CIRCUIT

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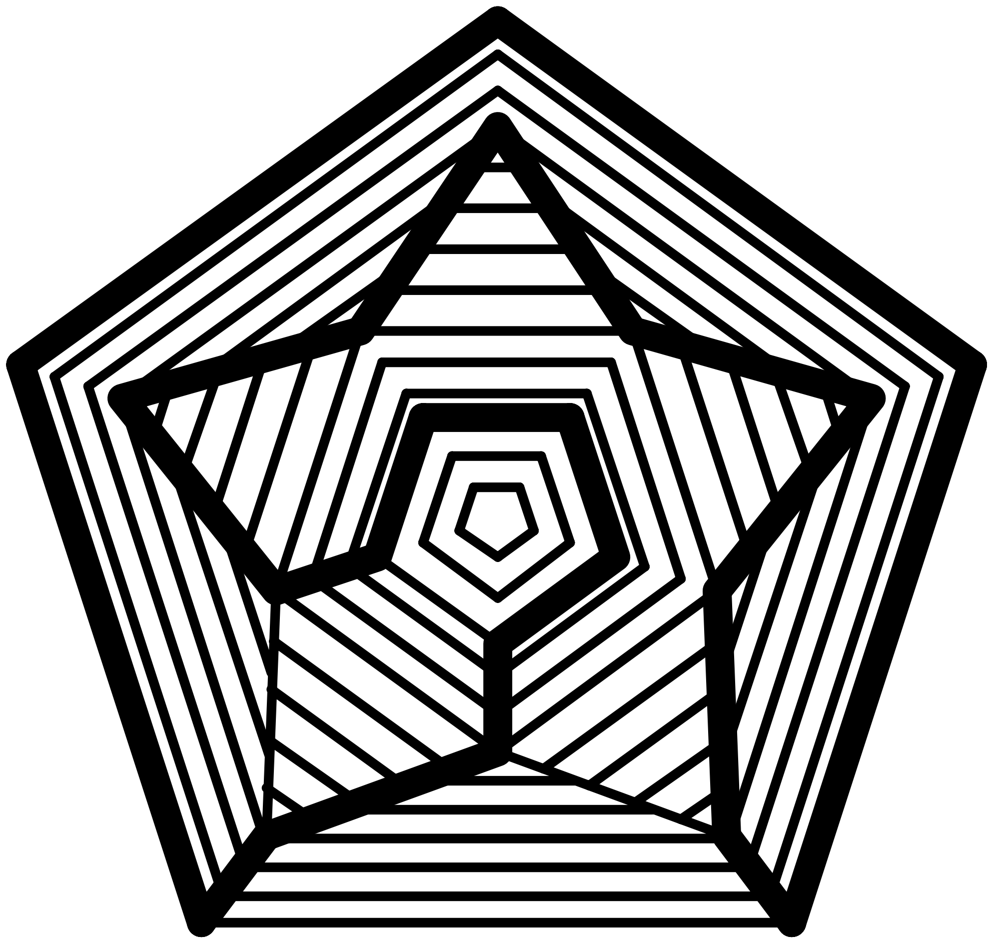

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The objects of the Canberra Mathematical Association are to promote interest in mathematics, to encourage improvements in the teaching of mathematics and its applications, to provide means of communication among teachers and students and to advance the views of the Association on any question affecting the study or teaching of mathematics and its applications.

The Canberra Mathematical Association Logo depicts a Hamiltonian Circuit on a dodecahedron.

CONTRIBUTIONS

Circuit is always keen to receive articles, notes, problems, letters and information of interest to members of the CMA. Please contact the Editor if you wish to contribute to Circuit. If possible contributions should be submitted as Word 97 documents.

FROM THE PRESIDENT

Welcome to World Mathematical Year! I attended the first official celebration, which was the Mathematics 2000 Festival at the University of Melbourne, in January. The Festival's aim was to celebrate the important role of mathematics in our society and the enduring message for me was that mathematics was about making the invisible, visible. Dr Keith Devlin, St Mary's College, California, spoke about mathematics being the study of patterns, and emphasised that in its applications mathematics makes things visible that are invisible. He made reference to a TV series developed by PBS in USA (my rough notes suggest called Life by the Numbers) which he recommended we watch if it appears on TV here. (Editor's note: A review of this series was published in Thumbnail Reviews in the December 1999 *Circuit*.)

Several presentations by mathematicians made visible, through mathematical models, the mathematics behind many sporting activities. They showed, in a manner readily understood by non-mathematicians, the role mathematics can play in explaining and improving sporting performance.

Dr Roger Penrose from Oxford University, a renowned mathematician, gave a keynote address on the scope, beauty and depth of mathematics. The part of his message that was particularly refreshing to hear, and pertinent for teachers of mathematics was that, *"the crucial though difficult thing is the understanding…one has to strive for this."*

Membership and AAMT 2001 Conference

Letters have been sent to all school Principals in the ACT and surrounding region outlining the benefits to schools of membership of CMA.

Included in the correspondence was a brochure about the AAMT 2001 Conference to be held in Canberra next January, and a registration form. Please assist in prominently displaying this material so that all teachers are made aware of it. A reminder that this is an outstanding professional development opportunity, both for teachers giving presentations and those attending presentations. It is timely to encourage your school to maximise the use of this opportunity through appropriate budget allocations. Attendance at the Conference contributes towards teachers' PD requirements.

Individual teachers are also encouraged to take out membership of CMA.

The Council of CMA welcomes involvement by members. We need continual renewal to remain relevant to members. Please consider making a contribution by contacting a Council member.

Professional Development

We have held two successful PD opportunities in first term. The first to the Canberra Space Dome and Observatory at the Downer Club was informative and teachers were impressed with the range of programs available for school groups. The recently revised program has features for children from years 2 to 12. A visit to www.cfmeu.asn.au/planetarium will fill you in.

Alastair McIntosh from the University of Tasmania provided food for thought and many practical activities for encouraging mental computation in students at all levels of schooling. Some of Alistair's suggestions are included elsewhere in *Circuit*. Teacher feedback from this PD session indicated that teachers would very much like more PD on teaching approaches to cater for the range of abilities in the classroom. The CMA is planning a program on this topic for third term.

PD in second term will begin with a review of mathematics Internet sites. Further information will be communicated through schools.

Meeting with Mathematics Faculty Heads

A productive program of senior secondary school visits has been developed following concerns expressed about communications between senior secondary teachers and university personnel. It is hoped that the process will provide both teachers and students with a better understanding of what mathematics is required of prospective tertiary students.

Canberra Mathematics Quest (CMQ)

This provides an opportunity to showcase quality mathematics work presented by children at all levels of schooling while, at the same time, providing professional development for teachers. All teachers of mathematics are invited to the launch of CMQ. Join us at the Sky Lounge of the Yamba Sports Club on Tuesday 23rd May 4pm for afternoon tea and the launch.

Best wishes for a productive year.

Paulene Kibble
paulene.kibble@act.gov.au

HAPPENINGS

A PLANETARIUM IN CANBERRA - A WONDERFUL SCIENCE AND MATHEMATICS RESOURCE

Have you been there yet?

The Downer club doesn't just sell drinks. It offers the customer a unique experience probably unavailable anywhere else in Australia. It's called a planetarium and it's a none-too-cheap facility that you get to step into. Once inside, the doors are closed, and you take your seat, lean right back and go for the ride of your life. Over a twelve month period, I had gone out most clear nights and tried to locate the more obvious constellations and the movement of the moon and planets. When I looked up inside this planetarium dome, I was immediately taken back at how real it looked. The landscape of stars was instantly recognisable.

The great benefit of a planetarium is that a view of the heavens is afforded, without clouds and city lights, for any location on the Earth's surface, for any time of the year and for any year chosen. A planetarium can speed up the procession of stars across the sky so that their apparent motion makes sense. Constellations can be highlighted and even retrograde motions of planets can be observed.

In no time at all the planetarium can turn into the front end of a space ship, and occupants can, with a little imagination, feel like they are being catapulted into the solar system at speeds faster than the speed of light.

A group of mathematics teachers were given this special planetarium opportunity thanks to the noble efforts of Paulene Kibble and the CMA. I would certainly recommend a visit.

Ed Staples, Erindale College

NOTICE BOARD

The University of Canberra Maths Day 2000

UC Maths Day 2000 will be held on Friday 26 May in the Sports Centre at the University of Canberra.

Some 36 - 40 schools send teams of five year 12 students plus a teacher, who acts as supervisor for another school's team.

Before Maths Day, schools are given a scenario with a problem to be solved. Each team produces their solution in poster form and these are judged on Maths Day for their mathematical accuracy, graphic, and overall presentation.

During the day the teams compete in four events with a break for lunch.

Group Contest Each team works together or separately on ten questions.

Swiss Contest Here two teams pit their skills against each other, as, given the top "X" row of numbers, they have to call out the "Y" row (Yes, a couple of "Y" squares are filled in to get them going!).

Lunch We provide sandwiches, cakes, fruit etc for the students in the Refectory, and a more sedate sit down buffet lunch for the teachers in the Balcony Restaurant.

Cross Contest Following lunch each team splits into two at one table and three at another. One group gets the "across" clues and the other the "down" clues.

Relay Contest or Maths On the Run A fun finale. Again the teams are split, this time into pairs with a runner, whose speed can be the determining factor in a good final score!

And for what do the teams do all this challenging maths? At the end of the day it's back to the Refectory for afternoon tea and the Presentation of the UC Maths Day Trophy, City Schools Shield and Country Schools Shield, Poster Contest awards and a Lucky Door Prize for the teachers.

Interested teachers are most welcome to come and see how Maths Day works!

FROM THE EDITOR

Welcome to another year of *Circuit* where I still find myself as sole cook and bottle washer. Needless to say, I would prefer to have some other CMA members to work with in a small editorial team, sharing ideas, support, expertise and enthusiasm, and making the production task a little less daunting. My hope is that *Circuit* would benefit by being more representative of the broad membership of CMA. It's certainly never too late to volunteer your services, so please get in contact with me if you would like to be involved with producing *Circuit.*

At last it seems that ACT Government teachers have concluded an Enterprise Bargaining Agreement in which modest salary increases over 3 years have been funded from the budget, and student learning environments and teachers' working conditions have not been traded off. Perhaps the most important aspect of this Agreement is that the ACT Government, in partnership with the Australian Education Union, is committed to funding and implementing Professional Development programs for teachers to ensure high quality outcomes for all students. Hopefully CMA can make a useful contribution in what one hopes will be new times for mathematics and numeracy PD in the ACT, with AAMT 2001 the icing on our cake in the short term.

This *Circuit* is again a mix of original and reprinted material. Margaret Carmody contributes an interesting and timely reflection on aspects of the mathematization of motherhood. From government, banking, finance and international markets, through health, transport, communication, entertainment, sport, design and planning, to technology, science, architecture, engineering and construction, global mathematization is the order of the day. By way of contrast, the age old question "Is mathematics discovered or invented?" is discussed in a relatively short and accessible piece by Paul Ernest, and touched on again in my review of Stanislas Dehaene's book *The Number Sense - How the Mind Creates Mathematics.* Thanks to all the other writers this issue for their efforts and remember that we always welcome your contributions and suggestions. Don't forget to visit the new CMA Homepage which is subject to ongoing development: http://www.ash.org.au/canberramaths

SOME FORMATS FOR MENTAL COMPUTATION LESSONS

1. Today's Number Is …

The teacher writes a number on the board. Children are asked to give calculations to which this is the answer. Various conditions can be set by the teacher to elicit more or less sophisticated strategies.

2. How Did You Do It?

The teacher gives a mental calculation. Children explain and discuss their strategies for obtaining an answer.

3. Find My Number

The teacher thinks of a number. Children try to find the number by asking questions to which the answer can only be Yes or No. A number line can be used to keep track of the range of numbers remaining as possibilities are eliminated. Magnets could be used on a magnetic surface.

4. Compatible Numbers

The teacher provides a collection of numbers. Children look for pairs of numbers that total a given amount, say 10, 20, 100.

5. Sum Stories

The teacher writes a calculation on the board. Children offer stories that embody that calculation.

6. Within Limits

The teacher gives a measure to be estimated. Children, in groups, agree on upper and lower limits within which they are confident the measure lies. These limits are presented and discussed

Paulene Kibble, inspired by Alastair McIntosh's presentation in Canberra earlier this year, adapted these formats from McIntosh, A., De Nardi, E. and Swan, P., *Think Mathematically*, Longman, Melbourne, 1994.

PAULENE'S TEACHING TIPS

Activities that can easily be adapted to cater for the range of children in a class

Calendar Activity

This can be a whole class activity.

Take a collection of cards that have numbers to 31 and the names of the days of the week. Establish criteria for arranging the numbers for the month, such as the 4th of the month is on Wednesday and the month has 30 days. Ask people with other days and dates to place them into the correct position(s).

Take a Length of Yarn

This activity can involve the whole class and the number of participants in each group is not critical.

Measure about 6 metres of yarn using your body to estimate the length. A practical suggestion here is to take the distance from fingertip to nose when the head is turned away from the outstretched hand to be about 1 metre.

Specify the rules for the groups. For example, both hands must remain on the yarn and each group member must agree with the construction before it can be submitted for appraisal, acknowledgment or debate.

Ask the groups to make shapes with: 3 sides; 3 equal sides; 3 sides, two of which are equal; 9 sides; one side twice as long as another side; 3 diagonals; etc.

Paulene Kibble

MATHS AND MOTHERS

We are grateful to the editors of *Literacy and Numeracy Studies,* University of Technology, Sydney, for their permission to reprint this paper.

Introduction

It is surprising to discover what a major part explicit mathematics plays in motherhood in our society. Not only do we measure the length of a pregnancy in weeks, or even days, count the contractions of labour in minutes and the dilatation of the cervix in centimetres, but we describe the birth of a baby, that momentous occasion, as the second stage of labour, with the passing of the placenta as the third stage, implying equivalence of those events, merely by giving them numbers. Mothers are weighed, the contents of their blood are counted and estimated, and their pelvises are measured and described as adequate or inadequate.

But that is just the beginning. Once the baby is born, the explicit mathematics starts in earnest. The baby is weighed and measured. It is given a score out of ten at birth, the apgar score. It is fed and the lengths of, and intervals between, feeds are recorded. The baby is weighed again, sometimes before and after feeds, to ascertain the success of breastfeeding. The nappies are counted, the progress is assessed and supplementary feeds may be prescribed, or requested. Then we enter the world of measured feeds, given with regulated frequency, of determined strength and composition, using equipment of latest design to ensure comfort, efficiency and hygiene. Equipment must be cleaned and sterilised using temperature or chemical methods. Knowledge of temperatures, time and solutions is essential.

The explicit maths involved in being a mother is significant; however, the use of maths is generally for the benefit of other people in making assessments of the baby's progress and the mother's abilities. Maths has been used in particular ways by health professionals to influence mothers in the care of their babies, specifically in the area of infant nutrition and in the assessment of the well-being of the baby. Mothers have been (and still are, at times) regarded by health professionals as '... usually not the most sensible of people.' (Gaisford and Lightwood, 1953:291)

The purpose of this paper is to describe and examine advice on infant nutrition given to mothers by health professionals since the 1940s, and to examine the ways in which mathematics has been used to position women and health professionals and to influence their behaviour. This paper argues that a particular, limited kind of mathematics, in combination with an authoritarian, transmission mode of learning have been used by many health professionals, particularly doctors, and infant welfare nurses when communicating with mothers. This communication around maths has had the effect of often disempowering and confusing mothers, of misleading them into obeying, or trying to obey, a few set, explicit mathematical formulas for feeding, timing and weight gain. Rather than creating a constructivist, (ie socially negotiated) reflective learning situation, health professionals have sought to have mothers obey their dictates, and have placed considerable pressure on them to conform.

This paper draws on paediatrics texts, journals, newspapers and newsletters, as well as an interview with a general practitioner-obstetrician who has worked in Sydney and is still in practice. It seeks to provide an account of the historical constructions of measured motherhood, one which, it is argued, continues to have its effects today.

Expert Instruction

Infant nutrition and the general management of infants have been seen as an area of knowledge which is objective, and premised on some sort of external reality constructed as ‘scientific fact’ rather than as social constructions (Johnston, 1994). As far as health professionals were concerned in the 1940s, their role as teachers of mothers was one where learning relied on the transfer or transmission of facts from the health expert to the mother.

Learning was considered to have taken place when the mother had absorbed these facts, and when she could repeat the actions as instructed, whether it was folding a nappy, making up a bottle of formula, or responding to crying. Integral to this model involving health professional/mother/baby is the power relationship of expert/students; of telling, instructing, reinforcing and correcting. The mother is seen as an empty receptacle into which the health professional pours knowledge that she is expected to understand and practise. The hospital system of infant care became the dominant guide for parents. It was perpetuated outside the hospital by means of written regulations given to the mother at the time of her discharge and subsequently through the regulation of community health agencies and the advice of many paediatricians based, in general, on hospital practice (Jackson and Trainham, 1950:12).

Schedules

In Australia in the 1940s, mothers were advised to breastfeed according to a schedule. This advice was based on the theories of Dr Truby King, a New Zealand doctor who first promoted his ideas in the 1920s. Truby King proved to be a powerful influence and his ideas were implemented in many places, including the Tresillian organisation in New South Wales, (Harper, 1955, Armstrong, 1939), an organisation which assists new mothers to develop feeding and sleeping routines with their babies.

In response to concerns about the high infant mortality in the early part of this century, Truby King, along with others, maintained that babies should be fed four-hourly because it took four hours to digest a feed. Truby King argued that babies must be brought up to be regular and disciplined (Rich, 1998, Kociumbas, 1997, Dick, 1987). Mothers were told by health professionals in the 1940s to time the length of the feeds, starting with two minutes per breast, every three hours, and gradually increasing the length of feeding and the interval between feeds, so that the baby was fed for 20 minutes, five times a day. Mothers were advised to regulate the feeds and to establish a routine, usually with feeds at 6am, 10am, 2pm, 6pm and 10pm. It was thought possible to get any baby into such a routine. Part of the routine was to ignore the baby if it cried between feeds, to not rock it or cuddle it. ‘Rocking chairs and cradles were banned as these would over-excite the baby and addle its brain’ (Rich, 1998). Spoiling was actively discouraged, for example, ‘things to be particularly avoided are too much playing, tossing, or jumping him up and down ... tickling him to make him laugh’ (Harper, 1955:87).

Malnourished babies

Inevitably, with such a strict regimen, breastfeeding frequently failed. The babies were not fed frequently enough and some probably had food intolerances. Not many mothers could produce sufficient milk with such little stimulation. The malnourished, breastfed baby presented as a common problem in doctors’ rooms. As well, many of these mothers were themselves malnourished and overworked, making breastfeeding very difficult. This was during World War II, when many women were often working in non-traditional roles (Kociumbas, 1997:197). It is true that, because of the War, families were separated and fathers were away or may have been killed; many women were caring for their children in difficult circumstances, lacking the emotional and practical support of their husbands and perhaps even taking responsibility for a family farm or business.

Women were not encouraged to breastfeed. Doctors told mothers they could try if they wanted to, but many babies were automatically put on the bottle in the hospitals. Mothers wanted to breastfeed, because women in their families had done it, but health professionals did not actively encourage it (Rich, 1998). The message was: breastfeeding is good but probably impossible to achieve. Since the 1930s, when it became possible to fortify dried milk directly with vitamins to bring them up to breastmilk levels, health professionals felt that bottles were safe (Dick, 1987:45).

Progress of the baby

The solution to the problems, given the assumed equivalence of human and cow’s milk, weakened and sweetened, was to supplement the breastfeeds or to bottle-feed the baby entirely. The way to assess the success of the feeding and the progress of the baby was to weigh the baby. No other measurements were taken – weighing was the criterion. Babies were weighed weekly at the local baby health clinic and some mothers even had their own scales and weighed the baby before and after each feed. Baby health clinics were very regimented. ‘I can remember a friend quoting the clinic sister as saying about the baby: “He’s coming on well. He can have an extra 1/2 ounce per feed”’ (Rich, 1998). There were formulas to gauge what a baby’s weight should be, according to the average, at any given age. It is true that mothers were told, ‘... no baby is average’ (Spock, 1951:103); however the weight gain, along with considerations such as crying, indigestion or vomiting were what the doctor relied on in assessing the baby’s progress:

> The best guide to an infant’s progress is afforded by regular weighing, which should be recorded once a week ... . (Sheldon, 1941:43)

Pernicious breastfeeding

Many health professionals in the 1940s had the attitude that breastfeeding according to the baby’s needs was uncivilised behaviour: ‘It was a great mistake’ (Harper, 1955:30). As well, they thought it was a time-consuming, unnecessary activity, suitable only for mothers who were content to be ‘just’ housewives. ‘[Demand feeding] is always pernicious to both mother and child’ (Kociumbas, 1997:33). Rich recalls an obstetrician advising her, after the birth of one of her own children: ‘You’ve got more important things to do with your life than staying at home and breastfeeding a baby'.

Science has the answers

Faced with the myriad problems of the health of infants and their mothers, health professionals sought to reduce infant nutrition to an exact science: a science with precise values and formulas, outcomes and expectancies. Using formulas and average weight gains as guides to good infant nutrition was based on the deductive method of reasoning. However, the understanding of physiology, in the 1940s and 1950s, was incomplete. For instance, the importance of colostrum – that mother's milk for a period of time after birth contains protective antibodies – was not fully understood (Gaisford and Lightwood, 1953:124, Sheldon, 1941:43-4, Harper, 1955:28) and there was absolutely no acknowledgment of the role of colostrum in the bonding of mother and baby.

Infant nutrition and the relationship between the mother and the baby are exceedingly complex. So many factors are involved in the progress of a baby that to single out just one – weight gain – is unrealistic and unreliable. Yet this is what happened. Mothers were advised that average weekly weight gain should be 8oz per week to 3 months, 4-6oz per week to 6 months, 4oz per week to 12 months (Rich, 1998). Reliance on weight gain as the measure did not necessarily take into account the accuracy of the scales, or the technique of weighing. If the clothes, the time of day or the position of the baby on the scales were different, then the results were not reliable. As well, the milk formula for bottle fed babies was adjusted according to the weight gain and this required complex calculations if it was to be followed accurately.

While it is true that, as a result of this scientific approach, there were undoubted advances in the provision of clean cow's milk to the community, hygienic storage and transport of milk and regulation of the industry, which benefited the whole community, the long term effect was to create significant health problems, and the immediate effect was to transform the home from a 'blessed sanctuary' into a 'quasi-military factory'. It demanded from the mother, '... unprecedented personal vigilance and domestic hygiene' (Kociumbas, 1997:133).

Measuring the formula

Reading these paediatrics books it would seem that (unsurprisingly) a considerable number of mothers did not comprehend how to carry out all these measurements accurately, and babies were fed weak or contaminated solutions and so failed to thrive. This has been of particular concern in relation to mothers who had few or no opportunities to seek further help due to their social and domestic circumstances, and/or to mothers with little exposure to western mathematics, for example, indigenous mothers (Rich, 1998). Some mothers may not have understood the nutritional importance of accuracy, and thought if it looked white, the feed was suitable. Thompson (1998:24) points out the cumulative effects of minor errors in measuring the components of milk formulas, which can lead to significant health risks for the infant.

Discourses of infant nutrition

In the area of infant nutrition, a way of describing the progress of the baby and mothering has evolved. The dominant discourse, the one used by the health professionals, names and defines the experiences and practices, thereby having a profound effect on the ways mothers cared for their babies. As Lee (1995:47), following Foucault, points out, discourse itself has the power to actually create reality: this power to create is always a 'distributive politics', that is, what is deemed 'real' and 'true' determines what is included and excluded, so that what cannot be named may not even be noticed. The discourse of motherhood, and specifically infant nutrition, has created a particular, limited knowledge and understanding of infant nutrition and of motherhood generally.

Precise motherhood

The precision of numbers and of scientific language has been used against mothers. The precision of numbers – their accuracy – made doctors feel this method offered perfection, assurance and satisfaction. The artificial method of feeding became equated with good health care, good infant nutrition and good maternal care and practice. The reliance on maths to assess the baby's wellbeing provided security for mothers and ease for doctors, but it also put a pressure on mothers to be constantly watching the clock, measuring bottles, counting feeds, and weighing the baby:

> ... the infant's average weight gain is the best measure of the success of lactation. (Gaisford and Lightwood, 1953:126)

The constant attention to things mathematical, whether it was weight gain or measuring formula, became an end in itself. It became not just a way of controlling and curing nutritional deficits, but a way of controlling mothers and babies and their relationships with each other. Furthermore, the discourse affected doctors' relationships with their patients; and indeed it affected whole families in their attitude to the role of the mother in the family: if the mother did as she was advised, by Spock and others, in the 1940s, the mother actually spent very little of her day handling and feeding the baby – sometimes even propping up the bottle.

> ... Some busy mothers with other children and a husband to take care of, have to prop at certain feedings ... it does no harm for a loving but busy mother to prop some of the bottles. (Spock, 1972:121-2)

Inherent in this scientifically-based advice is the expectation that the mother should meet everyone's needs and that her own needs to cuddle her baby should not be considered.

> ... the importance of regularity in feeding must be emphasised ... Of course a hungry infant will cry, but the proper management is to increase the amount at each meal, not to feed indiscriminately ... if he is asleep at feeding time he should be roused sufficiently to suck. (Sheldon, 1941: 43)

In the discourse of the doctors, the factors about motherhood that could not be quantified were irrelevant. Typically, in the paediatrics books of the 1940s and 1950s, there are brief general statements about contentment (Sheldon, 1941:41, Gaisford and Lightwood, 1953:118), but there is also considerable detail about measuring indicators such as weight gain. In any discussion of mathematics and motherhood, it is difficult to ignore the part infant nutrition plays in our overall expectations concerning childhood and infant behaviour. This goes far beyond the physiological relationship (Gaisford and Lightwood, 1953:123).

The literacy of infant nutrition

Inherent in the prescribing and making up of formulas are a variety of mathematical skills, but there are additional skills of reading and understanding the instructions, and the maths symbols for the operations involved: addition, subtraction, multiplication and division; and of reading and understanding the abbreviations for measurements, for example, 'oz' for 'ounce'. Much of the information regarding nutrition and feeding was given to mothers in a 'skill and drill' fashion (Freebody and Luke, 1990: 8).

There are also assumptions on the part of the health professionals about equipment, knowledge, domestic skills and circumstances, including income to spend on the formula and equipment. It is assumed that the mother has a measuring jug, that she has a standard tablespoon and bottles and teats. It is assumed that she knows how to scrub things clean and then sterilise them; that she has access to a clean kitchen and utensils, storage space and refrigeration. It is also assumed that she will understand the importance of temperature for storage and deterioration.

Maths and culture

Questions about interpreting the culturally laden material of baby care arise when considering the advice given to mothers in paediatrics texts, Texts often rely very heavily on prior knowledge, including cultural mathematical knowledge. For example, if we examine a formula suggested by Spock, an American, the material is problematical for Australian mothers,

> Evaporated milk ... 13oz + Water ... to make 1 quart + Corn Syrup ... 2 tablespoonfuls. This will make about 5 1/4 ounces in 6 bottles, 6 1/2 ounces in 5 bottles, 8 ounces in 4 bottles. (Spock, 1972:116)

The relationship between pints and ounces varies from the United States to Australia; therefore, an Australian mother could end up with 41 oz formula, that is milk 13 oz, water 27 oz syrup 1 oz. But this makes six feeds each of 6.7 oz. It does not add up. However, assuming that the mother knows that a pint in the USA is 16 oz, not 20 oz, and that a quart is therefore 32 oz and that she makes it accordingly, the formula does not take account of the syrup. How many mothers measured out 5 1/4 oz into each bottle and then wondered where they had gone wrong? How many threw out the whole lot and started again, only to make the same 'mistake'?

Ease of adjustment

There is, of course, a good deal of security in measuring the time, the interval, the baby's weight when breastfeeding, or measuring the formula, the amount in the bottle, and so on. These are all related to explicit signs of the baby's progress: they are interpreted as indications that the baby is being cared for and loved? There is the satisfaction for the mother in being able to see the milk being drunk and there is the ease of adjustment for the health professional in having measurable indicators on which to base decisions.

> The main reason why some regime is desirable is that it may be necessary to prescribe modifications which are not practicable unless the mother is able to say how many feeds are given in the day and at about what times. (Gaisford and Lightwood, 1953:125)

This is a very institutional view, and was in fact criticised by some writers at the time, (Jackson and Trainham, 1950:12). It is easier to weigh the baby and advise a change in formula than take the time to discover what is really going on by careful questioning and listening. Not only are there the problems associated with the mothers understanding the texts or the maths, or the verbal instructions, there is also the problem of the transfer of training from the classroom or clinic situation to daily life in the home (Perkins, 1980:93). Hence the aspects of institutionalisation of infant nutrition. Infant nutrition was seen as a 'body of knowledge', not a 'context of knowing' (McGuirk and Johnston, 1995)

Transmission learning mode

In the 1940's, mothers were given many formulas to remember by rote, with no questions asked:

> We used to stand at the foot of the bed and rattle off these complicated formulas to mothers – I have no idea how they ever followed them, or if

> indeed they ever did follow them. (Rich, 1998)

This was the ultimate transmission mode of learning. There were no choices for the mother, no sense of collaboration, of valuing differences or of reflection. There was certainly no exposure to different points of view. The health professional was cast in the role, whether they liked it or not, of the powerful, expert teacher and the mother was the obedient student in a completely non-negotiable learning situation. This applied equally in domestic science classes in schools, the doctor's surgery and the baby clinic. (Kociumbas, 1997:151) No doubt there were health professionals who rejected this role and there were mothers who defied the instructions. Nevertheless, the dominant learning mode was transmission:

> In a relationship where communication is controlled by the professional, the patient is less likely to play an active part in his own learning. (Perkins, 1980:164)

Critical understanding

The texts about infant nutrition imply impartiality, truthfulness and detached authority. The baby food manufacturers' products are listed in the same way as medications. (Gaisford and Lightwood, 1953:146, Kociumbas, 1997:161) Another remarkable feature of the 1940s paediatrics books is the brief attention given to breastfeeding, while a lengthy section is spent on bottlefeeding. In contrast, Thompson (1998) has 13 sections on breastfeeding and only five on bottle feeding and does not list the formulas. While the 1940s books say a child should be breastfed until nine months of age, they give no explanation for this advice, whereas Thompson (1998:59) is very comprehensive on weaning, and what constitutes a nutritionally adequate weaning diet, including breastmilk in that diet. It is doubtful that many health professionals, let alone mothers, questioned the authority of the earlier texts and the doctors who issued instructions. Freebody and Luke (1990) argue for the importance of a critical understanding of texts as:

> texts can effect both opportunity and exploitation: initiation into the roles of code-breaker, text participant, and text-user can open up new and powerful forms of bureaucratic colonisation, unless that initiation offers the tools of discourse analysis and critique. (14)

Desperate measures – transmission mode learning

Given the desperate situations that faced the health professionals in the 1940s, with babies presenting with scurvy, rickets, anaemia and malnutrition, (Rich, 1998) and the background of high infant mortality, (Kociumbas, 1997:154) it may have seemed that transmission mode learning was the only practicable method to effectively impart the information to the mother: that it was, under the circumstances, the most appropriate educational approach. This is, however, in contrast to much earlier methods and beliefs that '... the mother must educate herself' (Bull, 1851:11)

One could argue that surely transmission mode learning was the most effective way of changing practices for the better and as quickly as possible. However, there are two arguments against this. The first is that is that health professionals are not always 'right': they influenced mothers to adopt practices which are now considered to be at the least undesirable. The second reason is that this transmission approach to learning – to teaching mothers how to care for their infants, denies the individual and collective wisdom of women. It is, in the long term, ineffective because it creates mothers who feel they know nothing, can make no decisions and must always rely on 'expert' instruction. If women begin motherhood in this way, never having the opportunity to construct their own knowledge and determine their own actions, then it may become very difficult for these women to deal with problems as the child grows – tantrums, bullying, acne, whatever. Women become disempowered and the transmitted knowledge rather than the child is the focus.

Mothers construct their own knowledge

In contrast to the descriptions above, the learning model in evidence in the texts, classes and Nursing Mothers' Association of Australia booklets in the 1990s is a constructivist model: one where the information itself is seen as something socially negotiated. Mothers are introduced to the information in such a way that they can construct and reconstruct their own knowledge – they can find out for themselves, on the basis of a wealth of information, what suits them best. An essential ingredient of this constructivist approach to learning, in this case learning about infant nutrition and, by implication, about motherhood itself, is time. There needs to be time to listen, to ask questions, to propose solutions, ideas and feelings (*Nursing Review*, 1998:12). As well, there needs to be time for collaboration not only of mothers with health professionals, but also of mothers with mothers (*Nursing Review*, 1998:7). Finally, there needs to be an opportunity for mothers to engage in critical engagement in the problems and methods of mothering.

Today, mothers are given quite different, in fact radically different advice, and they are given the opportunity to construct their own knowledge in the area of infant nutrition (Dugard, 1997). In parent education classes, for instance those at The Canberra Hospital, they are told the benefits – nutritional, health and emotional – of breastfeeding. They are told about the physiology of lactation, so they can understand how milk is made, what is in it and why it is so good for the baby (NHMRC, 1996). We now know much more about the physiology of breastmilk and this knowledge questions the previous assumptions of the equivalence of formulas and breastmilk. When assessing progress of a baby, more data is taken into account. Today, health professionals take into account factors other than measurements, such as hereditary patterns of growth (Thompson, 1998:32-33). Finally, the days of test feeding, with a special 'test feed room' (Crisp and Ruddock, 1979:78, 84) at every baby clinic in Canberra, are gone. No longer do health professionals weigh the baby before and after a feed and on that basis judge the adequacy of the milk supply. If there are reasons to advise formula feeding, then the guidelines are specified very clearly and discussed with the mother.

Informed decisions

Rather than transmission mode learning today there is a feeling of partnership between health professional and mother: gone is the attitude of blaming the mother, instead, there is respect for her informed decisions. An example is Thompson's guidelines on feeding goat's milk to babies. She first sets out the reasons for not using goat's milk, and then she lists suggested practices for parents who do want to use goat's milk. There is a strong feeling that the mother is in charge and the scientific facts are there to help her to come to the 'right' decision – the health professional is there to support her:

> [Cow's] Milk is a dietary staple of major nutritional significance throughout infancy, but mothers need advice concerning the role of different milks. (Thompson, 1998:71)

Mathematical simplification

Much of the mathematical difficulty of the past has been removed by the standardisation of formulas. (Thompson, 1998:24) While there is still an onus on the mother for accuracy, that accuracy is easier to achieve. The complications of the 1940s formulas no longer exist. Rather than the complicated scales of the 1940s, they use as a guideline 150ml/kg bodyweight per day. Mothers are often advised to feed frequently and on demand. Health professionals no longer use language like 'established', 'feeding routine' or 'management', as Gaisford and Lightwood (1953) did. Most significantly gone is an attitude of doctors and other health professionals, controlling 'unreliable' mothers'. Maths is no longer being used as a way of controlling, as a way of medicalising, infant nutrition, and by implication, the mothers' relationships with their babies.

Mothers' resistance

There was gradual resistance to the restrictive and prescriptive style of mothering in Australia. This took various forms, including many doctors and infant welfare nurses coming to question the validity of the earlier practices as they observed that the demand-fed, breastfed babies did as well as, or much better then those on schedules, and/or formula fed (Rich, 1998). The establishment by Mary Paton of the Nursing Mothers' Association in 1964 was also an attempt to say to mothers that there were other ways of doing things. Current approaches to infant nutrition, as evidenced in the literature of the Nursing Mothers' Association of Australia, (Brodribb, 1995) and in current textbooks, seem to be more effective and more desirable. Mothers are encouraged to use their own knowledge and imagination, rather than substituting them for obedience to explicit mathematics, as instructed by the medical 'experts' (Dunham et al, 1991).

Final remarks

It is important to understand the origins of the mathematics and motherhood nexus, and to acknowledge that doctors, other health professionals and mothers did what they did with the knowledge available to them and within the cultural context of the times. It is also important to realise that the attitudes and approaches to learning of earlier times and their discourses, still exist in our society today, vying with the negotiated approaches to knowledge and practices around motherhood.

Doctors in Australia no longer see 'marasmic' babies, but they do see babies with failure to thrive and with infections despite the changes that have occurred in the information given to mothers and in the learning approaches employed by health professionals. Discourses of the 1940s, about controlling infant nutrition and by implication, the relationship of mother and baby, by means of weighing, measuring, counting and timing, still exist. The practices of the 1940s and 1950s involving transmission mode learning with the mother as obedient student also still exist.

Mothers should be aware of the use of mathematics in the discourses of motherhood, specifically infant nutrition, and the part that the discourses play in regulating (or not) their own practices. Understanding the complex nature of how explicit maths and the transmission mode of learning have been integrated and the ways this has dominated the lives of mothers caring for their babies is essential if taking control of their own practices is to become more widespread.

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IS MATHEMATICS DISCOVERED OR INVENTED?

This article appeared in the *Philosophy of Mathematics Education Journal 12*, November 1999. An abridged version was originally published as *New Angles on Old Rules* in the Times Higher Educational Supplement, 6 September 1996. For those with an interest in the philosophy of mathematics education, other articles are available on the POME website at http://www.ex.ac.uk/~PErnest/

Recently a heated debate between realists and relativists in science has erupted. The conflict is between those who see science as a rational description of the world converging on the truth, and those who argue that it is a socially constructed account of the world, and just one of many possible accounts. Typically scientists and philosophers of science are realists, arguing that science is approaching a true and accurate description of the real world, whereas social and cultural theorists support a relativist view of science, and argue that all knowledge of the world is socially constructed.

What has gone unnoticed in this debate is that there is a parallel and equally fundamental dispute over whether mathematics is discovered or invented. The absolutist view of mathematics sees it as universal, objective and certain, with mathematical truths being discovered through the intuition of the mathematician and then being established by proof. Many modern writers on mathematics share this view, including Roger Penrose in *The Emperor’s New Mind*, and John Barrow in *Pi in the Sky*, as indeed do most mathematicians. The absolutists support a ‘discovery’ view and argue that mathematical ‘objects’ and knowledge are necessary, perfect and eternal, and remark on the ‘unreasonable effectiveness’ of mathematics in providing the conceptual framework for science. They claim that mathematics must be woven into the very fabric of the world, for since it is a pure endeavour removed from everyday experience how else could it describe so perfectly the patterns found in nature?

The opposing is view often called 'fallibilist' and this sees mathematics as an incomplete and everlasting 'work-in-progress'. It is corrigible, revisable, changing, with new mathematical truths being invented, or emerging as the by-products of inventions, rather than discovered. So who are the fallibilists? Many mathematicians and philosophers have contributed to this perspective and I will just mention a few recent contributions. First of all, the philosopher Wittgenstein in his later works such as *Remarks on the Foundations of Mathematics* contributes to fallibilism with his claim that mathematics consists of a motley of overlapping and interlocking language games. These are not games in the trivial sense, but the rule-governed traditional practices of mathematicians, providing meanings for mathematical symbolism and ideas. Wittgenstein argues that we often follow rules in mathematical reasoning because of well-tried custom, not because of logical necessity. So Wittgenstein's contribution is to point out that it is what mathematicians do in practice, and not what logical theories tell us, which is the engine driving the development of mathematical knowledge.

Imre Lakatos is another fallibilist, and he argues that the history of mathematics must always be given pride of place in any philosophical account. His major work *Proofs and Refutations* traces the historical development of a result in topology, the Euler Relation, concerning the number of faces (F), edges (E) and vertices (V) of mathematical solids. For simple flat-sided solids, the relationship is F+V=E+2. However, proving this fact took over a hundred years as the definitions of mathematical solids, faces, edges and vertices were refined and tightened up, and as different proofs were invented, published, shown to have loopholes, and modified. Lakatos argues that as in this example, no definitions or proofs in mathematics are ever absolutely final and beyond revision.

Philip Kitcher offers a further refinement of fallibilism in his book *The Nature of Mathematical Knowledge*. He argues that much mathematical knowledge is accepted on the authority of the mathematician, and not based on rational proof. Furthermore, even when mathematical results are proved much of the argument is tacit and draws on unspoken mathematical knowledge learned through practice, as opposed to being completely written down explicitly. Since the informal and tacit knowledge of mathematics of each generation varies, mathematical proof cannot be described as absolute.

In my book *Social Constructivism as a Philosophy of Mathematics*, I argue that not only is mathematics fallible, but it is created by groups of persons who must both formulate and critique new knowledge in a formal 'conversation' before it counts as accepted mathematics. These conversations embody the process that Lakatos describes in the evolution of the Euler Relation, as well as what goes on in Wittgenstein's mathematical language games. Knowledge creation is part of a larger overall cycle in which mathematical knowledge is presented to learners in teaching and testing 'conversations' in schools and universities, before they themselves can become mathematicians and participate in the creation of new knowledge. This perspective offers a middle path between the horns of the traditional objective/subjective dilemma in knowledge. According to social constructivism, mathematics is more than a collection of subjective beliefs, but less than a body of absolute objective knowledge, floating above all human activity. Instead it occupies an intermediate position. Mathematics is cultural knowledge, like the rest of human knowledge. It transcends any particular individual, but not all of humankind, like art, music, literature, religion, philosophy and science.

Although fallibilist views vary, they all try to account for mathematics naturalistically, that is in a way that is true to real world practices. Unfortunately, fallibilism is too often caricatured by opponents as claiming that mathematics may be part or all wrong; that since mathematics is not absolutely necessary it is arbitrary or whimsical; that a relativist mathematics, by relinquishing absolutism, amounts to 'anything goes' or 'anybody's opinion in mathematics is as good as anybody else's'; that an invented mathematics can be based on whim or spur of the moment impulse; and that if social forces are what moulds mathematics then it must be shaped by the prevailing ideology and prejudices of the day, and not by its inner logic.

However these claims and conclusions are caricatures, and no fallibilist I know would subscribe to them. Fallibilism does not mean that some or all of mathematics may be false (although Gödel's incompleteness results mean that we cannot eliminate the possibility that mathematics may generate a contradiction). Instead, fallibilists deny that there is such a thing as absolute truth, which explains why mathematics cannot attain it. For example, 1+1=2 is not absolutely true, although it is true under the normal interpretation of arithmetic. However in the systems of Boolean algebra or Base 2 modular arithmetic 1+1=1 and 1+1=0 are true, respectively. As this simple example shows, truths in mathematics are never absolute, but must always be understood as relative to a background system. Unlike in physics, in which there is just one world to determine what is true or false, mathematics allows the existence of many different interpretations. So an assumption like Euclid's Parallel Postulate and its denial can both be true, but in different mathematical interpretations (in the systems of Euclidean and non-Euclidean geometries). Mathematicians are all the time inventing new imagined worlds without needing to discard or reject the old ones.

A second criticism levelled at fallibilism is that if mathematics is not absolutely necessary then it must be arbitrary or whimsical. Relativist mathematics, the criticism goes, by relinquishing absolutism amounts to 'anything goes'. Therefore an invented mathematics is based on whims or spur of the moment impulse. For example, Roger Penrose asks, are the objects and truths of mathematics "mere arbitrary constructions of the human mind?" His answer is in the negative and he concludes that mathematics is already there, to be discovered, not invented.

Plausible as this view seems at first, it is often argued on mistaken grounds. Mathematicians like Penrose often contrast necessity with arbitrariness, and argue that if relativist mathematics has no absolute necessity and essential characteristics to it, then it must be arbitrary. Consequently, they argue, anarchy prevails and anything goes in mathematics. However as the philosopher Richard Rorty has made clear, contingency, not arbitrariness, is the opposite of necessity. Since to be arbitrary is to be determined by chance or whim rather than judgement or reason, the opposite of this notion is that of being selected or chosen. I wish to argue that mathematical knowledge is based on contingency, due to its historical development and the inevitable impact of external forces on the resourcing and direction of mathematics, but is also based on the deliberate choices and endeavours of mathematicians, elaborated through extensive reasoning. Both contingencies and choices are at work in mathematics, so it cannot be claimed that the overall development is either necessary or arbitrary. Much of mathematics follows by logical necessity from its assumptions and adopted rules of reasoning, just as moves do in the game of chess. This does not contradict fallibilism for none of the rules of reasoning and logic in mathematics are themselves absolute. Mathematics consists of language games with deeply entrenched rules and patterns that are very stable and enduring, but which always remain open to the possibility of change, and in the long term, do change.

The criticism that relativism in mathematics means that "anything goes" and that "anybody's opinion is as good as anybody else's" can be countered by using William Perry's distinction between the positions of Multiplicity and Contextual Relativism. *Multiplicity* is the view that anyone's opinion is valid, with the implication that no judgements or rational choices among opinions can be made. This is the crude form of relativism in which the opposite of necessity is taken as arbitrariness, and which frequently figures in 'knockdown' critiques of relativism. It is a weak and insupportable 'straw person' position and does not represent fallibilism. *Contextual Relativism* comprises a plurality of points of view and frames of reference in which the properties of contexts allow various sorts of comparison and evaluation to be made. So rational choices can be made, but they always depend on the underlying contexts or systems. Fallibilists adopt a parallel position in which mathematical knowledge is always understood relative to the context, and is evaluated or justified within principled or rule governed systems. According to this view there is an underlying basis for knowledge and rational choice, but that basis is context-relative and not absolute.

This position weakens the criticism from absolutists that an invented mathematics must be based on whims or spur of the moment impulses, and that the social forces moulding mathematics mean it can blow hither and thither to be reshaped accorded to the prevailing ideology of the day. The fallibilist view is more subtle and accepts that social forces do partly mould mathematics. However there is also a largely autonomous internal momentum at work in mathematics, in terms of the problems to be solved and the concepts and methods to be applied. The argument is that these are the products of tradition, not of some externally imposed necessity. Some of the external forces working on mathematics are the applied problems that need to be solved, which have had an impact on mathematics right from the beginning. Many examples can be given, such as the following. Originally written arithmetic was first developed to support taxation and commerce in Egypt, Mesopotamia, India and China. Contrary to popular opinion, the oldest profession in recorded history is that of scribe and tax collector! Trigonometry and spherical geometry were developed to aid astronomy and navigational needs. Later mechanics (and calculus) were developed to improve ballistics and military science. Statistics was initially developed to support insurance needs, to compute actuarial tables, and subsequently extended for agricultural, biological and medical purposes. Most recently, modern computational mathematics was developed to support the needs of the military, in cryptography, and then missile guidance and information systems. These examples illustrate how whole branches of mathematics have developed out of the impetus given by external needs and resources, and only afterwards maintained this momentum by systematising methods and pursuing internal problems.

This historical view of fallibilism also partly answers the challenge that John Barrow issues to 'inventionism'. He asks if mathematics is invented how can it account for the amazing utility and effectiveness of pure mathematics as the language of science? But if mathematics is seen as invented in response to external forces and problems, as well as to internal ones, its utility is to be expected. Since mathematics studies pure structures at ever increasing levels of abstraction, but which originate in practical problems, it is not surprising that its concepts help to organise our understanding of the world and the patterns within it.

The controversy between those who think mathematics is discovered and those who think it is invented may run and run, like many perennial problems of philosophy. Controversies such as those between idealists and realists, and between dogmatists and sceptics, have already lasted more than two and a half thousand years. I do not expect to be able to convert those committed to the discovery view of mathematics to the inventionist view. However what I have shown is that a better case can be put for mathematics being invented than our critics sometimes allow. Just as realists often caricature the relativist views of social constructivists in science, so too the strengths of the fallibilist views are not given enough credit. For although fallibilists believe that mathematics has a contingent, fallible and historically shifting character, they also argue that mathematical knowledge is to a large extent necessary, stable and autonomous. Once humans have invented something by laying down the rules for its existence, like chess, the theory of numbers, or the Mandelbrot set, the implications and patterns that emerge from the underlying constellation of rules may continue to surprise us. But this does not change the fact that we invented the ‘game’ in the first place. It just shows what a rich invention it was. As the great eighteenth century philosopher Giambattista Vico said, the only truths we can know for certain are those we have invented ourselves. Mathematics is surely the greatest of such inventions.

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QUOTABLE NOTES AND NOTABLE QUOTES

A Design for Learning

The person who does the thinking does the learning.

Anon

Derek Holton’s Problem Solving Mathematics (à la Picasso)

A lesson plan for Problem Solving Mathematics:

1. Whole class introduction.
2. Group work.
3. Reporting back.
4. Topic notes.

The teacher’s role is to be a ‘guide on the side’ not a ‘sage on the stage’, and the aim is to establish solid foundations which facilitate deeper student understanding later.

To Holton “... the traditional way that we have taught mathematics is like painting by numbers.” What we’d like are more mathematical Picassos.

Adapted from Derek Holton,
Mathematical Gazette, July, 1999

Writing For Learners, In a Nutshell

"I (planned to) write so that the learner may always see the inner ground of the things (s)he learns, even so that the source of the invention may appear, and therefore in such a way that the learner may understand everything as if (s)he had invented it ..."

G. W. Leibriz

The Religion of Mathematics

"If religion is defined to be a system of thought which requires belief in unprovable truths, then mathematics is the only religion that can prove it is a religion."

Wry remark attributed to John Barrow
by Paul Davies in *The Mind of God*

An Example of The "Shut Up, I am Telling You" Approach to Mathematics Teaching

"The theory of limits entered my mind mechanically and crudely, not in a refined way but rather in a forced, policelike manner, in accordance with the formula:
'shut up, I am telling you'.

Russian mathematician N. N. Luzin, 1931

An Alternative Mathematics Education Lexicon

Assessment System
The major means of destroying learners' morale, aspirations and self esteem.

Classroom
Any inconvenient space unable to accommodate those persons allocated to it.

Education
A concept unknown in government circles.

Mathematics
A word much used by those who don't want schools to teach mathematics for that which they do want schools to teach.

Off Task
In the real world.

Planning
The principal method of ensuring that what is best for a student on a particular occasion is not allowed.

Private Schools
Where the teaching isn't free – as opposed to state schools, where the teachers aren't free.

Report
(1) That which, written by teachers will often give pain to a pupil.

(2) That which, written by government officials, will often give pain to teachers.

(3) That which, written by education researchers, won't be given a toss about by government officials.

Responsibility, Delegation of
Means whereby a teacher on a low salary does the work for which someone on a high salary gets paid.

Voluntary
What mathematics ought to be at all ages – as opposed to banned, which is what numeracy ought to be below the age of 14.

Wrong
Ideally what a teacher always is; in practice, what a pupil often is; by definition what a government minister never is.

Yesterday
The time by which everything a teacher has to do has to be done, as opposed to tomorrow, the time by which everything a teacher needs to do won't be done.

Extracted and adapted from The Politically Incorrect Mathematics Teacher's A to Z of Education, by Jonathan MacKernan, published in *Mathematics Teaching 167*.

When is a Problem Not a Problem?

In mathematics of course. A good problem is what mathematicians yearn for; it signifies interesting work.

Adapted from Phillip A. Griffiths

Two John von Neumann Anecdotes

The Brilliant Mathematician ...

Von Neumann was held in awed respect by his colleagues who claimed that 'While he was indeed a demi-god, he had made a detailed study of humans and could imitate them perfectly'.

Quoted in *The Pleasures of Counting* by T. W. Körner

and The Butt of a Limerick

Von Neumann had proved the existence of invariant measures on compact topological groups, but tried to discourage Alfred Haar from working on the locally compact case on the grounds that it seemed unlikely to be true in that generality. Fortunately Haar persisted:

> Said a mathematician named Haar,
> Von Neumann can't see very far.
> He missed a great treasure:
> They call it Haar measure.
> Poor Johnny's just not up to par

Paul R. Chernoff, *The American Mathematical Monthly, November 1999*

PROBLEMS AND ACTIVITIES

Remember that we include a coding system which attempts to indicate in terms of Year levels the suitability range for each item. Thus 6–8 suggests an item accessible to students from Year 6 to Year 8.

(1) Shopping with Brian and Magda

6–12

Brian and Magda want to buy the same music CD which costs a whole number of dollars. However Brian is $24 short of the price of the CD and Magda is $2 short. They decide to pool their funds and buy one copy of the CD. When they do, they find they still do not have enough money. How much does the CD cost?

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(2) Reciprocating Equations

9–12

Solve this system of equations to find x, y and z.

$$\frac{x+y}{xy} = \frac{1}{3}$$

$$\frac{y+z}{yz} = \frac{1}{7}$$

$$\frac{z+x}{zx} = \frac{1}{5}$$

(3) Arrayngement

10–12

The rectangular array

1	3	6	10	15	…
2	5	9	14	20	…
4	8	13	19	26	…
7	12	18	25	33	…
11	17	24	32	41	…
16	23	31	40	50	…
⋮	⋮	⋮	⋮	⋮	⋮

is constructed by placing the integers 1,2,3,... in order starting from the top-left corner and working along bottom-left to top-right diagonals in the (row, column) position sequence (1,1), (2,1), (1,2), (3,1), (2,2), (1,3), (4,1), (3,2), (2,3), (1,4), (5,1)... .

In what row and column does 2000 occur?

(4) A Mediated Triangle

10–12

Construct a triangle given the lengths a and b of two sides and the length m of the median to the third side

(5) Factorial of a Product Inequality

11–12

Show that if p and q are positive integers then $(pq)! \geq (p!)^q\,(q!)^p$,
where for m a positive integer, m! denotes the product of all positive integers up to m.

THUMBNAIL (and longer) REVIEWS

Readers are welcome to contribute to this section. Reviews can cover books, periodicals, videos, software, CD ROMs, calculators, mathematical models and equipment, posters, etc.

The Number Sense
How the Mind Creates Mathematics
by Stanislas Dehaene
Published by Oxford University Press, 1997.

Dehaene is a mathematician turned neuropsychological researcher at the Institute de la Santé et de la Recherche Médicale à Paris. He became fascinated with numbers. “Where did they come from? How was it possible for my brain to understand them? Why did it seem so difficult for most people to master them? …Why did all languages have at least some number names? Why did everybody seem to find multiplication by 7, 8 or 9 particularly hard to learn? Why couldn’t I seem to recognise more than four objects at a glance? …What tricks allowed lightning calculators to multiply two three digit numbers in a few seconds?” As he “learned increasingly more about psychology, neurophysiology and computer science, it became obvious that the answers had to be looked for…in the very structure of our brains.” Dehaene became an active participant in a new field of science at an exciting time with functional imaging tools becoming available. This field was mathematical cognition, “the scientific enquiry into how the human brain gives rise to mathematics.” This book provides a summary of what researchers into mathematical cognition have discovered.

The final paragraph of the Introduction summarises Dehaene’s biological and cultural tour. Chapters 1 and 2 summarise current knowledge about animal and human abilities in arithmetic, suggesting that our mathematical abilities have biological precursors. Chapter 3 shows how many traces of the animal mode of processing numbers are still at work in adult human behaviour. Chapters 4 and 5 move from observation of how children learn to count and calculate to attempt to understand how children's initial approximate system can be supplanted, and the difficulties that the acquisition of advanced mathematics raises for our primate brain. There are asides here which examine current approaches to maths teaching and the extent to which they fit our mental architecture. Chapter 6 examines the characteristics which distinguish a young Einstein or a calculating prodigy from the rest of us. Chapters 7 and 8 lead us into the fissures of the cerebral cortex, where the neuronal circuits which support calculation are located, and from which, alas, they can be dislodged by a lesion or vascular accident, thus depriving otherwise normal people of their number sense.

This book is treasure trove for all mathematics learners. The emerging science of mathematical cognition is only at a rudimentary stage, but I don't believe an interested reader could emerge from this book without new perspectives and much food for thought, follow up and investigation. For example, as regards animal arithmetic Dehaene points out "an undisputed and widespread ability to apprehend numerical quantities, to memorise, to compare and even to add them approximately." Also " a considerably lesser ability, probably confined to a few species, for associating a repertoire of more or less abstract behaviours, such as pointing to an abstract digit or to numerical representations". Dehaene's simple working hypothesis is that "we are endowed with a mental representation of quantities very similar to...that...found in rats, pigeons or monkeys." And babies less than a year old can count too. Piaget's notion of conservation and assertion that babies are devoid of a genuine concept of number are confounded because the tests he favoured "do not enable children to show what they are really capable of". Mathematical entities like $\sqrt{2}$ and $\sqrt{-1}$ "are so difficult for us to accept and so defy intuition because they do not correspond to any pre-existing category in our brain.

There is much practical wisdom here for all maths educators. "When there is a passion for mathematics, talent does not lag behind it. If, conversely a child develops maths anxiety, this phobia can prevent even the simplest of mathematical concepts from falling into place." Dehaene explains complex material in a generally accessible manner with fascinating examples such as animal mathematicians, idiot savants, newborn infants and split brain patients all as a means of elucidating aspects of our sense of number. From the innate endowment of numeracy, via the origin of Roman numerals, to the latest MRI results and "the unreasonable effectiveness of mathematics", this book, complete with notes, references and index is timely and important.

Peter Enge

Mathematical Encounters of the 2nd Kind
by Philip J. Davis
Published by Birkhauser, 1997.

Reading this book is like a long, pleasant conversation. You can remember what you talked about, and what people said, but sometimes it is hard to figure out how you got from one topic to another. There are four tales in this conversation, bound together by the same smooth and invisible threads that make friendly discourse seem to flow so effortlessly.

Davis admits that he is "fond of listening to and telling stories," and boasts that he has been called a "tangentialist." This is a book of tangents, encounters of the "second kind." Mathematics itself constitutes the "first kind."

There is a little bit of the mathematics of the "first kind" in the first of the four sections of the book, "Napoleon's theorem." The theorem states that the centers of equilateral triangles drawn on the sides of a given triangle are the vertices of yet another equilateral triangle. The theorem is an example of some of the geometry of triangles that was not known to Euclid. As a youth, Davis came across this theorem as an exercise, and he found that he was unable to solve it. He eventually gave up, but, like a bad penny or a seasonal allergy, he kept coming across it again and again.

There are some who suspect that Napoleon's theorem is not really Napoleon's, but, perhaps LaGrange's, or someone else's. (Just as there are some who claim that William Shakespeare didn't write all those plays, but someone else who had the same name wrote them.) The second section seems to be an account of Davis futile search for the true roots of Napoleon's theorem. It only seems to be that. It is really a sketch of the New England humanist and classicist Alexander Sedgewick Carpenter, Davis' friend and sometime collaborator in the search for the genealogy of Napoleon's theorem.

The third and shortest section of the book describes Davis' graduate education, his relationship with his advisor, Stefan Bergman, the crush Davis had on movie star Liesel Bergner, and how both Bergman and Bergner endured the loneliness of exile from their homelands. Davis repeats a story about Bergman and his wife, Dr. Hilda Geiringer, and how they missed the European ways with which they had grown up. Davis quotes Clifford Truesdell: "In later years Hilda Geiringer told my wife and me a story about Bergman. When all America had come to call everybody Bill and Jane, he asked her if he could speak to her alone in complete secrecy. She agreed. He said: I know that now in public we have to be Hilda and Stefan, but when we are alone together, may I still call you Frau Dr. Geiringer and will you call me Herr Dr. Bergman?"

The last and longest section, "The Rothschild I Knew," is an account of Davis' friendship with Lord Victor Rothschild. The friendship began in January, 1986, when Rothschild sent Davis a note about a small error he had found in one of Davis' books, and it ended with Rothschild's death in March, 1990. In just four years, they managed to become old friends.

The book concludes with some notes about the number theory that provided some of the fuel for their friendship. Thus, the book closes a number of circles, from Euclid as a geometer to Euclid as a number theorist, from Napoleon to Rothschild as amateur mathematicians, from Davis New England friend Alexander Sedgwick Carpenter to his English friend Lord Victor Rothschild.

Mathematical Encounters of the 2nd Kind is a very pleasant book, an appealing tale of the life and friends of a mathematician. It is well worth the time it takes to read it.

This review by **Ed Sandifer**,
Western Connecticut State University,
is reprinted from MAA online.

Note: Professor Emeritus Philip Davis will be a keynote speaker at the CMA hosted AAMT 2001 Conference next January.

The Mathematical Olympiad Handbook: an introduction to problem solving based on the first 32 British Mathematical Olympiads 1965–1996
by A. Gardiner
Published by Oxford University Press, 1997.

"This is unashamedly a book for beginners." So begins this wonderful book by A. Gardiner. The goal of this book is to introduce students to the world of problem solving, and it does so marvellously. The preface indicates that the book is aimed at students aged 15 or 16 and above, but this should not prevent older students from picking up this book and learning how to tackle Olympiad type problems. Indeed, problem lovers of all ages and ability levels will find much in this book to entertain and educate.

The book consists of three main sections. Part I contains over thirty pages which set out to teach and/or review much of the mathematical background needed to tackle the problems in Part II. Part II contains the problems themselves. All of the problems from the British Mathematical Olympiad Round One papers dating from 1965 through 1996 are included. Part III contains hints and outline solutions to the problems in Part II.

While it is impossible to fit a review of all necessary mathematics into 34 pages, Gardiner does a remarkable job of covering the basics. Topics in Part I include Numbers, Algebra, Proof, Elementary Number Theory, Geometry, and Trigonometric Formulae. Some of the topics within these sections are explained in some detail so a student who has little or no familiarity with the topic will be able to grasp the material. Other topics consist mostly of the basic facts (sometimes requiring the reader to supply the proofs or derivations of formulae -- good preparation for the problems in Part II). Gardiner has made very good decisions on what to include in the limited space allotted for this material. Also, he includes an additional 10 pages devoted to a categorized list of books on mathematics and/or problem solving which will certainly help satisfy an interested reader's curiosity.

There is not much to say about Part II of the book other than to say that it contains an entertaining and challenging set of problems.

Part III is where the book truly shines. It does NOT contain solutions to the problems from Part II. Instead, it contains hints and outlines of solutions. Thus, a reader finding herself stuck on a problem can turn to the hints to get help, but will not have the fun entirely spoiled by seeing the fully worked out solution. The hints and outlines are complete enough that a bright student (who is willing to do some work) will be able to fill in the gaps and solve the problem herself. Additionally (and importantly), the solutions are *not* slick proofs which leave students thinking "I would never have thought of that." Instead, the solutions are at a level appropriate for the intended audience, and they make use of the material contained in Part I of the book. This will likely increase the confidence of the reader and make the book that much more exciting to work through.

As an illustration of the structure of the book, I include the following simple example from the 21st British Mathematical Olympiad, 1985:

> Given any three numbers a, b, and c between 0 and 1, prove that not all of the expressions a(1-b), b(1-c), and c(1-a) can be greater than 1/4.

The first hint encourages the reader to experiment a little bit ... to note that for various choices of a, b, and c, it is always true that at least one of the expressions is less than or equal to 1/4.

The second hint asks the reader to prove that if a is between 0 and 1, then a(1-a) ≤ ____. The reader needs to fill in this blank *and* complete the proof. It is pointed out that the AM-GM inequality (discussed in Part I) can help here.

The third hint tells the reader to consider the **o*u** of the three expressions a(1-b) * b(1-c) * c(1-a) and to apply the result from the second hint (after rearranging the terms) to complete the proof.

The hints in the book are somewhat wordier than this. They give the reader the chance to discover what she needs without seeing too much too soon. While not (quite) completely giving it away, the hints are designed to carefully lead the reader to discover the proof for herself.

I would highly recommend this book for anyone interested in this sort of problem solving. It would be a particularly valuable resource for those who participate in mathematics competitions at the high school or college level.

This review by **Carl D. Mueller,** Georgia Southwestern State University, is reprinted from MAA Online.

SOLUTIONS TO PROBLEMS AND ACTIVITIES

(1) Since Magda is \$2 short, Brian can only have nothing or \$1. So, assuming Brian actually does have some money to contribute, it must be \$1, and the CD costs \$25.

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(2) We rewrite the system of equations in the form

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{3} \qquad (1)$$

$$\frac{1}{y} + \frac{1}{z} = \frac{1}{7} \qquad (2)$$

$$\frac{1}{z} + \frac{1}{x} = \frac{1}{5} \qquad (3)$$

Subtracting (2) from (1) gives

$$\frac{1}{x} - \frac{1}{z} = \frac{4}{21} \qquad (4)$$

Adding (1) to (3) gives $\frac{2}{x} = \frac{41}{105}$,

$$\text{so} \quad x = \frac{210}{41}.$$

Substitution in equations (1) and (2) yields the solution

$$x = \frac{210}{41}, \ y = \frac{210}{29}, \ z = 210$$

for the system.

---

(3) The numbers along the first row are the triangular numbers

$$1, 3, 6, 10, 15, ..., \frac{n(n+1)}{2}, ...$$

These numbers are also the last numbers on each of the bottom-left to top-right diagonals. If we can find the largest triangular number less than or equal to 2000 we will be able to locate 2000 in the array.

The largest integer satisfying

$\frac{n(n+1)}{2} \leq 2000$ is $n = 62$, and

$\frac{62 \times 63}{2} = 1953$. Hence 2000 is on the 63rd bottom-left to top-right diagonal.

Since 2000 − 1953 = 47, it is in position (63 − 47 + 1, 47) = (17, 47).

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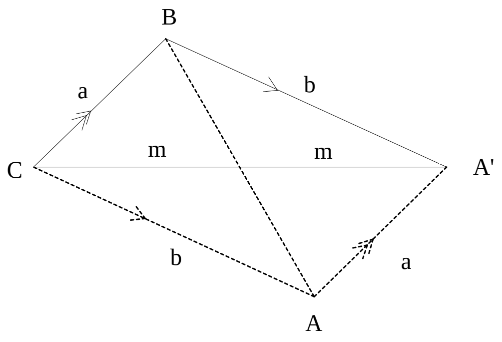
(4) Our solution depends on the fact that the diagonals of a parallelogram bisect.

Construct a triangle BCA' with sides of length a, b and 2m as shown in the diagram. On CA' construct the triangle CA'A so that CA has length b, AA' has length a, and hence BCAA' is a parallelogram



Then each of the triangles ABC and BA'A satisfies the requirements of the problem.

(5) We can split (pq)! into the product of p blocks of q consecutive integer factors as follows.

$$\begin{aligned}(pq)! = \ & 1 \times 2 \times ... xq \\ \times \ & (q+1) \times (q+2) \times ... \times 2q \\ \times \ & (2q+1) \times (2q+2) \times ... \times 3q \\ \times \ & ... \\ \times \ & ((p-1)q+1) \times ((p-1)q+2) \\ & \times ... \times pq\end{aligned}$$

A typical line of q factors on the right hand side of this last expression has the form $(mq + 1) \times (mq + 2) \times ... \times (m + 1)q$, with m an integer such that $0 \leq m \leq p - 1$.

But for $1 \leq n \leq q$,

$$mq + n \geq mn + n = (m + 1)n,$$

and so we have the inequality

$$(mq + 1) \times (mq + 2) \times ... \times (m + 1)q$$
$$\geq 1(m + 1) \times 2(m + 1) \times ... \times q(m + 1).$$

Making repeated use of this last inequality in the previous expression for (pq)! gives

$$\begin{aligned}(pq)! \geq \ & 1 \times 2 \times ... \times q \\ & \times 2 \times 4 \times \quad \times 2q \\ & \times 3 \times 6 \times ... \times 3q \\ & \times ... \\ & \times p \times 2p \times ... \times pq \\ = \ & p! \times 2^p p! \times ... \times q^p p! \\ = \ & (p!)^q (q!)^p.\end{aligned}$$