many years experience the teacher had. This question provided some interesting results. Ninety-four percent of public college teachers have been teaching for 15 years or more whereas only one third of public high school teachers have been teaching for a similar length of time. Sixty percent of public high school teachers have been teaching for less than 10 years. However there were no respondents from the public college sector who had been teaching for such a short period of time. Finally, eleven out of the seventeen public college school teachers have 20 or more years teaching experience but only one of the fifteen public high school teachers had an equivalent length of teaching experience. The figures lead us to some conclusions. Firstly, there are obviously more experienced teachers in the public colleges. Secondly, the lack of experienced teachers in the high school system could indicate a high burnout or departure rate of these teachers. Are the colleges becoming an environment for teachers to wait out their retirement or is it a method of rewarding experienced teachers by getting them to "do time" in high schools to learn their trade and then move onto college?

Questions four and five asked if the respondents wanted to do teaching when they left school and if they were satisfied with teaching as a career. Fifty-two percent of teachers did not want to teach when leaving school, however a high seventy-three percent of those questioned were satisfied with teaching as their chosen profession. It is noteworthy that only forty-four percent of teachers with twenty or more years experience found satisfaction with their career. Many respondents indicated that they actually had higher career aspirations than teaching, for example as an engineer, chemist/pharmacist or surveyor, however due to circumstances they were unable to fulfil these aspirations. The fifty-two percent of teachers who did not want to teach when leaving school might have enrolled in a university degree, for example Bachelor of Arts, then drifted into a post graduate teaching course (many with a scholarship!) because there was nothing else they could have used the degree for.

The sixth question was included in the questionnaire to determine whether the teachers were interested in teaching for only a short time or whether they wanted to continue until retirement. A staggering eighty percent of those teachers with less than five years teaching experience and fifty-seven percent of teachers with between five and ten years experience could not see themselves teaching in ten years time. This early departure of teachers from the profession could be a result of many factors including insufficient support from experienced teachers in the high school sector, lack of professional development to help these teachers cope with the stresses of teaching, teaching not meeting their expectations or seeing teaching as merely a stepping stone to other careers. This exodus of young teachers will lead to a dire shortage of qualified mathematics teachers early next century. We will soon have people with no knowledge or background in mathematics teaching our students.

Question seven asked the respondents whether they thought society appreciated their work. A resounding eighty-six percent of teachers thought they were not appreciated by society. The largest category of teachers who felt that they weren't appreciated were public school high school teachers (87%), followed by private school teachers (83%) and then college teachers (71%). Many teachers were quite passionate with their response to this question. It was as if they had been waiting for such an opportunity to arise to vent their opinions. The reasons why teachers thought they weren't appreciated varied, both in length and detail. Those teachers with only a few years experience gave short answers and their reasons were basically that society does not realise just how much work teachers have to do outside school hours and that society views them as "lazy workers with too many holidays". All these teachers (that is 100%)

thought that society does not appreciate their work. Eight-six percent of teachers with between five and ten years experience thought they weren't appreciated by society and mentioned the above reasons and included such factors as low pay, being blamed for social ills, budget cuts, high stress levels and limited resources as possible reasons why society does not appreciate them. Eighty percent of those teachers with between ten and fifteen years experience and seventy-two percent of teachers with between fifteen and twenty years experience thought they weren't appreciated by society. The reasons given for the lack of appreciation were very similar to those in the previous two categories of teachers.

Seventy-five percent of the most experienced teachers expressed the thought they weren't appreciated by society. This category of teacher gave the most detailed and at times emotive answers. Once again the reasons for this lack of respect were similar to previous categories of teachers however there were some teachers who also included reasons such as low TER university entrance scores and lack of status for our knowledge and expertise.

The consensus of opinion is that the public thinks that teachers work from 9 until 3.30, receive long holidays, good pay and are to blame for low numeracy/literacy rates and every other ill of young society. Further, the respondents suggest society does not realise the long hours put in by the average teacher outside school time, the stresses associated with teaching adolescents (many of whom do not want to be there!), the demands put on classroom teachers with regard to administrative, counselling, classroom and management duties, the difficulties of working with limited resources and the comparatively low wage for those at the top of the teaching scale.

Question eight asked the teachers if they thought that the students appreciated their efforts. Fifty-seven percent of respondents answered in the positive. Once again, surprisingly, it was the high school teachers who felt most strongly with 12 out of the 15 respondents answering in the positive. Seventy one percent of the public college teachers felt appreciated by the students. The most surprising statistic is that only 2 of the 12 private teachers felt fully appreciated by their students. Once again the reasons for this lack of appreciation varied according to the number of years teaching experience. Those teachers with less than ten years experience cited reasons such as lack of understanding of how much time and effort actually goes in to preparation and teaching and that many students feel that teachers are only in this profession because they couldn't get a "real job". Those teachers with more classroom experience thought that generally most students did appreciate their work, but there was a small pocket of students who were not motivated, too ego-centric, too ready to blame teachers for their own failings and who didn't treat teachers with respect. This type of student is unfortunately found in our schools, public or private, high school or college and often they tarnish the attitudes and behaviour of otherwise good students.

Question nine asked if you would encourage someone to become a mathematics teacher and the results were split fairly evenly. Fifty percent of respondents answered in the affirmative, forty three percent answered in the negative and seven percent had mixed feelings. As long as people keep encouraging others to pursue a career in Mathematics teaching then perhaps the shortage of qualified teachers will not be as drastic. There needs to be a push from schools, universities and governments to encourage suitable applicants into this career.

The results of this questionnaire raise some important issues that need to be addressed. Firstly, there is obviously a problem in our schools; not just with the badly behaved student or the limited resources or with the politics of education but with teacher morale. Teachers are undoubtedly very dedicated professionals. How many other people would stay in a profession with so little appreciation from their clients and the public, where they are constantly being blamed for the ills of society and where budget cuts mean increased administrative tasks? The fact that nearly three-quarters of those teachers surveyed indicated that they were satisfied with teaching, yet 86% of respondents felt unappreciated by society would seem to indicate a great deal of dedication amongst our profession. This problem of low levels of teacher appreciation must not be ignored. Education departments, schools, unions and the media must work together to inform the public of the real situation in our schools; to enlighten those in the dark about the dedicated professionals teaching our young.

Secondly, this survey points out the ageing teaching population in ACT colleges. There appears to be a real shortage of experienced teachers in the high school system. Every opportunity must be made to make college teaching accessible to all teachers, not just those select few. Younger, less experienced teachers have a lot to offer in the new idea and enthusiasm department. If these teachers are not being given the opportunity to use these skills in the college sector, but rather being kept in the high school system where there are more discipline problems and less scope to use their expertise, then these teachers will simply leave the teaching profession altogether. This will lead to a mass shortage of qualified Mathematics teachers in our schools, a problem which is already prevalent in many schools in the territory. As shown in the outcome of question six eighty percent of those teachers with less than five years teaching experience and fifty-seven percent of teachers with between five and ten years experience could not see themselves teaching in ten years time. Consequently, there will be many unqualified Mathematics teachers in our schools. These teachers may well have only limited experience themselves with the subject and yet they will be instructing our students in the concepts of Mathematics.

Robyn Gibson, final year M.Ed. student in 1997 at University of Canberra Phone 6288 5204

The AAMT Mailing List Community on the Internet

Introduction

The real power of the internet for teachers lies in its role, not as a "look it up" place, but as a place where people go to communicate.

Do you sometimes feel isolated? Do you have questions you need help with? Would you like to talk about teaching mathematics?

The AAMT has set up a mailing list community for mathematics teachers.All you need is an email address and you can be a part of this initiative.The list is open to teachers at all levels across Australia and beyond and is devoted to teachers of mathematics sharing their views, questions and resources. Debates and discussions have included topics such as use of the internet, the role of algorithms in a technological era, and the problems and possibilities of teaching periods of 90 minutes duration. There have also been a number of mathematics problems shared and attempted solutions argued about.

To subscribe, just drop an email to Will Morony at **wmorony@nexus.edu.au** and you will soon be part of this new list community. You can of course,unsubscribe at any time.

A Sample Interaction from the AAMT Mailing List Community

The following extracts were part of a recent discussion on the use of graphing calculators in assessment. Contributors to the discussion were from different States of Australia and their contributions have been edited slightly for printing here.

Message 1:

'I would like to start a discussion about the implications of graphics calculators in assessment and use a summary of the ideas to present to an assessment panel with which I am associated. Your ideas will make a difference so please add your thoughts.

Here are some of mine to get the discussion going.

Assessment needs to be reliable, valid and equitable. My interpretation of these terms in this context follows.

Reliable means the results are reproducible and are some "true" measure of what is being assessed. In most assessments such as tests or examinations, we are looking to measure the performance of the individual on some syllabus criteria. Assuming that the syllabus criteria encourage the use of graphic calculators, a reliable assessment should include their use.

Valid means that assessment reflects the learning that it is attempting to measure. If graphics calculators are encouraged or required in the learning,then they should also be used in the assessment.

Equitable means that one candidate should not be advantaged over another because of the technology available to them. This is particularly so if the technology is not essential to the subject however useful it might be.

Technology changes the nature of what is assessed. A text capable calculator such as the TI 92 allows students to bring electronic summaries of work as well as programs and pre-installed graphs. I am not comfortable with the option of pressing the reset button as students' graphics calculators may contain several hundred hours worth of programs and other intellectual effort. The personalisation of the calculator by the user is something I believe we must respect and do not have the right to interfere with.

Options such as open book exams level the playing field for students without text capable calculators and probably are appropriate as the learning focus moves from content to processes.

Enough of the starter ideas. Please react to the above and add more big picture ideas if you thing the reliability, validity and equity dimensions do not cover the debate adequately. A useful debate will have breadth as well as depth on each dimension and lead towards functional policy and procedures for the use of graphics calculators (and possibly hand held computers) in examinations.

Thank you for your anticipated input.'

Message 2, Replying to Message 1:

'As this is an AAMT list, a good place to start might be with recent papers in AAMT journals. Many of the assessment issues are raised in these two papers:

Graphics calculators, equity and assessment, ASMJ, 8, 2, 31-44 (1994)

Graphics calculator use in examinations: Accident or design?, ASMJ, 10,1, 36-50 (1996)

We would be interested in reaction to these, and hope that they help discussion.'

Message 3, Replying to Message 1:

'The original post asked if other dimensions to assessment (in addition to reliability, validity and equity) might be applicable to the debate about the use of graphic calculators in assessment. The following then should be viewed as an addition to the original discussion.

One other dimension might be that of "strategic rationality". My understanding of this term is that we want assessment to promote those attitudes and practices that are held important by the discipline. In the case of mathematics I would translate this into:

Choosing assessment tasks that:

1. engender, in the students, a positive attitude to mathematics;
2. illuminate the nature and/or usefulness of the discipline of mathematics;
3. encourage the students to think and act mathematically.

If you accept that strategic rationality is an important dimension to assessment then the question has to be asked "How well do examinations do in terms of this dimension?"

I came across the idea of strategic rationality in the following reference:

Andresen, L.W. (1993) *Three Criteria of a Good Assessment System*, Professional Development Centre course paper, University of New South Wales.'

Message 4, Replying to Message 3 (the same author as message 2)**:**

'I have three reactions to this question:

1. It seems to be three dimensions and not one, as described above. Although clearly not independent, they are certainly different.
2. Examinations do extremely poorly on all three, except for an insignificant minority of (very successful) students, who seem able to endure them without lasting damage. Usually, the more important the examination, and the older the students, the worse the examinations do, in my experience. While I understand the practicalities of examinations for large groups of students at once (such as external examinations and large undergraduate courses), I am a bit more mystified at the continuing use of examinations with smaller groups, such as smallish undergraduate classes. Yet in my experience, mathematics seems to prize and take notice of examinations more than most other disciplines.
3. We need to be careful not to stray too far from the original topic, related to the use of personal technologies in assessment, and get too distracted by

the examinations issues. IMHO, if we must have examinations,it doesn't make much sense to exclude from them what are rapidly becoming tools of the mathematical trade.'

And the debate continues!

Steve Thornton
Director Teacher Enrichment Program
Australian Mathematics Trust
University of Canberra
tel 06 201 2017
email SteveT@amt.canberra.edu.au

The Mathematics Challenge for Young Australians Enrichment Series

In the days when the philosophy of mathematics teaching has changed, the term mathematics enrichment takes on a new meaning. Certainly, in an attempt to make mathematics learning a more positive experience for more students of all standards, the curriculum has changed in a way that less of the structural, rigorous parts of the subject survive at the school level. Of course this is a subject in itself for much debate, but I do not wish to enter that here.

What we do notice however, is that there is an increasing market for what we call mathematics enrichment. Who would have thought, for example, 30 years ago, that each week of the school term, up to 50 Canberra high school students would voluntarily turn up after a full week of school to an extra session of mathematics on Friday nights (as they certainly do now)?

There are many definitions of "talented" and "elite". For the sake of this article I am going to assume that "elite" means in the top 2% of the population and "talented" means in the top 10%. The Friday night groups which run during term time in Canberra can generally be considered to be suitable for the top 2% (although no-one is turned away). Obviously many students in this centile ranking choose not to go. Teachers who do have students in that ranking generally have little difficulty in identifying them and we are fortunate that in Canberra there are enough sufficiently interested academics from both ANU and UC to be able to cater for those who turn up.

In this article I wish to discuss the more general issue of the "talented" student, who may not be catered for in the Friday night group. The experience of the Australian Mathematics Trust is that there is a demand from these students for extension work. We now have a regular enrolment of about 6,000 students annually throughout the country doing one of the Mathematics Challenge for Young Australian Enrichment Courses: **Euler**, **Gauss**, **Noether** or **Polya**. Even these have come so far from only about 20% of Australian Schools, i.e. those schools with teachers who feel they have the resources to be able to support the students (this support being most desirable). The Trust is attempting to help teachers develop these resources through the Australian Mathematics Teacher Enrichment Program, directed by Steve Thornton.

There are occasionally pressures, especially from parents, to accelerate such students. Acceleration has some drawbacks, both socially and academically when subjects get out of phase. Mathematics is a vast subject, and the enrichment series, written by some of Australia's leading educators, gives the talented student an opportunity to tap into it and broaden their mathematical experience.

Each of the four non-overlapping courses caters for a different standard, and students in Years 7 to 10 can take a different one each year, preferably in the order Euler, Gauss, Noether and Polya. The work in each book is designed to take about 6 months' study, generally about April to October, followed by submission of problem solutions after which the student is recognised with a certificate for their work. So what is in these books? They range from extending class-room techniques to developing skills in various problem solving techniques. A brief survey of the courses shows the following topics:

Euler: Some number theory (primes, composites, LCM, HCF, congruences), sequences, a little geometry and discrete maths (counting, pigeon-hole).

Gauss: More geometry, spreadsheets, diophantine equations, more counting and continued fractions.

Noether: Expansion and factorisation, sequences and series, number bases, inequalities, methods of proof, circle geometry.

Polya: More expansion and factorisation, polynomials, more series and inequalities, advanced geometry.

Whatever the case many students are enjoying this work and benefiting.Those who do them will be even better equipped for their eventual university study, particularly in the scientific, engineering and technological area. Whatever the case many students are simply enjoying the challenge of new problems, in the same way that they enjoy the challenge of sport, music and the arts.

Peter Taylor
Australian Mathematics Trust
and University of Canberra
email: PeterT@amt.canberra.edu.au

QUOTABLE NOTES AND NOTABLE QUOTES

An Examination Parable

A student was sitting a degree exam - one that had to be passed. After three hours it was pens down time, but the student hadn't finished and carried on writing. Next a warning of disqualification was issued unless the writing stopped. Nevertheless the student continued, finished 10 minutes later and walked up to the invigilator to hand in the paper. The invigilator refused it.

The student, cool as a cucumber, asked: "Do you know who I am?" The invigilator was unmoved and reiterated that the student was disqualified. The student again asked whether the invigilator knew his identity. Again the invigilator replied: "No, and I don't care." The student picked up all the papers on the invigilator's desk, added his own, threw them up into the air and walked out. The student passed.

Feedback, New Scientist, 27 September.

Is Mathematics a Science?

Apparently Not!

"For most countries, even though gender differences were minimal in mathematics, they were pervasive in science. Boys outperformed girls, particularly in physics, chemistry and earth science."

Third International Mathematics and Science Study, the Middle School Years, 1997.

The Poetry of Mathematics

"Mathematics and poetry have a lot in common ... they are both metaphorical ... they are ... concerned with the precise use of images in one sense ... metaphors in relation to poetry and symbols in mathematics. The equal sign in mathematics is certainly equivalent to the metaphor in poetic language."

Tom Petsinis, Mathematics Lecturer at Victoria University of Technology, discussing his recently released book *The French Mathematician*, a "fictional biography" of Evariste Galois.

Limericks

We would welcome readers' contributions in this form with mathematical themes - an excellent holiday activity for spare moments. Suitable efforts will be published in future editions of *Circuit*. Here, to whet your appetite, are some taken from the American Physical Society Physics Limericks Archives on the World Wide Web at

http://www.aps.org/apsnews

A simple improvement I've found:
Let troublesome numbers be round
And both *pi* and *e*
Be equal to three
And *kgm* = 2 *lb*.

Edward H. Green

A lady was once on a horse
She rode it with much too much force
Her scalar went vector
The horse tried to protect her,
But the impulse had killed her, of course.

Valerie Lesh

The skill to do math on a page
Has declined to the point of outrage.
Equations quadratica
Are solved on Mathematica,
And on birthdays we don't know our age.

Harvard team

The Perils of Education and Information

"Thank God there are no free schools or printing ... for learning has brought disobedience and heresy into the world, and printing has divulged them ... God keep us from both."

Sir William Berkeley, Governor of Virginia, died 1677.

Mathematicians at Work - A Contrast

"Mathematics had always been the jewel in the Cambridge academic crown. In the good old days you had to acquit yourself first in mathematics before you were permitted to proceed to other subjects of study... At the very summit (of competition) were dazzlingly able students... With respect to the problems which constituted a mathematics examination, it was said of Maurice Pryce that every time he looked at the question paper you knew he'd done another one, which was surely depressing for the other candidates who happened to be sitting near him."

Fred Hoyle in *The Small World of Fred Hoyle*

Before appointing Julia Robinson to a part-time job as a teaching assistant in statistics at the University of California, Berkeley in 1940, "Personnel" asked for a description of what she did each day. Julia complied: "Monday - tried to prove theorem, Tuesday - tried to prove theorem, Wednesday - tried to prove theorem, Thursday - tried to prove theorem, Friday - theorem false.

quoted in *JULIA - A Life in Mathematics* by **Constance Reid.**

Variation on a theme

"There's lies, there's damned lies and there's statistical significance."

PROBLEMS AND ACTIVITIES

Remember that we include a coding system which attempts to indicate in terms of Year levels the suitability range for each item. Thus **6 - 8** suggests an item accessible to students from Year 6 to Year 8.

(1) Now You See Them, Now You Don't

5 - 12

A counting problem from "The Last Word" page of a recent issue of *New Scientist*.

Read the following sentence and count the Fs.

FINISHED FILES ARE THE RE-
SULT OF YEARS OF SCIENTIF-
IC STUDY COMBINED WITH
THE EXPERIENCE OF YEARS.

(2) Did You Know?

5 - 8

There are two varieties of dice: right-handed and left-handed. A left-handed die has 1 - 2 - 3 circulating anticlockwise around a corner. My quick survey of dice to hand at home revealed that left-handed dice seem to significantly outnumber right-handed dice.

Make a stack of three left-handed (or three right-handed) dice and cover the top face with a coin before you look at it. Can you deduce the numbers on the top face and the bottom face of the stack?

This interesting snippet and the associated puzzle came from Prof. Mike Gore's very interesting CMA Public Lecture given at Canberra Grammar School Lecture Theatre on Monday, 20 October 1977. Mike entertained the audience with anecdotes, history and captivating demonstrations.

(3) Five Points

10 - 12

Five lattice points (points with integer components) are chosen on the number plane. Why can we be certain that at least one mid-point of a line joining a pair of the chosen points is also a lattice point?

(4) An Escalating Problem

9 - 12

"How many steps does that escalator have?"

"While they are moving they are difficult to count. But if you will walk up it and count the number of steps from bottom to top, I think we can find the number. We will start together, but I will walk twice as fast as you. Watch me and take one step every time I take two."

By the time the slower person reached the top she reported that she had taken 21 steps, while the faster person had taken 28. How many steps were in sight at one time on the escalator?

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(5) Evaluation of the sin and cos functions using a calculator

9 - 12

Many students wonder how trigonometric functions are evaluated. We suggest a way students in the later years of high school or in college can use a calculator to evaluate the cos and sin functions (and hence the other trigonometric functions). At points in the discussion calculus is touched on, but students without knowledge of calculus should still be able to follow the discussion and perform the calculations.

Students who go far enough with calculus will eventually meet Taylor polynomials. This activity is based around the Taylor polynomial relationship

$$\cos x = 1 - x^2 / 2! + x^4 / 4! - \ldots + (-1)^n x^{2n} / (2n)! + R_{2n+2}$$

which is valid for all real x and all integers $n \geq 1$.

R_{2n+2} is called the remainder term and it can be shown that

$$|R_{2n+2}| \leq |x|^{2n+2} / (2n+2)!$$

The evaluation of mathematical functions by means of polynomial approximations is an important application of calculus.

(a) Put n = 2 in the Taylor polynomial relationship for cos x and show that for $|x| < 0.2$, the remainder (error) term

$$|R_6| < 5 \times 10^{-7}.$$

So provided $|x| < 0.2$, the approximation

$$\cos x \cong 1 - x^2 / 2! + x^4 / 4!$$

is accurate to six decimal places, since an error less than 5×10^{-7} cannot affect the sixth decimal place.

(b) Show that

$$1 - x^2 / 2! + x^4 / 4! = ((x^2 - 6)^2 - 12) / 24.$$

Notice that the expression on the right hand side of this relationship is in convenient form for calculator evaluation.

Now use this expression and your calculator to evaluate cos 0.1 accurate to 6 places of decimals. To find sin 0.1, instead of using the Taylor polynomial expansion for sin we use

$$\sin 0.1 = (1 - \cos^2 0.1)^{1/2}$$

Do the calculation and see that it gives sin 0.1 accurate to six places of decimals.

(c) Remembering that $1° = \pi / 180$, use the ideas of parts (a) and (b) to evaluate both $\cos 10°$ and $\sin 10°$ to six decimal place accuracy.

(d) In order to calculate $\cos x$ when $x \geq 0.2$, we repeatedly halve x until we are below 0.2, evaluate the Taylor polynomial approximation at this x, then double angles using the formula

$$\cos 2\theta = 2 \cos^2\theta - 1$$

until we arrive back at the required $\cos x$. (Maintaining precision during this process requires analysis of the way errors are propagated as the angle doubling process is repeated, and will not be dealt with here.)

Calculate $\cos 1$ by beginning with the Taylor polynomial approximation for calculating $\cos 0.125$ and working to seven decimal places.

Now evaluate

$\cos 0.25 = 2 \cos^2 0.125 - 1$, $\cos 0.5$

and finally $\cos 1$.

Notice how the initial accuracy deteriorates through the angle doubling process. The final answer is accurate to only five places of decimals.

(e) Invert

$\cos x \cong ((x^2 - 6)^2 - 12) / 24$

to obtain

$\cos^{-1}x \cong (6 - (24x + 12)^{1/2})^{1/2}$

with error less than 10^{-5} provided $0.92 < x < 1$. Use this expression to find $\cos^{-1}0.95$ and $\sin^{-1}0.95$.

THUMBNAIL REVIEWS

Readers are welcome to contribute to this section. Reviews can cover books, periodicals, videos, software, CD ROMS, calculators, mathematical models and equipment, posters, etc.

Mental Maths Years 2-7
by Peter Cribb and Anita Straker
Published by Cambridge University Press, 1997

This series of mental maths workbooks presents as bright and attractive. However, a closer examination shows that the size of print is the same from Year 2 to Year 7.

Consequently, the books for the senior year groups do not contain enough work on each page. As one teacher of Year 6 said, "it's not meaty enough for my class". Another concern about the books for the senior year groups was that some pictures were too large; it would be better if the pictures were reduced to provide space for more practice. Additionally some of the concepts were inappropriate for these groups.

There is a good variety of tasks within the workbooks and it is good to see that basic maths tasks are followed by practical applications. But the linkage between the basic maths and practical applications could be improved.

Macquarie Primary School Staff

Working Mathematically: Space
CD ROM & support book
Published jointly by Curriculum Corporation and DECS, SA, 1996.
Single user $110, multiple packs available.

This is a wonderful Australian package for exploring, learning and doing mathematics across the space, number and algebra strands of the mathematics curriculum. Although I have not used it with younger students, I am confident that its transparency, power and versatility make it suitable across the range from primary school to tertiary level

There are three work environments, a 3D Constructor in which structures can be built from cubes, a 2D Builder which allows construction of nets and orthographic drawings, and a Journal word processor which enables student and teachers to record their mathematical work, make notes and interact. Teachers are able to track student journal entries and individually comment on them. The Support Book, written by Kevin Olssen and Steve Walsh who were part of the CD ROM development team, explains the package environments and features. It also includes a useful tutorial which demonstrates the major details of the package to the user, and a detailed index for locating information.

This package creates a marvellously rich mathematical environment for the user. Polyominoes and the Soma cube, as discussed in this *Circuit* by Yvonne Wisbey, can be investigated, along with spatial visualization, patterns and transformations. Animations can be created. Also included are 134 supported investigations along with extra mathematical support material, mathematical strategies, and a Teacher menu. At the click of a mouse button, complex nets and views of 3D structures can be printed and all the usual tools of a Windows environment are available.

As always with powerful new curriculum materials, teachers would benefit from Professional Development in the ways of this package, plus plenty of opportunity to play with it. Many parents will want it too. In schools, even with multiple copies, careful planning and organisation would be required to guarantee equitable student access. Fortunately the August AAMT Mathematics Resources form mentions that trials of a Windows NT network version are under way. Schools need the widest possible access to this truly brilliant production.

Peter Enge

More Mathematical Challenges
Problems from the UK Junior Mathematical Olympiad 1989-95
by Tony Gardiner
Published by Cambridge University Press, 1997, 140 pages

This book contains over 100 thought-provoking problems for students aged 11-15, taken from the UK Junior Mathematical Olympiad. Its value is increased by the inclusion of additional problems in similar style. It complements the book *Mathematical Challenge*, by the same author, a collection of multiple choice problems from the UK Schools Mathematical Challenge.

The second section of *More Mathematical Challenges* gives detailed comments and hints, and the third section presents outline solutions. Notes on additional resources and a booklist are included, along with details of UK JMO award winners. The problems are about how to reason correctly and how to calculate (without a calculator) under a severe time constraint.

Success in these circumstances requires insight and the problems are intended to stretch able youngsters whatever their background. A sample problem: "In Lower Polygonia the unit of currency is the gon. Only three kinds of coins are in circulation: 5 gons, 9 gons and 12 gons. What amounts of money cannot be made up exactly by a suitable combination of these coins?"

This is certainly a most useful resource for teachers in high schools seeking materials to extend able pupils or for use as challenge problems. Taken together *Mathematical Challenge* and *More Mathematical Challenges* make an excellent problem solving package.

Peter Enge

SOLUTIONS TO PROBLEMS AND ACTIVITIES

(1) Strangely, on first reading most people see only three Fs, although there are actually six. Various explanations have been advanced to explain why many people miss the Fs in the ofs.

(2) Knowing that all three dice in the stack are left-handed (or right-handed) and that the numbers on opposite faces of a die sum to seven should enable you to deduce the required numbers.

(3) Using O to represent an odd integer and E to represent an even integer, lattice points can only take the form (O,O), (O,E), (E,O) or (E,E). So if five points are chosen, by the pigeon-hole principle at least two of them must take the same form and yield a lattice mid-point because their respective components have the same parity.

(4) Let *n* be the number of steps from bottom to top. Let *t* be the time a step takes to displace the one immediately above it. A person standing still on the escalator would take a time *nt* to go from bottom to top. The faster person walked up 28 steps to reach the top on the step which was *n-28* from the top when she began at the bottom. Therefore the time of this trip was *(n-28) t*. Since she took 28 steps in this time, she walked at a rate of *(n-28) t / 28* per step , that is two steps in a time

$2\,(n - 28)\,t / 14 = (n-28)\,t / 14.$

Using similar reasoning, the time per step for the slower person was
$(n-21)\,t / 21$.

Since these two times are equal,

$(n-28)\,t / 14 = (n-21)\,t / 21$
and $n = 42$.

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(5)

(a) Substituting n = 2 and
$|x| = 0.2$ gives
$|R_6| = (0.2)^6/6! \cong 8.9 \times 10^{-8}$
$< 5 \times 10^{-7}$

(b) $\cos 0.1 \cong (((0.1)^2 - 6)^2 - 12) / 24$
$\cong 0.099\ 833$

(c) $\cos 10° = \cos(\pi / 18)$
$\cong (((\pi / 18)^2 - 6)^2 - 12) / 24$
$\cong 0.984\ 808$
$\sin 10° = (1-\cos^2 10°)^{1/2}$
$\cong 0.173\ 648$

(d) $\cos 0.125 \cong 0.992\ 1977$

$\cos 0.25 \cong 0.968\ 9126$

$\cos 0.5 \cong 0.877\ 5833$

$\cos 1.0 \cong 0.540\ 3049$

Comparing these results with the values from a calculator's built in cos function we see that the angle doubling calculation has caused a loss in precision.

(e) $\cos^{-1} 0.95$
$\cong (6 - (24 \times 0.95 + 12)^{1/2})^{1/2}$
$\cong 0.31756$

$\sin^{-1} 0.95 = \pi / 2 - \cos^{-1} 0.95$
$\cong 1.25323$