# SHORT CIRCUIT

Newsletter of the Canberra Mathematical Association INC

VOLUME 14 NUMBER 9 SEPTEMBER 2023

## NEWS AND COMMENT

On the Saturday morning workshop held at Namadji School last month, Valerie Barker writes:

*Thank you to those CMA members who helped us present another successful workshop yesterday.*

*We had 11 attendees including both primary and secondary teachers, permanent and casual, and early career and retirees, and the three presentations not only complemented each other but addressed the range of needs within this group.*

Thanks are due to Namadji School and to the presenters Chris Wetherell, Theresa Shellshear and Bruce Ferrington.

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Pages 2 and 3 in this edition are mostly given over to advertisements for initiatives and programs; one from the Australian Association of Mathematics Teachers, and the other from Early Learning STEM Australia.

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A longish article about the origins of arithmetic begins on page 6.

CANBERRA MATHEMATICAL ASSOCIATION

## MEMBERSHIP

Memberships run from 1 Jan to 31 Dec. each year. Membership forms may be downloaded from the CMA website: http://www.canberramaths.org.au

The several benefits of Membership of CMA may be found on the website.

## SHORT CIRCUIT

The CMA newsletter, Short Circuit, is distributed monthly to everyone on our mailing list, free of charge and regardless of membership status.

That you are receiving Short Circuit does not imply that you are a current CMA member.

CMA welcomes all readers.

**Inside:**

# FOCUS ON MATHS

a professional learning initiative from

## What is Focus on Maths?

- Focus on Maths is an initiative of the Australian Association of Mathematics Teachers (AAMT) to build capacity in maths teaching by funding professional learning for primary and secondary teachers of mathematics
- Schools identify the areas of their maths PL need and with their principal's approval apply for funding
- Grants of between $2,000 and $30,000 are available

## Who can apply for Focus on Maths funding?

- The program targets areas of need, so applications are most welcomed from:
  - schools in low socioeconomic areas (ICSEA<1000)
  - schools in rural / remote areas
  - schools with significant numbers of Aboriginal and Torres Strait Islander students
  - out of field secondary teachers
  - teachers lacking confidence in teaching mathematics

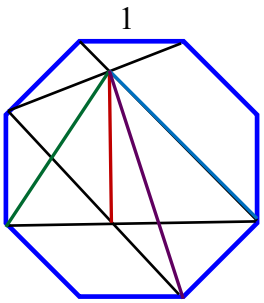
Visit our website **www.aamt.edu.au** to find out more and to apply using the online form

# ELSA

The **ELSA: STEM in Schools Program** builds upon the success of the Early Learning STEM Australia (ELSA) preschool Program, developed by the STEM Education Research Centre (SERC) team at the University of Canberra (UC).


# PUZZLE

**Amazing octagon**




The coloured line segments drawn in this regular octagon represent an interesting collection of lengths. Can you identify them?

NEWSLETTER OF THE CANBERRA MATHEMATICAL ASSOCIATION INC

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We're on the Web!
http://www.canberramaths.org.au/

# ABOUT THE CMA

The Canberra Mathematical Association (Inc.) is the representative body of professional educators of mathematics in Canberra, Australia.
It was established by, among others, the late Professor Bernhard Neumann in 1963. It continues to run - as it began - purely on a volunteer basis.

60 years ago

Its aims include

* the promotion of mathematical education to government through lobbying,
* the development, application and dissemination of mathematical knowledge within Canberra through in-service opportunities, and
* facilitating effective cooperation and collaboration between mathematics teachers and their colleagues in Canberra.

# THE 2023 CMA COMMITTEE

| | | |
|---|---|---|
| President | Aruna Williams | Erindale College |
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| | Roisin Boadle | Erindale College |

Theresa Shellshear is CMA's COACTEA representative.

Sue Wilson is CMA's AAMT representative.

Joe Wilson is the website manager.

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**Find us on Facebook**

http://www.facebook.com/pages/Canberra-Mathematical-Association/110629419011275

# PUZZLE SOLUTIONS from Vol 14 No 8

**1 Enough information?**

If the area of a right-angled triangle is known, and the length of its hypotenuse, is it possible to determine the lengths of its short sides and the size of the angles? In particular, consider a right-angled triangle with area 2 units and hypotenuse 4 units.

In the given case, if the perpendicular sides are $a$ and $b$, we have $ab/2 = 2$ for the area and $a^2 + b^2 = 4^2$ for the hypotenuse. From the first equation we can write $a = 4/b$. After substituting for $a$ in the second equation we obtain a quadratic equation in $b^2$ with the positive solution $b^2 = 8 + 4\sqrt{3}$. Then, $a^2$ must be $8 - 4\sqrt{3}$. If $\alpha$ and $\beta$ are the angles opposite $a$ and $b$ respectively, we find that $\sin\alpha = \frac{1}{2}\sqrt{(2-\sqrt{3})}$ which means $\alpha = 15°$, and $\beta = 75°$.

Thus, $a = 4\sin\alpha \approx 1.0353$, and $b = 4\sin\beta \approx 3.8637$.

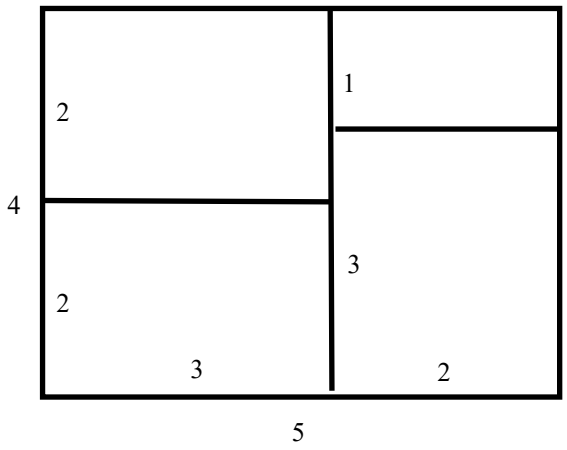
**2. Covering**

You have a good supply of identical blocks in the shape of rectangular prisms, with dimensions 1, 2 and 3 units.

You would like to arrange blocks, in a single layer, so that they fit into and just cover a rectangle that has been drawn on a piece of paper. The dimensions of the rectangle are 4×5 units

How can this be done? Can it be done for rectangles with dimensions 3×7, 2×11, and 1×23 ?

The prisms can have any of their three faces upwards so that areas 2, 3 and 6 units are available. The 4×5 rectangle has area 20 We note that 20 can be partitioned into 6 + 6 + 6 + 2, and by experiment a solution like the following can be found. There are other possibilities.




Solutions exist for all the other rectangles in the question.

# CAREERS AND MATHEMATICS

## Auditor

Detailed information can be found here: Auditor - Career Advice - Banking, Finance and Insurance - On The Job

**Context and relevance**: The first activity will help students understand how their families can get value for money with simple things like toilet paper, while the second activity looks at their community – their school and the cost of rubbish removal. The final activity encourages the students to envisage an issue of trees within their total ecology.

**On the Job Activities for the Classroom**

**Activity 1: Household Toilet Paper – An Audit**

Toilet paper buying during the Pandemic went through the roof. Is it still the same today? Students are to develop a checklist of the type, thickness, cost, and number of rolls used over a two-week period. This timeline gets extended to one month. Individually students are to work out the number of rolls and thereby the cost to the household over the year. Comparing with others in their group, they reflect on changing products.

**Activity 2: The Cost of Rubbish Removal at School – Any alternatives?**

This is an Action Research Project for students to undertake that could be of great benefit to the whole school community. Students are to find out the costs to the school for landfill each fortnight, each term, each year. They are to take the contents of a rubbish bin after lunch and divide it up into recycle, green waste, plastic and landfill and work out how much money could be saved by having different bins for different purposes; having a worm farm; having a compost heap; and, reducing the amount of landfill and thereby the cost for the school.

**Activity 3: The Dead Tree Detective**

Students are provided with stimulus material from "The Conversation" and "The Dead Tree Detective" websites. This project involves identifying the dead trees within their area, using a smartphone with a camera and GPS, using a free Clinometer to work out the height of the tree or work this calculation out manually. The data collected is then uploaded to the University of Western Sydney's database as an ongoing project for the Atlas of Living Australia. Students are then asked to think about planting some trees in their local area.

**Careers & Mathematics can be found at** https://onthejob.education/teachers_parents/Mathematics_Teachers/Careers_Mathematics_Index.htm

## Contact Information

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# ARITHMETIC HAS BIOLOGICAL ROOTS

By **Randolph Grace**
Professor of Psychology, University of Canterbury, NZ
Reprinted from The Conversation

Everyone knows that arithmetic is true: 2 + 2 = 4. But surprisingly, we don't know *why* it's true.

By stepping outside the box of our usual way of thinking about numbers, my colleagues and I have recently shown that arithmetic has biological roots and is a natural consequence of how perception of the world around us is organised.

Our results explain why arithmetic is true and suggest that mathematics is a realisation in symbols of the fundamental nature and creativity of the mind.

Thus, the miraculous correspondence between mathematics and physical reality that has been a source of wonder from the ancient Greeks to the present — as explored in astrophysicist Mario Livio's book Is God a mathematician? — suggests the mind and world are part of a common unity.

**Why is arithmetic universally true?**

Humans have been making symbols for numbers for more than 5,500 years. More than 100 distinct notation systems are known to have been used by different civilisations, including Babylonian, Egyptian, Etruscan, Mayan and Khmer.


Different cultures have developed their own symbols for numbers, but they all use addition and multiplication. Wikimedia Commons, CC BY-SA

The remarkable fact is that despite the great diversity of symbols and cultures, all are based on addition and multiplication. For example, in our familiar Hindu-Arabic numerals: 1,434 = (1 x 1000) + (4 x 100) + (3 x 10) + (4 x 1).

Why have humans invented the same arithmetic, over and over again? Could arithmetic be a universal truth waiting to be discovered?

To unravel the mystery, we need to ask why addition and multiplication are its fundamental operations. We recently posed this question and found that no satisfactory answer – one that met standards of scientific rigour – was available from philosophy, mathematics or the cognitive sciences.

The fact that we don't know why arithmetic is true is a critical gap in our knowledge. Arithmetic is the founda-

tion for higher mathematics, which is indispensable for science.

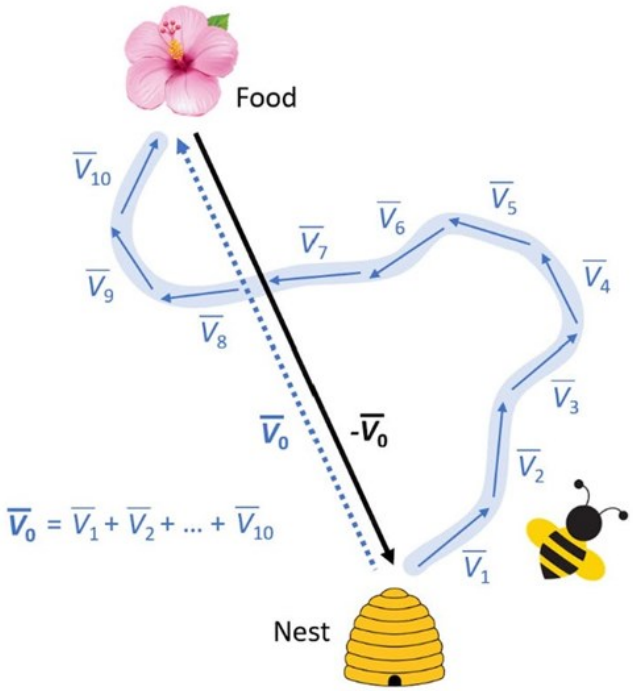
Consider a thought experiment. Physicists in the future have achieved the goal of a "theory of everything" or "God equation". Even if such a theory could correctly predict all physical phenomena in the universe, it would not be able to explain where arithmetic itself comes from or why it is universally true.

Answering these questions is necessary for us to fully understand the role of mathematics in science.

**Bees provide a clue**

We proposed a new approach based on the assumption that arithmetic has a biological origin.

Many non-human species, including insects, show an ability for spatial navigation which seems to require the equivalent of algebraic computation. For example, bees can take a meandering journey to find nectar but then return by the most direct route, as if they can calculate the direction and distance home.




Bees can integrate their zig-zag flight path to calculate the straightest route back to the hive. Nicola J. Morton, CC BY-SA

How their miniature brain (about 960,000 neurons) achieves this is unknown. These calculations might be the non-symbolic precursors of addition and multiplication, honed by natural selection as the optimal solution for navigation.

Arithmetic may be based on biology and special in some way because of evolution's fine-tuning.

**Stepping outside the box**

To probe more deeply into arithmetic, we need to go beyond our habitual, concrete understanding and think in more general and abstract terms. Arithmetic consists of a set of elements and operations that combine two elements to give another element.

In the universe of possibilities, why are the elements represented as numbers and the operations as addition and multiplication? This is a meta-mathematical question – a question about mathematics itself that can be addressed using mathematical methods.

In our research, we proved that four assumptions – monotonicity, convexity, continuity and isomorphism – were sufficient to uniquely identify arithmetic (addition and multiplication over the real numbers) from the universe of possibilities.

- **Monotonicity** is the intuition of "order preserving" and helps us keep track of our place in the world, so that when we approach an object it looms larger but smaller when we move away.

- **Convexity** is grounded in intuitions of "betweenness". For example, the four corners of a football pitch define the playing field even without boundary lines connecting them.
- **Continuity** describes the smoothness with which objects seem to move in space and time.

**Isomorphism** is the idea of sameness or analogy. It's what allows us to recognise that a cat is more similar to a dog than to a rock.

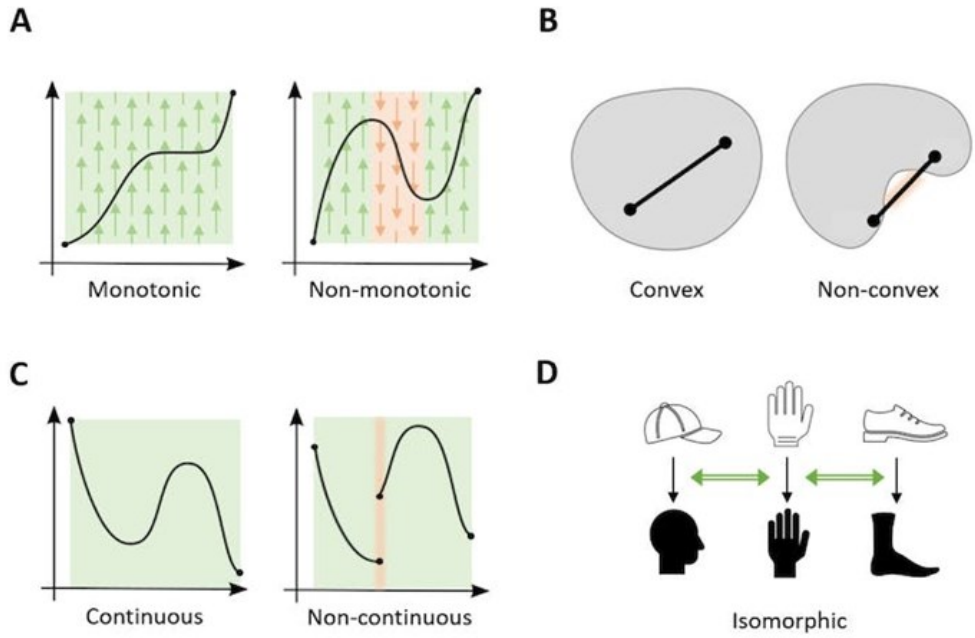
Thus, arithmetic is special because it is a consequence of these purely qualitative conditions. We argue that these conditions are principles of perceptual organisation that shape how we and other animals experience the world – a kind of "deep structure" in perception with roots in evolutionary history.

In our proof, they act as constraints to eliminate all possibilities except arithmetic – a bit like how a sculptor's work reveals a statue hidden in a block of stone.

**What is mathematics?**

Taken together, these four principles structure our perception of the world so that our experience is ordered and cognitively manageable. They are like coloured spectacles that shape and constrain our experience in particular ways.

When we peer through these spectacles at the abstract universe of possibilities, we "see" numbers and arithmetic.




These four principles structure our perception of the world and, collectively, point to arithmetic as an abstract symbol system that reflects that structure. Psychological Review, CC BY-SA

Thus, our results show that arithmetic is biologically-based and a natural consequence of how our perception is structured.

Although this structure is shared with other animals, only humans have invented mathematics. It is humanity's most intimate creation, a realisation in symbols of the fundamental nature and creativity of the mind.

In this sense, mathematics is both invented (uniquely human) and discovered (biologically-based). The seemingly miraculous success of mathematics in the physical sciences hints that our mind and the world are not separate, but part of a common unity.

The arc of mathematics and science points toward non-dualism, a philosophical concept that describes how the mind and the universe as a whole are connected, and that any sense of separation is an illusion. This is consistent with many spiritual traditions (Taoism, Buddhism) and Indigenous knowledge systems such as mātauranga Māori.