SHORT CIRCUIT

Newsletter of the Canberra Mathematical Association INC

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NEWS AND COMMENT

In this edition we continue a sequence of articles by Heather Wardrop about language strategies in the mathematics classroom. Language is an issue in the classroom because of the need for understanding the special meanings of mathematical words. Facility in the language is needed for precision in communicating mathematical ideas.

Some readers might argue, from the opposite direction, that since mathematics deals in logic it reinforces a student's ability to think clearly and therefore to express ideas convincingly in written or spoken language.

Perhaps the humanities and the sciences are not so different after all.

As usual, your thoughts and observations on this and similar issues would be very welcome as Short Circuit items.

On page 3 you will find information about a photographic competition organised by the Australian Academy of Science, and, also needing quick attention, the AAMT webinars for August.

The Puzzle page contains some history and some old problems. Several suggestions came from Ed Staples library of venerable maths books. If you come across an interesting puzzle or a good solution, please let us know.

CANBERRA MATHEMATICAL ASSOCIATION

Inside:

Coming Events:

CMA conference: CANCELLED.

CMA AGM: 11 November, 2020.

Wednesday Workshop:

MEMBERSHIP

Memberships run from 1 Jan to 31 Dec. each year. Membership forms can be accessed from the CMA website: http://www.canberramaths.org.au

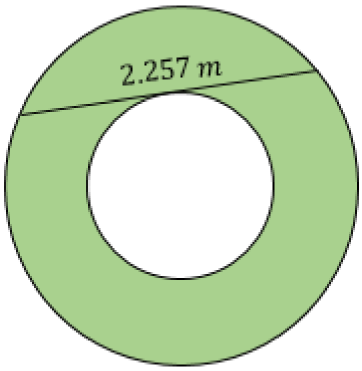
Membership of CMA includes affiliation with the Australian Association of Mathematics Teachers and a subscription to one of two AAMT journals.

As a member, you are entitled to attractive rates for the CMA annual conference and CMA professional development events.

CMA members may attend conferences of the AAMT affiliates in other states, MAV, MANSW, etc. at member rates.

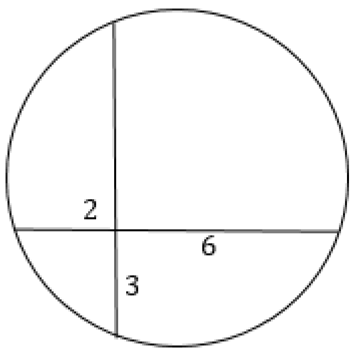
PUZZLES

1 First Circle



An annulus is formed by a pair of concentric circles. This is an old but interesting problem. In practice, it could be a circular garden bed around a tall tree. The two radii are hard to measure but we do know the length, to the nearest millimeter, of the chord drawn tangent to the inner circle. Is it possible to find the area of the garden bed?

2 Second Circle



The two chords are mutually perpendicular. As a first step, find the length of the missing segment. Then, if possible, find the diameter of the circle.

3 Algebra

$$2^x - 3^x = \sqrt{6^x - 9^x}$$

What value of the unknown makes this a true statement?

4 Factors

What two numbers multiply together to give one billion (1 followed by 9 zeros) but neither ends with the digit zero?

5 Third Circle

The 1st century historian Flavius Josephus, in Book 3, Chapter 8, Part 7 of *The Jewish War*, tells a story of survival from a deadly elimination scheme. Here is a sanitised version:

Forty-one children in a playground are arranged in a circle. Another child is 'it' and has to run around the circle tapping every third person. To be consistent with Josephus, we suppose the first person to be tapped occupies position 3 in the circle. Those tapped are then out of the game and have to sit down. Having travelled around the circle once, the tapping process continues, skipping the children who are 'out'. Eventually, just one of them is still 'in'. What position in the circle did that person occupy? What position did the second last person occupy?

6 Between the lines

I counted the number of lines on a page in a book I am reading. Counting in threes I find a remainder of 2, in fives also a remainder of 2, but in sevens there is a remainder of 5. How many lines are on the page?

7. En vacances

Maurice Kraitchik wrote a book *La Mathematique des Jeux*. From the revised 1953 edition, in English, we surmise that recent immigrants to the USA from a troubled Europe sometimes crossed the Atlantic again for a holiday. Kraitchik gives the following problem:

"Two steamers simultaneously leave New York for Lisbon, where they spend 5 days before returning to New York. The first makes 30 miles an hour going and 40 miles an hour returning. The second makes 35 miles an hour each way. Which steamer gets back first?"

8. Bad cancelling

A student cancels the 6s in 16/64 to get the correct answer 1/4. Find three other instances like this where both the numerator and the denominator are two-digit numbers.

PHOTOGRAPHIC COMPETITION

To celebrate mathematics and its prominence in science and society, the Australian Academy of Science's National Committee for Mathematical Sciences is hosting scienceXart: spot the maths, a photographic competition for school students of all ages. A collaboration with reSolve and supported by the Australian Mathematical Society and the Statistical Society of Australia, this initiative is part of the Academy's celebration of the International Mathematical Union's Centennial.

Open for entries from 28 June to 25 September, the competition engages students with the mathematical sciences and highlights the inherent creativity of maths.

Competition website: https://www.science.org.au/sciencexart/spot-the-maths

AAMT

AAMT WEBINARS

When: Thursday, August 6

Title: Targeted Teaching

Register here

When: Wednesday August 12

Title: Using data to improve mathematics outcomes

Register here

AAMT e-news bulletins: subscribe.

Download a catalogue of resources sold by AAMT from www.aamt.edu.au/Webshop/Catalogue.
CMA members can order items from the AAMT catalogue at a discount.

MORE RESOURCES

LET'S COUNT PARENT MATHS BOT

The Smith Family, children's education charity, has launched a new learning tool on Facebook Messenger to help children with their maths skills.

Dr Alan Finkel, Australia's Chief Scientist, has a launched a national Storytime Pledge campaign to encourage young students to read more. For further information, see here

AMT—Australian Maths Trust
https://www.amt.edu.au/

CSIRO—STEM professionals in schools program
www.mathematiciansinschools.edu.au

reSolve—Maths by inquiry
https://www.resolve.edu.au/teaching-resources

NEWSLETTER OF THE CANBERRA MATHEMATICAL ASSOCIATION INC

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ABOUT THE CMA

The Canberra Mathematical Association (Inc.) is the representative body of professional educators of mathematics in Canberra, Australia.
It was established by, among others, the late Professor Bernhard Neumann in 1963. It continues to run - as it began - purely on a volunteer basis.

Its aims include

* the promotion of mathematical education to government through lobbying,
* the development, application and dissemination of mathematical knowledge within Canberra through in-service opportunities, and
* facilitating effective cooperation and collaboration between mathematics teachers and their colleagues in Canberra.

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http://www.facebook.com/pages/Canberra-Mathematical-Association/110629419011275

LANGUAGE STRATEGIES FOR MATHEMATICS, THE FIRST TWO STRATEGIES

By Heather Wardrop

The two strategies presented here follow the introductory article I wrote for the July edition of *Short Circuit*. I have used these extensively myself and have found them to be very beneficial.

1. Newman Questioning Technique

Too often teachers read and interpret the question for the student and reduce the rich context into just another "drill and practice" question. This approach takes less time in a busy classroom but is completely disempowering for the student who never learns to successfully tackle the worded problems.

The strategy: Say to the student

1. Would you read the question to me? Stop at any words you do not understand.
2. What do you think the question means? What do you have to do?
3. Can you do that? What mathematics will you use?
4. Complete the calculations now.
5. What does your answer mean? (later, when you come back.)

I found that perhaps 90% of students who could not do the question stumbled with part 1 and could not even read the question to me without hesitating. This happened with all levels of mathematics because the mathematical language is much more complex at the higher levels so that even very capable students falter with worded questions.

Initially, this technique was frustrating for me and for the students who often just blurted out "Why don't you just tell me how to do it?" I explained my reasoning, and within a few lessons, students were raising their hand and saying things like: "I've read the question. I don't know what *northerly* means – is it straight up or just somewhere North? What does *due East* mean? I think I need to use trigonometry or Pythagoras' theorem. Can you help me to start the diagram?"

Later, when I came back, they would tell me that 14.2 meant "The boat travelled 14.2 km", changing their answer back into a worded, real-world context.

Several other techniques need to be applied to this question. After a while I found myself doing this automatically. Read on…

2. Glossaries

Have a student compile a glossary in the back of their book. This can include technical words like *hypotenuse* but also the "slippery" words like *due* and *northerly* that seem insignificant but are crucial in drawing the diagram.

Other examples are: the direction *of* the boat *from* the port is different from the direction *of* the port *from* the boat. The order is key to interpreting the angle that needs to be found.

Rationalise the denominator does not involve the normal use of the word *rational.* This less obvious information should also be in the glossary, as well as the technical word *denominator.* It may be helpful to search these out before each topic is taught and deal with them as a class group, discussing the confusing meanings openly and making students feel more comfortable when reading a worded problem.

Encourage students to use their own diagrams and write in their first language in the glossary. When using the Newman Questioning, a term should be added to the glossary on the spot if it is not understood. In the end, whilst slow at the start, it is efficient for the teacher. It is unlikely you will be asked it again and the process speeds up. It is easily applied to all other topics and the students become familiar with it.

Prior to assessment tasks I often examined students' glossaries, ensuring there were no worked examples, and then let students take them into assessment tasks. International students found this especially helpful but over time all students relied on these less often. They learned the language.

PUZZLE SOLUTIONS

First Circle

There are many ways to adjust the diameters of the circles while keeping the length of the chord constant. Therefore, if there is to be a solution, it would have to be true that whenever the chord is fixed, so is the area. To check whether this is the case, call the radii R and r, and the chord c. Deduce that

$$c = 2\sqrt{R^2 - r^2}$$

The area of the annulus is

$$\pi(R^2 - r^2) = \frac{\pi c^2}{4}$$

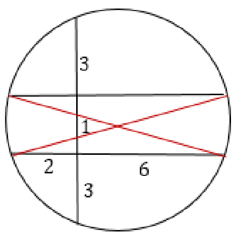
which is independent of the two radii. (Check what happens when r approaches 0.)

In this case, the area is 4.00 m^2.

Second Circle

Via similar triangles we obtain the power of a point or intersecting chords theorem. Thus, the products of the segments of each chord are equal, and the missing piece is 4.

There always exists a diameter through the end point of a chord and by a symmetry of the circle we can make a construction like the following.



Thus, by Pythagoras, the diameter is

$$\sqrt{8^2 + 1^2} = \sqrt{65}$$

Another intriguing idea is to use the fact that if the segments of the chords are taken to be diameters of circles, the sum of the areas of the four small circles is equal to the area of the large circle. An article in ASMJ from 2009 by Ed Staples delves further into this.

Algebra

The exponent x is about –1.7095.

Factors

$$\begin{aligned} 1\,000\,000\,000 &= 10^9 \\ &= 2^9 \times 5^9 \\ &= 512 \times 1\,953\,125 \end{aligned}$$

There can be no other solutions.

Third Circle

By brute force, the last person occupies position 31 and the second last occupies position 16. There are generalisations and rather complicated but efficient ways of finding solutions. See Wikipedia etc.

Between the lines

Since division by 3 and by 5 gives the remainder 2, so does division by 15. So, the required number must be one of 17, 32, 47, 62, … . Some of these give a remainder of 5 on division by 7. So, we examine 12, 27, 42, 57, … for divisibility by 7. Thus, the number of lines could be 47 or 152 or 252 or … . Let's say 47.

En vacances

Kraitchik gives the answer that the second steamer gets back first because

$$\frac{x}{30} + \frac{x}{40} > \frac{2x}{35}$$

But this too may need some unpacking. Consider the times taken for the journeys when x is the distance from New York to Lisbon.

Bad cancelling

26/65, 19/95, 49/98