CIRCUIT

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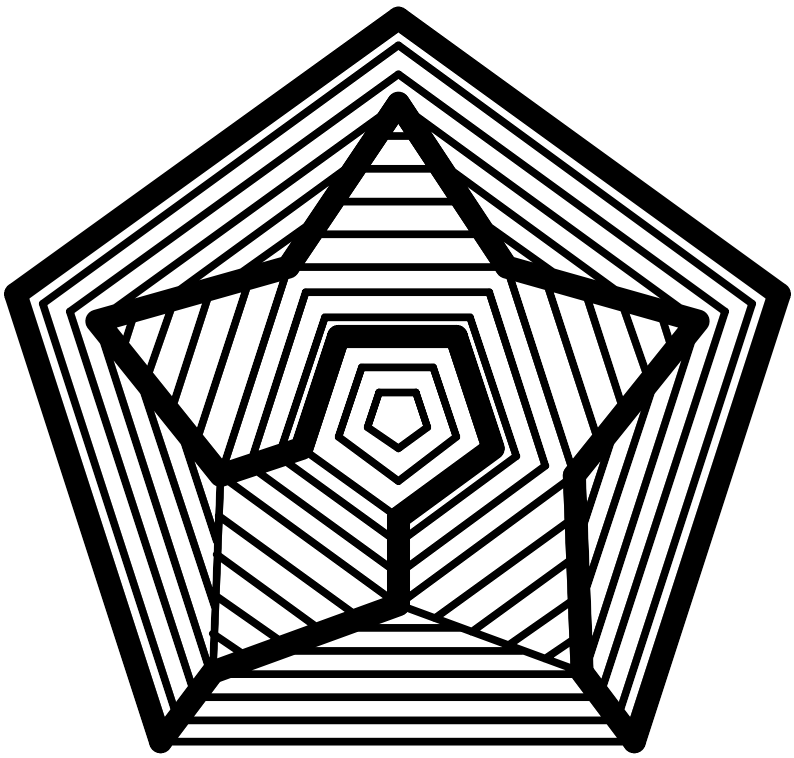

CANBERRA MATHEMATICAL ASSOCIATION

OFFICE BEARERS 1998

EDITORIAL AND PUBLISHING TEAM FOR *CIRCUIT* 1998

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CMA INTERNET HOME PAGE

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The objects of the Canberra Mathematical Association are to promote interest in mathematics, to encourage improvements in the teaching of mathematics and its applications, to provide means of communication among teachers and students and to advance the views of the Association on any question affecting the study or teaching of mathematics and its applications.

The Canberra Mathematical Association Logo depicts a Hamiltonian Circuit on a dodecahedron.

CONTRIBUTIONS

The Circuit publishing team is always keen to receive articles, notes, problems, letters and information of interest to members of the CMA. Please contact a member of the publishing team if you wish to contribute to Circuit.

FROM THE PRESIDENT

The Canberra Mathematical Association (CMA) seeks to support high quality mathematics teaching and learning K to 12 and to make 1998 a very supportive year for teachers of mathematics in the ACT. A recent CMA workshop attracted nearly 50 primary and secondary teachers. It was a stimulating and practical way to get 1998 off to a good start.

CMA Membership

In 1997 the CMA worked hard to increase its membership, particularly in the primary sector. In 1998 the CMA again hopes that teachers of mathematics' will support mathematics teaching and learning by:

- becoming members of CMA, either as individuals or through institutional membership;
- attending and perhaps contributing to practical workshops;
- engaging students at your schools in a number of exciting projects coming up in 1998.

Remember that CMA members receive 10% discount on mathematics resources through CMA's affiliation with AAMT. Individual members may attend CMA workshops at no cost, whereas non-members pay $5 each time. Institutional membership allows the attendance of two participants at no cost.

Professional Development Activities 1998

A draft program for CMA workshops in 1998 is included in this Circuit. Activities will be advertised in more detail later in the Government Bulletin as well as by broadcast fax.

Very Special Activities in 1998

This year will see two special events take place for teachers and learners of mathematics P - 12:

- National Mathematics Day to be held Friday 8 May, during Science Week.;
- the National Mathematics Talent Quest, (NMTQ) 1998;

Flyers have been sent to principals and mathematics coordinators in all ACT schools. Entries are due in Term 3 and we hope many entries will come in from ACT schools. Please contact me if you have any questions.

I hope the CMA's program tempts your school and teachers to contribute. We look forward to seeing you at workshops or social activities.

We would also like to draw your attention to the professional development Enrichment Course for teachers of mathematics being run first semester at UCAN by Steve Thornton on behalf of the Australian Mathematics Trust. Teachers may attend for single sessions. Please contact Steve on 6201 2017 for more details. You can also find details on: http://www/amt.canberra.edu.au/~sjt/amtep.htm

National Mathematics Day

Mathematics will be afforded a high profile nationally on Friday May 8. AAMT has asked each affiliate to plan some activities for that day under the theme "Mathematics in the Mall". The aim is to showcase mathematics education through student activities and displays. The exact nature of the "Mathematics in the Mall" is to be determined by each area network, but we hope teachers K to 12 will contribute. If you want to discuss ideas please email me on beth@mail.eddirect.com or phone me on 6282 5962.

Finally, a date for your diary: **15-19, January 2001**. No, we are serious! The CMA will host the National Biennial AAMT Conference in 2001 and this will afford a wonderful opportunity for all ACT teachers to participate or contribute. We are already planning a strong and cohesive program, with varied forms of registration.

Beth Lee , March 17, 1998

FROM THE PUBLISHING TEAM

Welcome to the first issue of *Circuit* for 1998. Hopefully many good first term mathematical things are still happening in all our classrooms. Apart from the regular sections, this *Circuit* consists of reprinted material from other publications. The reason is not only because we did not have original material to hand, but also because we believe the chosen pieces are relevant, still important, and will not previously have been seen by most readers. Thanks to Annabelle Cassells and Beth Lee for suggesting and supplying the reprinted material in this issue.

As always, we welcome comments and reactions, original material for publication, and suggestions, ideas or copy for reprinting in future issues. Our hope this year is to consolidate *Circuit*'s position as a stimulating and useful mathematics and education journal, which publishes original contributions from CMA members and teachers and, space permitting, reprints selected material from other sources.

WAS LIFE SIMPLER THEN?

This is a reprint of a reprint. The original article was from Mathematics Teaching No. 45, 1968 *and was reprinted in* Mathematics Teaching No. 145, 1993.

Over the past few years, the teaching of mathematics by the use of calculators has become widely accepted by teachers at all levels. In fact, during the past ten years, Britain has become the leading nation for this method of teaching.

One of the reasons for this has been the well known range of Brunsviga calculators and the latest in this line, the hand-operated 13RM is continuing this tradition.

Children learn easier, faster and with more enjoyment with the 13RM, which is manufactured to withstand the wear and tear of school and college operation is simple and quickly understood, while a wide range of problems can be tackled, ensuring everyone obtains a thorough appreciation of the many facets of mathematics.

With full tens transmission; back transfer; capacity of 10 x 8 x 13; convenient one-hand operation and a special educational price of 39 pounds, it represents excellent value, and the large number of Brunsvigas already in use is the testimonial to this robust, low-cost machine.

Could they have imagined the new generation of graphic calculators?

THE NEW MAGIC OF INFORMATION TECHNOLOGY

This article is based on a paper by Chris Hopkins, which was printed in Mathematics Teacher *No. 149, 1994. Hopkins was a keynote speaker at the 1994 conference of The British Association of Teachers of Mathematics.*

Mathematics, magic and mystery.

Many of us are caught up in the wonder and mystery of magic. But there is a danger that the 'magic of mathematics' can be the magic of a priesthood, a secret and exclusive knowledge. The mathematician, with an abracadabra, can produce logarithms that turn multiplication into addition. However, 'many years ago', mathematics teachers could turn that magic into hard slog with fifty 'interesting' examples.

So what of the new magic of IT?

Computers are likely to be seen as powerful and magical. How might we exploit that power to do mathematics and draw more people into mathematics?

The educational concern with an educational innovation is constant: will understanding be lost? If our students do not learn as we learnt, will they really understand? This concern is a strongly expressed one even though the general level of mathematical understanding in the general population is not impressive.

Hopkins reminds us that learning and understanding are very active processes - "Mathematics is condensation; to learn with understanding is to unpack, and then accept, the power". (1994, p. 14)

The British Association of Teachers of Mathematics has advocated that:

- IT is to be welcomed and exploited in the task of teaching;
- any new advance requires a re-examination of the knowledge base to see what knowledge is needed to operate effectively with the new tools;
- the trend is to mental mathematics + IT (which emphasises the importance of students' confident use of and estimation with numbers).

Hopkins suggests we need to look at the power and potential of IT - to look forwards, not backwards.

What are the substantial and important ideas in the study of mathematics which would make full use of IT? Hopkins proposes four ways:

1. Experimentation

There has been considerable development both in the class of problems and of a pedagogical style that will encourage students to experiment, to hypothesise, to generalise. In this environment learners are encouraged to develop their own ideas about a mathematical situation rather than to identify and apply a standard algorithm. Experimentation is meant to be slightly broader than 'investigation' which some identify with particular types of problems.

Some software such as Logo can be used in an experimental way, but much more needs to be done on developing suitable problems and pedagogical approaches in a computer-rich environment.

2. Visualisation

A shift to the visualisation and interpretation of results, not just emphasis on the development of a visual model. The visual image can provide a powerful impetus for further research. Much of the 'calculation' can be entrusted to the computer but it is the reader who must 'see' the global picture. Algorithms can be taught, but it is less obvious how to teach problem-solving and develop the insights inherent in problem solving. Working on imagery is a powerful way of encouraging involvement in a problem.

We need teaching techniques to encourage thinking, yet it is so much easier to encourage action, often the carrying out of a written routine.

A renewal of the geometry curriculum would provide a natural base for working on imagery that could then be developed as a general problem-solving tool. Robotics, Geometer's Sketchpad and Logo are useful here. Programs such as Logo work on the modelling process, on seeing how to draw a hexagon or a spiral. There needs to be a shift from hoping the software increases understanding of triangles and angles, to a valuing of the construction of a logical sequence of actions that will produce the required result.

Analytical and imagery approaches complement one another. They are not opposed.

3. Recursion

Solutions to a range of mathematics and computing problems can be based on the repetition of sequences of steps. Iteration is becoming a part of the mathematics curriculum, and graphic calculators and spreadsheets encourage or require the use of iterative procedures.

For instance, one of the great satisfactions of spreadsheets is seeing how to create a sequence such as 1, 4, 7, 10, ... by entering a single formula and filling down. A Logo procedure can elegantly, and with little effort, produce impressive results.

Seeing how to use an iterative procedure to achieve a desired result seems to be increasing in importance. A teacher's emphasis on certain recurring themes can give direction to a mathematics curriculum. What computers offer is control of a core procedure that can be repeated over and over again. Fractal images make a powerful impact which can be firmly linked to mathematics by explaining that the pictures are based on a square grid, and an iterative calculation is performed on the co-ordinates, the resulting calculation being colour coded.

4. Proof? Are you sure?

Are computers undermining the sense students have for a need for proof? Was proof, with the exception of Euclidean geometry, ever a strong feature of school mathematics?

Do students confuse proof with conjecture? The minimum requirement is to know when something has not been proved. Logic and reason can be exercised in a much wider variety of situations than formal proofs. Instead of mathematics teachers striving to encourage students to be aware of solutions to numerical problems that are totally ludicrous in nature, it may be more effective to encourage logical reasoning, and the development of the capacity to present an argument.

Hopkins suggests working with computers can make two important contributions to the idea of proof. The first is the chains of logical reasoning in correcting a drawing or model, or detecting the errors in a spreadsheet display. The second is in the opportunities for discussion of certainty. The teacher is crucial in asking "Are you sure?", in placing an emphasis on seeing why something has happened, and in encouraging discussion. The computer makes it possible to generate many examples, for instance of behaviour, which looks as though it has stabilised, but then changes.

"Proof is unlikely to be valued if the intuitively obvious never turns out to be false". (Hopkins: 1994, p. 19)

QUOTABLE NOTES AND NOTABLE QUOTES

Tensions at the Heart of Schooling

"I had been to school most of the time, and could spell, and read, and write just a little, and could say the multiplication table up to six times seven is thirty-five, and I don't reckon I could ever get any further than that if I was to live forever. I don't take no stock in mathematics, anyway."

Huck Finn in *The Adventures of Huckleberry Finn* by Mark Twain.

"One of the roles of schools in a democracy is surely to enable the younger generation to understand the world and that requires theoretical tools ... understanding of the world by the general population (...includ(ing) the ...decision makers of the era) is crucial. Without it, the door is open to all kinds of bigotry and obscurantism and with them, tyranny.

Professor Peter Schapira, Institut de Mathematiques, University of Paris, quoted in *Mathematical Intelligencer,* Volume 19 # 2, 1997.

Hunting Down Primes

John Friedlander of the University of Toronto and Henry Iwaniec of Rutgers University have established that there is an infinite number of primes of the form $a^2 + b^4$ where a and b are integers. Their result narrows the search for large primes and will be published in the journal *Annals of Mathematics*.

Adapted from IN BRIEF, *New Scientist*, 17 January 1998.

A Remembrance of Mathematician Paul Erdös

Hungarian Paul Erdös was one of the brightest mathematical luminaries of our century. He spent most of his later years travelling the world, living out of suitcases. As mathematicians would put it, Paul was everywhere dense on earth. If you wanted to see him, you just stayed in one place and waited - he soon appeared.

Remarkably, although Erdös started to study English at the age of eight and spent most of his life in English speaking countries, he had a very heavy accent. He often spoke to his mother in English. At the age of eighty-four she began to accompany her son on his trips, and decided then to learn English. She would turn to Paul and ask: "Palko, how do you call the fruit szilva in English?"
"Plimm, mother, plimm!" was his answer.

Adapted from Two Places at Once: A Remembrance of Paul Erdös by Janos Pach, *Mathematics Intelligencer,* Volume 19 # 2, 1997.

Power from the Parabola

An Australian oceanographer, Tom Denniss, is developing a Wave Energy System in which a bay in the shape of a parabolic reflector is used to focus a wide front of waves onto one point. Wave energy is concentrated and amplified at the focus in a shaped chamber and drives air across a turbine controlled by gears so as to spin in only one direction.

So far the parabolic bay has only been tested in the Unisearch Water Research Laboratory at the University of New South Wales using a 1:25 scale parabola in a 3 metre wide wave tank. Testing involved a variety of wave conditions, water depths and chamber shapes and aimed to maximise the parabola's wave amplification and energy available to drive the turbine. The waves at the focus of the parabola in the model were two to four times higher than those of the incoming waves, enough to provide a useful energy source.

Denniss estimates that a parabolic bay 40 metres by 20 metres could generate from 250 kilowatts to 1 megawatt of power, sufficient for up to 2000 households, at a similar cost to present-day coal fired generators. Building a full-sized parabolic bay would be relatively cheap says Denniss, as it could be made of prefabricated concrete or steel slabs.

Adapted from Bay Wash by Bill Clayton, *New Scientist*, THIS WEEK, 1 November 1997.

Probability, Common Sense and British Justice

In Britain, a bizarre ruling last November from the Appeal Court specifically forbade juries access to expert evidence concerning estimation of probabilities. The laws of probability, it declared, are “a recipe for confusion, misunderstanding and misjudgment”. Instead, juries should rely on their “individual commonsense and knowledge of the world”.

Yet many court cases involve interpretation of complex probabilistic evidence derived from forensic work on DNA or blood groups. Relying on common sense can lead juries into a minefield of fallacies, says David Balding, a Professor of Statistics at Reading University who has served as an expert witness in DNA trials. “Studies have shown that (ordinary) people cannot reason very well at all when faced with probabilities”, he says.

Adapted from Tipping the Scales of Justice by Robert Matthews, *New Scientist*, December 13, 1997.

Another challenge for us educators!

Limericks on Mathematical Themes

With no contributions received so far from readers responding to the plea in the last *Circuit*, your columnist had no option on this occasion but to include two of his own efforts as further provocation. Please get busy readers, so that we can publish your limericks on mathematical themes as a regular part of Quotable Notes and Notable Quotes.

The sublime Karl Friedrich Gauss
Made the observatory at Göttingen
famous.
His commitment to rigour,
And lifelong creative vigour,
Personify mathematical nous.

Peter Enge

The digital age is not new,
With origins in fingers and toes it’s true.
Calculation still advances,
By the seats of many pantses,
Now tedium’s mostly for CPUs.

Peter Enge

PROBLEMS AND ACTIVITIES

Remember that we include a coding system which attempts to indicate in terms of Year levels the suitability range for each item. Thus **6 - 8** suggests an item accessible to students from Year 6 to Year 8.

(1) A Special Long Multiplication

6 - 10

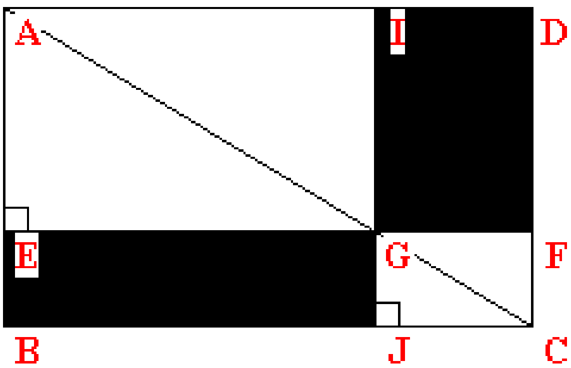
Describe how to multiply

31 671 425 307 by 125 in your head.

(2) An Area Surprise

7 - 10

ABCD is a rectangle. Which of the shaded rectangles has larger area?



(3) From the Specific to the General

9 - 11

Observe that

$$3^2 + 4^2 = 5^2$$
$$5^2 + 12^2 = 13^2$$
$$7^2 + 24^2 = 25^2$$
$$9^2 + 40^2 = 41^2$$

Find a general result in accord with these examples and prove it.

(4) A 1998 Recurrence

10 - 12

Define $f_0(x) = (1 - x)^{-1}$,

and $f_n(x) = f_0(f_{n-1}(x))$,

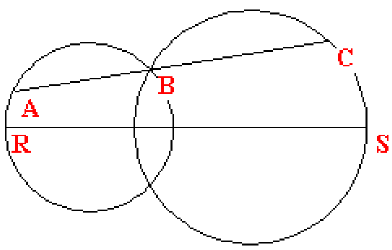
for $n = 1, 2, 3, \ldots$

Evaluate $f_{1998}(1998)$.

(5) The Longest Segment

9 - 12

Draw any two circles which intersect in two points as shown.



AC passes through B, one of the points of intersection of the circles, and RS is the line of centres. Which choice of line through B makes segment AC longest?

(6) A Legal Probability Conundrum

10 - 12

This is a follow up to the Quotable Note Probability, Common Sense and British Justice in this issue on page 6 . Its source is the *New Scientist* piece mentioned there.

Imagine you are a member of the jury in the trial of a man accused of murdering his wife. Evidence is given that the accused had regularly beaten his wife - but the defence presents statistics supposedly showing that only 1 in 1000 wife beaters go on to kill their wives. Should you be swayed by the defence argument and discount the wife beating evidence?

THUMBNAIL (and longer) REVIEWS

Readers are welcome to contribute to this section. Reviews can cover books, periodicals, videos, software, CD ROMs, calculators, mathematical models and equipment, posters, etc.

100% Mathematical Proof
by Rowan Garnier and John Taylor
Published by John Wiley and Sons, 1996, 317 pages

This is a usefully comprehensive and stimulating book at an intermediate level about one of the essential ingredients of mathematical activity: proof. The authors, who both work in universities in England, bemoan the fact that it has not usually "been the custom to teach the principles of mathematical proof systematically". They therefore set out to explore these principles and to examine "how proofs may be discovered and communicated".

The logical and structural features which characterise mathematical proofs are spelt out and a wide range of more specific proof techniques is discussed. The early chapters cover inductive and deductive reasoning and basic propositional and predicate logic. Then follows a discussion of axiom systems and formal proof. The remainder of the book deals with the major methods of direct and indirect proof, including constructive and non-constructive proofs, proofs by contradiction, existence and uniqueness proofs, and mathematical induction.

Many examples of proofs are presented, discussed and occasionally re-presented, in order to elucidate points of structure, layout, language, economy or intelligibility. Surprisingly, elegance does not rate a mention, but common pitfalls are pointed out. The numerous exercises are well-chosen and assume mathematical background to roughly Year 12 level, apart from some which touch on analysis or modern algebra. They build on the examples and help students practice and develop their theorem proving, while reinforcing understanding of concepts. Hints and solutions to selected exercises are included.

The material in the book is up-to-date and never stuffy. For instance, the opening chapter backgrounds Gödel's Incompleteness Theorems, Fermat's Last Theorem, The Four Colour Theorem, and the Classification of Finite Sample Groups to illustrate the scale and scope of contemporary mathematics in regard to proof.

Throughout, the authors make an excellent fist of presenting complex material in clear, well-organised and interesting fashion. This would make a good text for a secondary college or beginning university course dealing with a systematic treatment of mathematical proof. It would also make a particularly useful resource for teachers of primary or secondary mathematics who wish to widen their experience and improve their understanding of the nature of mathematical proof.

Peter Enge

Strength in Numbers

Discovering the Joy and Power of Mathematics in Everyday Life

by Sherman K. Stein
Published by Wiley, 1996.

A book which I found in turn inspiring and infuriating: “You and I - all of us - can explore the inner and outer worlds far more than we imagine possible. Many of us close the doors too soon. I dedicate this book to all who are willing to open closed doors and open even wider the doors already open.” At the other extreme, in the chapter titled What is a Job, Really?, Stein suggests that we think of the economy as a balance between “income stations” and “production stations”. For me such an oversimplified model of the economy begs too many questions and does not lead to any very useful insights into work and jobs. Either way, Stein provokes a response in the reader.

The book has three Parts: About Maths; From High School to Kindergarten; Closer and Closer. Part 1 samples the elephant of mathematics: use and abuse of numbers; triumphs and limits of computers; myths about maths and its practitioners; applications; maths in trades and professions; maths instruction. Part 2 looks afresh at concepts often met in school: geometric series and applications; Pythagoras; dividing fractions; - x - = + ; how to picture equations. Part 3 delves into a technique for finding an unknown quantity; the measurement of the steepness of curves; the computation of areas bounded by curves; the computation of π (not a practical scheme). A section on further reading, chapter notes, and an index are included.

The positive aspect of the book is the variety of angles which Stein offers on mundane mathematics, angles which offer starting points for ordinary people wishing to know more mathematics. For example, his coverage of myths about mathematics, and of fraction and negative number arithmetic is interesting and generally informative. He also comments on the past 100 years of moves to reform maths teaching in the USA. His own reform proposals are modest. He accepts the givens of corporate America, preferring to stick with apple pie sentiments, and makes a plaintive appeal for parents to encourage their children in maths.

Stein’s list of myths about maths is worth browsing. Those that particularly appealed to me are: “There is a gene for maths”; “It’s a dead subject.”; “All that mathematicians do is study numbers.” “There is nothing new in maths.”; “Mathematicians spend all day at computers.”; “Mathematicians start with axioms, see what these follow, then look for examples.”; “Mathematicians are over the hill by age 30.” Sometimes the folksy American tone grates. In a chapter titled “Out of Thin Air” Stein exposes the bankers’ fraud (the illusion that all lenders could on demand turn their savings into cash), using results on the summing of series.

Certainly this book opens doors on many aspects and applications of mathematics. It is a book for the general reader which deserves a place in every library and has much to offer teachers and students of mathematics. Not many books about mathematics are so readable or range so widely.

Peter Enge

SOLUTIONS TO PROBLEMS AND ACTIVITIES

(1) Since 8 x 125 = 1000, all we need do is add three 0s on the end of 31 671 425 307 and divide by 8. Clearly multiplications by 25 and 5 can be performed in analogous fashion.

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(2) The areas of the rectangles AEGI, GJCF and ABCD are halved respectively by segments AG, GC and AC. Each shaded rectangle is what remains when half of rectangle AEGI and half of rectangle GJCF are subtracted from half of the original rectangle ABCD. Hence the areas of the shaded rectangles are equal.

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(3) Rewriting each of the given relationships by subtracting the second term on the left from both sides, then factorising the resulting difference of two squares gives

$3^2 = 5^2 - 4^2 = 5 + 4,$
$5^2 = 13^2 - 12^2 = 13 + 12,$
$7^2 = 25^2 - 24^2 = 25 + 24.$
$9^2 = 41^2 - 40^2 = 41 + 40.$

Hence

$$(2n + 1)^2 = 4n^2 + 4n + 1$$
$$= (2n^2 + 2n + 1) + (2n^2 + 2n)$$
$$= (2n^2 + 2n + 1)^2 - (2n^2 + 2n)^2$$
$$\text{for } n = 1, 2, 3, \ldots$$

Transposing, this gives

$$(2n + 1)^2 + (2n^2 + 2n)^2 = (2n^2 + 2n + 1)^2$$
$$\text{for } n = 1, 2, 3, \ldots$$

as one possible general result in accord with the original examples.

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(4) It is not difficult to see that

$f_0(x) = (1 - x)^{-1},$

$f_1(x) = f_0(f_0(x)) = (x - 1)\, x^{-1},$

$f_2(x) = f_0(f_1(x)) = x,$

and

$f_3(x) = f_0(f_2(x)) = (1 - x)^{-1} = f_0(x).$

Thus

$f_{n+3}(x) = f_n(x)$ for $n = 0, 1, 2, 3, \ldots$

and in particular

$$f_{1998}(1998) = f_{3 \times 666 + 0}(1998)$$
$$= f_0(1998)$$
$$= -(1997)^{-1}.$$

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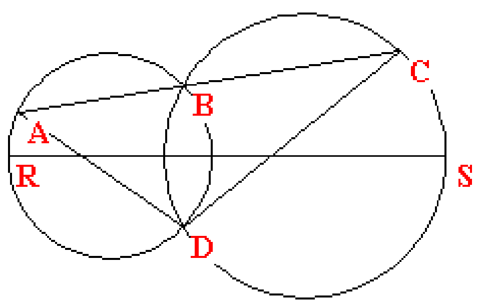
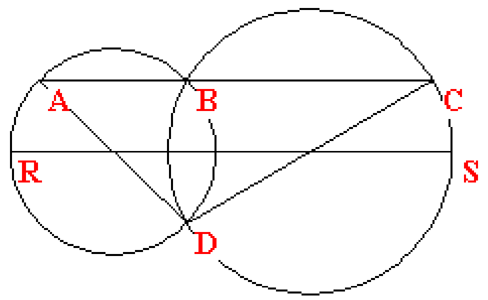
(5) Draw triangle ACD as shown, D being the other point of intersection of the two circles.



Chord BD subtends angle A in one circle and angle C in the other, so these two angles are independent of the particular choice of line through B. Hence all such triangles are similar. Clearly distance AC will be greatest when side AD (or DC) is longest, that is when AD (or DC) is a diameter. In this case (see following diagram), angle ABD, being the angle in a semicircle, is 90°, and since the line of centres RS bisects the common chord BD at right angles, AC is parallel to the line of centres.



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(6) No. Otherwise you fall into one of the traps awaiting those who rely on common sense to understand evidence based on probabilities. You should be focusing instead on the odds that a wife beater whose wife has been murdered is responsible for her death. These are not the odds, quoted by the defence, of wife beaters going on to kill their wives.