SHORT CIRCUIT

Newsletter of the Canberra Mathematical Association INC

VOLUME 13 NUMBER 2 FEBRUARY 2022

NEWS AND COMMENT

Welcome to the new school year!

A CMA welcome event in the form of a workshop is expected to be announced shortly to everyone on our mailing list.

In these uncertain times tentative plans are made for all sorts of initiatives but setting firm dates can be a last-minute thing. For this and other reasons, communication with CMA members and friends via the Short Circuit newsletter appears to be valued and this, at least, is likely to continue despite the pestilence.

With communication in mind, it would be good to hear from any readers who might put their recent observations and experiences related to teaching, on (digital) paper. Short Circuit is always open to contributions from you, the reader.

On page 2 there is information about the 2022 International Mathematical Modeling Challenge. The IM²C program in schools is set to occur from 8 - 24 March but now is the time to see about registering your team(s).

Stay tuned also for the National Maths Talent Quest which will be available again this year for ACT students.

The Easter break this year occurs on 15 - 18 April but we note that hot-cross-buns can be purchased in supermarkets already.

CANBERRA MATHEMATICAL ASSOCIATION

Inside:

Coming Events:

2022 CMA conference

Wednesday Workshops:

MEMBERSHIP

Memberships run from 1 Jan to 31 Dec. each year. Membership forms can be accessed from the CMA website: http://www.canberramaths.org.au

Membership of CMA includes affiliation with the Australian Association of Mathematics Teachers and a subscription to one of two AAMT journals.

As a member, you are entitled to attractive rates for the CMA annual conference and CMA professional development events.

CMA members may attend conferences of the AAMT affiliates in other states, MAV, MANSW, etc. at member rates.

PUZZLES

1. Devilish

This problem was posed by Fibonacci in the year 1225.

Given that p and q are integers, find a square of the form $(p/q)^2$, that remains a rational square when decreased or increased by 5.

2. Not quite so devilish

If $a^2 = b + c$, $b^2 = c + a$ and $c^2 = a + b$, then what value(s) can be taken by

$1/(a + 1) + 1/(b + 1) + 1/(c + 1)$?

IMMC

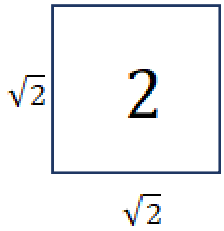
The International Mathematical Modeling Challenge (IM²C) is a mathematical modelling competition involving teams of secondary students from around the world.

The program is organised locally by ACER. Students from the ACT have participated successfully in past years. Registration is open from 3 February 2022 via the website.

INTRODUCING SURDS

When introducing surds, consider the following idea.

The square root of 2 is the length of the side of a square whose area is 2. It is written as √2 because the number cannot be written as a whole number ratio.

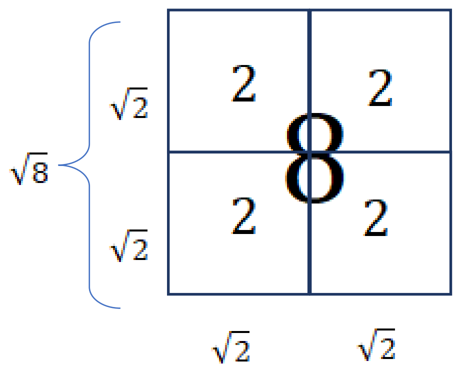


To simplify 2/√2 the standard method (known as rationalising the denominator) is to multiply the expression by √2/√2.

However, more intuitively, we can note that if the area of any rectangle is divided by the length of one of its sides, the length of the other side is immediately revealed.

It's a notion understood in primary years. So 2 divided by √2 is, quite plainly, √2.

Consider also √8, which by algebraic manipulation can be shown to be the same as 2√2. Thinking about this geometrically, the doubling of √2 to yield √8 becomes evident with a diagram like this.



In general, it may be that a geometrical representation of other expressions $a/\sqrt{b}$ will prove helpful.

The message: *Rushing to rules can compromise conceptual understandings.*

ON IRRATIONALITY

The Pythagoreans believed that all numbers could be expressed as the ratio of integers, and so the discovery of irrational numbers is said to have shocked them because this implied the existence of line segments that were *incommensurable* - they could not be compared in length by a common unit measure.

Two integers can be compared by their highest common factor, so that for example the numbers 3 and 11 can be compared by the common unit measure ***one***. The number 3 contains three ***ones*** and the number 11 contains eleven ***ones***. Likewise, the numbers 30 and 385 can be compared by using the common unit measure of ***five*** with 30 containing six ***fives*** and 385 containing seventy-seven ***fives***.

Every rational number expresses a ratio of two integers where each integer could be interpreted as representing the multiples of some common unit measure. For example, we can see that 30/385 is equivalent to 6/77 because $6 \times 5 = 30$ and $77 \times 5 = 385$ where 5 , the common measure, is the H.C.F. of 30 and 385.

Using Pythagoras' Theorem, the length of the diagonal of the unit square is $\sqrt{2}$, a number that we now know is not expressible as the ratio of two integers. The rational numbers 99/70 and 577/408 are close, but no matter how hard we try we will not find two integers p, q whose ratio corresponds to $\sqrt{2}$ exactly.

For if, among the infinite set of integers available to us, we found such a p and q so that $\sqrt{2} = p/q$, then we could write, by squaring and rearrangement, that $p^2 = 2q^2$.. Such an equation might look quite plausible at first, until one realises that there is a glaring problem with it.

The integer p can be broken into its primes (in only one way) and squaring p doubles up every one of them. To illustrate this, consider the number $18 = 2 \times 3 \times 3$. By squaring it we find $18^2 = 324 = (2 \times 2) \times (3 \times 3) \times (3 \times 3)$. The square of any integer will result in pairs of every prime it contains. Of course, the same applies to q and q^2, but the right-hand side of our equation contains an additional single factor of 2. There may well be other 2s in q but these will all be paired in q^2. The equation $p^2 = 2q^2$ now jumps out at us as nonsense because the 2 is unpaired!

What about the rationality of $\sqrt{3}$ or $\sqrt{5}$ or even $\sqrt{N}$ where N is a square-free number? Nonsense to all of these because, by setting $p^2 = Nq^2$ the prime factors in N must be unpaired. In fact, the only leading factors N that we can't rule out are the square numbers themselves, because they always come pre'pair'ed!

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MATHEMATICS IN NATURE

In the journal Virginnia Mathematics Teacher vol. 47 no. 2 there is a section on Mathematics in Nature edited by John Adam, professor of mathematics at Old Dominion University.

Serendipitously calling to mind Bruce Ferrington's miniMATHS series, although at a more advanced level, is a ten-page article by Adam himself: *Modeling* (sic) *Climate Change.*

The article was posted on LinkedIn and can be accessed at https://www.linkedin.com/posts/activity-6883819739701882880-kF6- .

More about Bruce's work can be found at www.minimaths.com.au .

ABOUT THE CMA

NEWSLETTER OF THE CANBERRA MATHEMATICAL ASSOCIATION INC

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We're on the Web!
http://www.canberramaths.org.au/

The Canberra Mathematical Association (Inc.) is the representative body of professional educators of mathematics in Canberra, Australia.
It was established by, among others, the late Professor Bernhard Neumann in 1963. It continues to run - as it began - purely on a volunteer basis.

Its aims include

* the promotion of mathematical education to government through lobbying,
* the development, application and dissemination of mathematical knowledge within Canberra through in-service opportunities, and
* facilitating effective cooperation and collaboration between mathematics teachers and their colleagues in Canberra.

THE 2022 CMA COMMITTEE

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Theresa Shellshear is CMA's COACTEA representative.

Sue Wilson is CMA's AAMT representative.

Joe Wilson is the website manager.

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PUZZLE SOLUTIONS from Vol 13 No 1

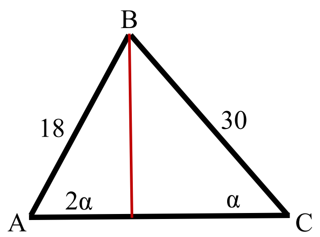
1. Days and dates

There is an ingenious but opaque algorithm for finding the day belonging to any date, invented by the author Lewis Carroll. It predicts that Christmas day will be on a Monday in 2028. However, by a step-by-step calculation we find that the very next time Christmas falls on a Monday will be in 2023.

The number of days in a normal year, 365, is one more than a number divisible by 7. Therefore, the day name advances by 1 in a normal year. Thus, since there are three normal years after 2020 before the next leap-year, Christmas day falling on Friday in 2020 leads to Christmas day on Saturday in 2021, Sunday in 2022 and then Monday in 2023.

After this, Christmas falls on Wednesday in 2024 because the leap-year causes the day to advance two steps. Then, we have Christmas on Thursday in 2025, Friday in 2026, Saturday in 2027, and then two steps to Monday in 2028.

2. Bothersome triangle



By right-angle trigonometry we have $30\sin\alpha = 18\sin2\alpha$ and it follows that $\cos\alpha = 5/6$. Then, $\sin\alpha = \sqrt{11}/6$. Thus, $\cos2\alpha = 14/36$.

With this information we can get the two parts of line AB. They are $7 + 25 = 32$.

The area must be $16 \times 30\sin\alpha$ or $80\sqrt{11} \approx 265.33$.

ANOTHER PUZZLE

Chevalier de Méré

A popular game in betting parlours in France in the early 17th century consisted of rolling at least one 6 in four rolls of a fair die. The gambler won as soon as a 6 was rolled, and so it was often the case that four rolls were unnecessary. A win returned double the gamblers stake, so for example if the bet was 100 francs, a win would return the gambler 200 francs. Otherwise, he would lose his stake. A renowned and wealthy gambler by the name of Chevalier de Méré made quite a bit of money playing this game.

At some stage a similar game was introduced that involved rolling *two dice* simultaneously a total of 24 times! The player would win if at least one double 6 was rolled in the 24 rolls. Chevalier de Méré was convinced that this game was just a longer version of the four-roll version and that the chance of winning was the same as the simpler four roll game.

He reasoned as follows: The probability of obtaining ‘two sixes’ when rolling ‘two dice’ is 1/36. This is one sixth of the probability of obtaining a ‘six’ rolling ‘one die’. To compensate for this probability difference, the player should be allowed to roll the two dice six times. Thus, because the simpler game allows four rolls of the one die, the ‘two dice’ game should allow the player $4 \times 6 = 24$ rolls. The probabilities of winning should then be the same in both forms of the game.

Alas, when Chevalier de Méré played the 24 two-dice version he seemed to lose more than he won. He wrote to Blaise Pascal and was astonished by his response.

Pascal pointed out that in the simpler game, the probability of obtaining at least ‘one six’ in four rolls of a fair die was

$1 - (5/6)^4 \approx 0.5177$ and the probability of obtaining at least one ‘double six’ in 24 rolls of ‘two dice’ was $1 - (35/36)^{24} \approx 0.4914$.

Where do you think the error is in the thinking of Chevalier de Méré?