Short Circuit

Newsletter of the Canberra Mathematical Association INC

VOLUME 12 NUMBER 9 SEPTEMBER 2021

NEWS AND COMMENT

The following statement has been added to the CMA website. It seemed appropriate to reproduce it here.

'CMA (and AAMT) exists to support all teachers of Mathematics in the ACT and surrounding regions.

'The current lockdown is a period of considerable uncertainty and stress for all of our teachers as they transition to on-line teaching – a challenge at all levels from Foundation to Year 12. As well, we have a significant number of teachers and school staff who are not only in lockdown, but also in quarantine, with this current COVID outbreak impacting a number of younger people including our students.

'Please know that we are here to support all of you.'

The website has links to some resources. We aim to update and add to this list. Perhaps you have a resource that you find especially useful when preparing for on-line learning? Let us know and we will be pleased to share any link with our community.

With the pestilence continuing, we have had to make some changes to our advertised programs.

The CMA conference for 2021 is cancelled; the Wednesday workshop with Kristen Tripet on September 8 is now online; and the workshop with Jono Adams set for September 9 has been moved to Term 4. See page 3 for details.

The AAMT virtual conference: *Future Proofing* Australia's mathematical capacity—is happening. See page 3 to make a booking.

There is also a link on page 3 to the ACU Mathematics Teaching and Learning Centre's series of webinars.

CANBERRA MATHEMATICAL ASSOCIATION

Inside:

Coming Events:

AAMT virtual conference 29-30 September. Theme: 'Future Proofing'

AGM: 10 November.

Wednesday Workshop:

September 8 with Kristen Tripet, online. See page 3.

MEMBERSHIP

Memberships run from 1 Jan to 31 Dec. each year. Membership forms can be accessed from the CMA website: http://www.canberramaths.org.au

Membership of CMA includes affiliation with the Australian Association of Mathematics Teachers and a subscription to one of two AAMT journals.

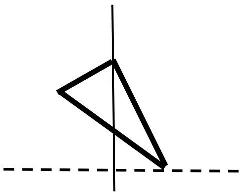
As a member, you are entitled to attractive rates for the CMA annual conference and CMA professional development events.

CMA members may attend conferences of the AAMT affiliates in other states, MAV, MANSW, etc. at member rates.

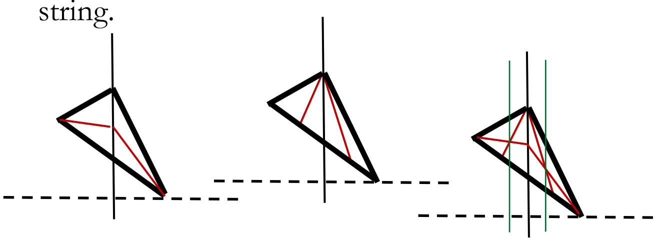
PUZZLES

1. Physics speaks maths—an investigation

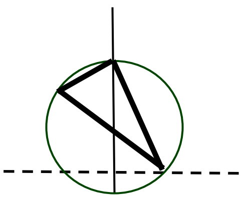
Imagine, or better still, construct, a triangular lamina freely suspended by a string passing through a vertex. At equilibrium, the string is vertical and the lower edge of the triangle makes a certain angle with the horizontal.



- Observe that the vertical string seems to coincide with a median of the triangle.
- Mathematically, this would imply something special about the two triangles on either side of the string.
- The physicist is making a study of balance in a triangular lamina. The 'median' observation inspires the physicist to insert further medians in the two parts of the divided triangle and then to deduce mathematically how far the resulting intersection points would be from the vertical string.



- In the special case of a right-triangle suspended at the 90° vertex there is another feature of the shapes of the two triangular parts of the lamina. It comes from the idea of fitting the triangular shape into a circle.



- In the right-triangle case, what can be said about the angle between the lower edge with the horizontal, in terms of the acute angles?

ONCE IN A HUNDRED

By Ed Staples

The Covid Pandemic has proven to be one of the toughest challenges the world has had to face in modern times. According to the World Health Organization figures, as at the end of July 2021, there have been more than 194 million confirmed cases and over 4 million deaths.

Recently we have heard state political officials in Australia proclaim that this pandemic is a 'once in a 100 year' event. But what does that mean and why are they mentioning it? How does that phrase fit with a pandemic that lasts, say, 4 years? Do the statistics back up such a notion anyway?

Throughout history, as humans have spread across the world, infectious diseases have been constant companions. The historical record includes devastating diseases such as the bubonic plague (1347-51), smallpox (1520), the European great plagues of the 17th and 18th Centuries, cholera (1817-1923), yellow fever (late 1800s), the Russian flu (1889-90), the Spanish flu (1918-19), the Asian flu (1957-58), the Hong Kong flu (1968-70), and HIV-Aids (1981 to the present) etc.

About 100 years ago, the Spanish flu infected 500 million people with fatalities estimated to be anywhere between 20 million and 50 million, and it's likely that it is this event that the authorities are using to substantiate the 100-year event statement. But is this claim reasonable?

While a flood or a meteor strike may happen at a sharply-defined time within a longer period, an epidemic has a duration. If we were to think about a 100-year pandemic in the same way as a 100-year flood, we might, in order to calculate a probability, focus on the moment the outbreak commenced. Then, we might define the event of interest to be the beginning of the pandemic and say that the probability is 0.01 of it occurring in any one year.

Assuming (heroically) that successive outbreaks are independent of one another, and that we live in a world whose attributes are unchanging, we could model the situation with either the Binomial or the Poisson probability distributions and show that there is a 63.4% chance of one or more of them occurring in any 100-year period. In other words, while there is a mathematical expectation of 1 pandemic per century, there remains a 36.6% chance that no such pandemic will occur at all in that time interval.

Alarmingly, according to the Poisson model, which is useful for rare events, and assuming no complications, if a pandemic commences on average once in 100 years, there is an 18% chance of two of them occurring in a 100-year period and a 6% chance of three.

We might surmise, generously, that the statement 'once in a 100 years' is being used by politicians to highlight the seriousness of the situation. Perhaps, more cynically, we might think that the phrase is being used to explain the general unpreparedness and why responses to the pandemic have been as though the situation were novel. After all, no one is alive today with any prior experience in dealing with such things, and that therefore mistakes are not only to be excused, but indeed expected.

What are your thoughts on the statement? Why not discuss this in your classroom.

Reference: WHO

FROM LOCKHART'S LAMENT

> 'Teaching is not about information. It's about having an honest intellectual relationship with your students. It requires no method, no tools, and no training. Just the ability to be real. And if you can't be real, then you have no right to inflict yourself upon innocent children.'

From Paul Lockhart's *A Mathematician's Lament* *(2002)* - a 25-page essay reprinted in 2008 in Keith Devlin's column Devlin's Angle in *MAA Online*.

Lockhart criticises mathematics education in the USA as it was 20 years ago. His comments may apply to aspects of current Australian practice.

CMA CONFERENCE 2021 CANCELLED

We had hoped to reschedule our cancelled August conference. However, this is no longer possible due to continuing venue restrictions.

We do, however, plan to bring you more online workshop opportunities in Term 4

AAMT VIRTUAL CONFERENCE

Future Proofing Australia's mathematical capacity

29 - 30 September—an event to engage and activate you in your profession, and deliver best practice mathematics teaching ideas and resources. AAMT Members $135.

ACU MASTERCLASS SERIES

The Australian Catholic University—Mathematics Teaching and Learning Centre (MTLC) is offering, at modest cost, a series of masterclasses by webinar.

There are 12 webinars, running from September to December with various target audiences within the primary and junior secondary areas.

For details click this link.

CMA WORKSHOPS

WORKSHOP 1 with Kristen Tripet

This workshop is now going on-line! Please join Kristen and CMA on **Wednesday 8th September from 4-5pm.**

Attend the workshop from the safety of your own space – all you need is pencil and paper. Register by email at **canberramaths@gmail.com**.

*Include your **email contact** so that we can send you the Zoom link. We look forward to having you join us*

WORKSHOP 2 with Jono Adams

This face-to-face workshop is **cancelled**. However, we will bring this to you in an on-line version in Term 4 (Week 2).

Watch this space for further information.

NEWSLETTER OF THE CANBERRA MATHEMATICAL ASSOCIATION INC

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We're on the Web!
http://www.canberramaths.org.au/

ABOUT THE CMA

The Canberra Mathematical Association (Inc.) is the representative body of professional educators of mathematics in Canberra, Australia.
It was established by, among others, the late Professor Bernhard Neumann in 1963. It continues to run - as it began - purely on a volunteer basis.

Its aims include

* the promotion of mathematical education to government through lobbying,
* the development, application and dissemination of mathematical knowledge within Canberra through in-service opportunities, and
* facilitating effective cooperation and collaboration between mathematics teachers and their colleagues in Canberra.

THE 2021 CMA COMMITTEE

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BENFORD'S LAW

One has to be approaching a certain age to remember when tables of logarithms were used to carry out calculations. Today, electronic devices hide this mathematical marvel from us, making the use of printed tables unnecessary. It was not so when astronomer Simon Newcomb published his *Note on the Frequency of Use of the Different Digits in Natural Numbers*, in the American Journal of Mathematics, 1881, Vol. 4, No. 1.

Newcomb's paper begins with the observation:

> That the ten digits do not occur with equal frequency must be evident to anyone making much use of logarithmic tables, and noticing how much faster the first pages wear out than the last ones. The first significant figure is oftener 1 than any other digit, and the frequency diminishes up to 9.

It seems natural to expect that in a more-or-less random collection of numbers, the initial digits 1 to 9 would occur about equally often. However, it turns out that the phenomenon of decreasing first digit frequencies is widespread, well beyond Newcomb's mathematical tables. It occurs in many naturally occurring and manmade sets of data. Dr Frank Benford, a physicist working for the General Electric Company, rediscovered this strange fact in 1938 and it came to be known as Benford's Law. A satisfactory explanation for Benford's Law was worked out only as recently as 1996 by Theodore P. Hill, an American professor of mathematics.

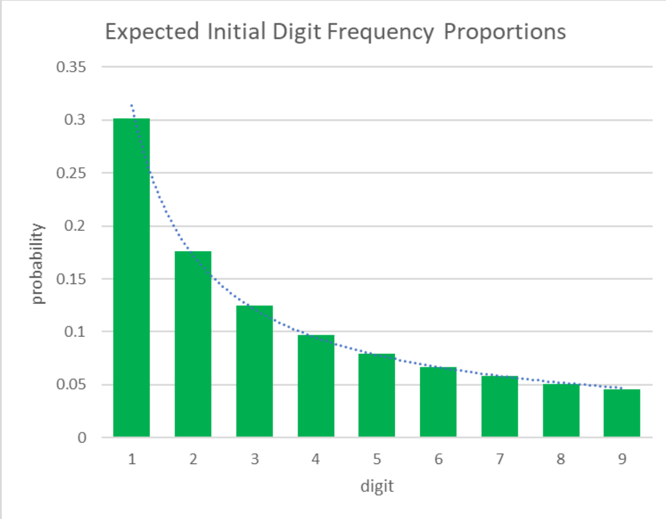
Forensic accountants can use Benford's Law as a tool in the detection of fraud. It must be said, however, that a criminal's knowledge of Benford's law might help them in concealing the crime.

As Newcomb and Benford discovered, in many data sets the distribution of the initial digits tends towards a precise rule. If d stands for the initial digit, 1 to 9, then the probability P_d that d is the initial digit in a number that is randomly selected from a given data set is given by the formula

$P_d = \log(1 + 1/d)$.

The 'log' in the formula is the base 10 logarithm. That Newcomb discovered the phenomenon in logarithm tables, is hardly coincidental.

The probabilities are interpreted as proportions so that for $d = 1$ the formula gives 0.301 meaning that of 1000 numbers, about 300 will begin with a 1. Similarly, when $d = 9$, the formula gives 0.046 meaning 46 out of a thousand numbers are expected to begin with a 9.



Fraudsters who have not heard of Benford's Law are likely to concoct data that does not follow the logarithmic distribution and so leave themselves open to exposure.

To begin to understand how data sets come to have the Benford distribution in their initial digits, there is an experiment one can do using a spreadsheet. Multiply each of 10 random numbers by each of 10 further random numbers. Count the occurrences of the initial digits 1 to 9 in the resulting 100 products. The proportions for each initial digit will be similar to the Benford distribution.

Such an experimental data set arises from a single process of multiplication. Experiments in which *three* or more sets of uniformly distributed random numbers are multiplied in pairs, approximate the logarithmic distribution of initial digits still more closely. In an accounting setting, numbers typically arise from chains of multiplications. Prices, for example, and hence payments, are often arrived at by combinations of multiples of the unit costs of various in-

Continued from page 5

puts. Thus, it is plausible that the distribution of initial digits in data from accounting and financial contexts should be skewed to the right as in the Benford distribution.

In a paper on Benford's law and random numbers, A. E. Kossovsky relates a fable that illustrates another way in which logarithmic initial digit data might be generated. To paraphrase Kossovsky:

> Members of an ancient society were permitted to have at most 9 gods, 8 olive trees, 7 fruits, 6 chickens, 5 sheep, 4 dogs, 3 slaves, 2 houses and 1 wife, and they had to have at least one of each. In conversations among the members of the community mentioning counts of these things, it is likely that the number 'one' would be mentioned most because it can come up in relation to any of the objects. 'Nine', on the other hand would only arise in relation to one kind of object: gods, and so would be mentioned least.

When one works out the likely frequencies of occurrence of each of the nine digits over the course of a conversation, a Benford-like distribution is obtained.

Consistent with the general explanation given by Theodore P. Hill, the act of combining samples drawn from separate distributions has led to the logarithmic distribution. In this case, each topic of conversation generates numerical data with its own characteristic frequency distribution, and their combination produces the logarithmic shape.

Kossovsky's idea suggests more realistic scenarios relating to data from accounting. Suppose, for example, that payments are made in, say, forty categories. Each category has a randomly determined upper limit somewhere between \$100 and \$100 000. Within each category several payments are to be made and the amounts are assumed to be evenly distributed over the category's range. Such a scenario can be simulated readily by Excel spreadsheet using the '=RAND()' function, and the initial digit distribution produced in this way does indeed tend to have the logarithmic shape.

Not all data sets have the Benford property, however. Numbers from a local telephone directory and numbers assigned to houses in a particular street do not, for example. So, while a surprising array of data sets do have the property, we cannot say for certain that failure to have it is evidence that a particular set of numbers has been falsified. On the other hand, such an observation might, and sometimes does, prompt a closer look at the source of the numbers.

PT

NMSS 2022

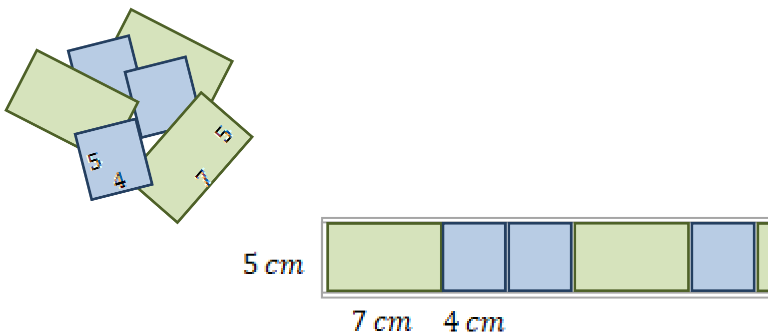
Three applications from Year 11 ACT school students have been forwarded to the organisers of the National Mathematics Summer School for 2022. This number is considerably less than in pre-Covid years. Selected students will be advised shortly.

The organisers noted the absence of female applicants. They are looking for ways to address the issue of gender equity in the ACT cohort for future years. Ideas from readers would be welcome.

A PUZZLE WITH TILES

You have a plentiful supply of ceramic tiles that come in two sizes, 5 × 4 cm² and 5 × 7 cm². The tiles are needed to cover a strip 5 cm high and 902 cm long. No tiles are to be cut and the strip is to be covered exactly. Any number of tiles of either size may be used.

- Can this be done?
- Assuming the tiling is possible, what is the smallest number of tiles required?
- What combination would use the most tiles to achieve the tiling?



PUZZLE SOLUTIONS from Vol 12 No 8

1. Billions factorised

In factorising the number 123,456,789 it is useful to know that there is a pair of 5-digit divisors. It is easy to determine that the prime factor 3 occurs exactly twice, and that there are no other small prime factors. After dividing by 9 we have 13,717,421 whose square root is about 3704. Trying 4-digit divisors less than this, ending with 1, 3, 7 and 9, we soon discover that 3607 × 3803 = 13,717,421. The two 5-digit divisors of 123,456,789 must be 10,821 (= 3 × 3607) and 11,409 (= 3 × 3803).

2. A fraction of a fraction

With Epsilon one pays 80% of 70% of the full price. At Omicron one pays 70% of 80% of the full price. There is no difference between them.

3. Vice versa, again

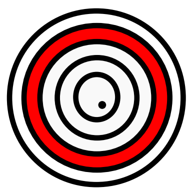
17.5% of $50 is the same as 50% of $17.50.

4. **Odd**

There is no odd number without an 'e' in its English spelling.

5. Bullseye

The white area inside the red region measures $3^2\pi$. The white area outside the red region measures $5^2\pi - 4^2\pi$ which is again $3^2\pi$.



6. Unlikely story

It is usually not true that $\sqrt{(x + x/y)} = x\sqrt{(x/y)}$. However, it is true in the infinitely many cases in which $y = x^2 - 1$.

7. Bermuda triangle

If the hypotenuse of a right triangle is 100 km, the altitude from that side is at most 50 km (by Thales theorem). An altitude of length 60 km is impossible. Hence the area does not exist.