SHORT CIRCUIT

Newsletter of the Canberra Mathematical Association INC

VOLUME 12 NUMBER 5 MAY 2021

NEWS AND COMMENT

Preparations are going ahead for the CMA conference in August. Previous presenters are about to be contacted and a call is out for prospective new presenters.

Short Circuit is always on the lookout for pieces about teaching, about education generally, and about mathematics itself.

For this edition, Peter Fox writes about an investigation he found particularly pleasing. Peter also spotted some slightly embarrassing errors in the last lot of puzzle solutions, leading your puzzle editor to reflect on the traditional assessment practices that require students in examination environments to obtain correct answers independently. Things are not quite like that in the real world. Everyone needs team insights and a proof-reader.

PRIMARY TEACHER STUDY

Are you a Primary/Elementary school teacher?

You are invited to take part in a **study on the teaching of fractions**. The purpose of the study is to explore what resources and documents teachers draw on in their planning for the learning and teaching of fractions. Findings from this study will further develop our understanding of how teachers plan. Participation in this study is voluntary.

The research is being conducted by Maria Quigley for the award of Doctor of Philosophy (PhD) under the supervision of Professor Janette Bobis at the University of Sydney.

This study will involve you completing a questionnaire. The questionnaire should take approximately 15-20 minutes to complete.

The questionnaire is anonymous. However, if you would like to be involved in a follow-up interview conducted via the internet (eg. Zoom) you have the option of providing your email/contact details at the end of the questionnaire.

You will find the questionnaire here: Planning for the Teaching of Fractions Survey

Please email me if you have any questions.

Your involvement would be very much appreciated.

Many thanks,

Maria

CANBERRA MATHEMATICAL ASSOCIATION

Inside:

Coming Events:

CMA conference: 7 August at ADFA

Theme: '19, 20, 21 – What's next?'

AAMT virtual conference 29-30 September.

AGM: 10 November.

Wednesday Workshop:

MEMBERSHIP

Memberships run from 1 Jan to 31 Dec. each year. Membership forms can be accessed from the CMA website: http://www.canberramaths.org.au

Membership of CMA includes affiliation with the Australian Association of Mathematics Teachers and a subscription to one of two AAMT journals.

As a member, you are entitled to attractive rates for the CMA annual conference and CMA professional development events.

CMA members may attend conferences of the AAMT affiliates in other states, MAV, MANSW, etc. at member rates.

PUZZLES

1. Eight nines

How can the symbols + and/or × be placed in the gaps between some of the eight nines

9 9 9 9 9 9 9 9 9

in such a way that the expression evaluates to the number 9999. Digits without a sign between them are taken to be a number.

2. Board game

A board is divided into 15×15 cells into which two players take turns to place the symbols 0 or ×. Neither player is restricted to one or the other symbol. The first player to complete a string of three of the same symbol in a row, column or diagonal wins. If this does not happen, the game is a draw. Is there a winning strategy for the first player?

3. Quotition

Economical Ania has divided the segment AB into 20 identical segments and called each of them a *big* one. Ambitious Tonay has divided the same segment into 2019 identical segments and called each of them a *small* one. They agree to call a small segment a *good* one if it lies completely inside one of the big ones. How many good segments are there?

4. Cycling

The word STABLE has the property that you can cycle the first letter to the end to make a new word, and you can do it again to make another new word. There is an analogue with numbers. If you use the letters in TABLES to represent six particular digits, then TABLES × 3 = ABLEST and TABLES × 5 = STABLE. What is TABLES × 7 (written as a number)?

5. Dogs

Two dogs belonging to breeders Harriet and Matilda have had litters of 10 and 9 puppies respectively. In the long run, 50% of all puppies are female. Harriet prefers female puppies and wonders what the probability is that her dog's litter has more female puppies than does Matilda's.

AUSTRALIAN CURRICULUM REVIEW

From
Mandy Kalyvas
Senior Director
Learning and Teaching Policy and Service Design
Education Directorate

In June 2020, education ministers agreed that it was timely to review the Foundation to Year 10 Australian Curriculum. ACT Education Directorate Learning and Teaching staff and classroom teachers have been participating in extensive review discussions with the Australian Curriculum and Reporting Authority (ACARA).

From 29 April until 8 July 2021, a consultation website will be available for interested stakeholders and the public to review the proposed revisions to the curriculum and to have their say via a survey format.

This will give the **Canberra Mathematical Association** the opportunity to provide feedback on the proposed revisions to the F-10 Australian Curriculum.

All learning areas, the general capabilities, and the cross-curriculum priorities are included in the review.

Readers who are interested in finding out more about the Australian Curriculum review, should visit the 'Curriculum review' page of the ACARA website.

MAV PRIMARY CONFERENCE 2021

Mathematical Association of Victoria

2021 Primary and Early Childhood Conference - Virtual

AAMT CONFERENCE 2021

Key dates:

e-Conference: 29th-30th September

Conference Proposals Open: 20th March-30th April

Early Bird Ticket Sales: 25th March-31st May

Theme:

Future Proofing Australia's Mathematical Capacity

Conference Website

CMA ANNUAL CONFERENCE 2021

SAVE THE DATE

Saturday 7th August 2021, at ADFA

"19, 20, 21, ...What's next?"

Teachers and educators in all sectors are warmly invited to attend this year's conference.

The 2021 Conference committee is also pleased to call for **expressions of interest in being a presenter** at the conference. All presentations to do with the teaching, learning or use of Mathematics are welcome. If you can incorporate the conference theme or respond to it, so much the better.

For further details, please contact Valerie Barker: vnwb@internode.on.net .

A BEAUTIFUL PROBLEM

By Peter Fox

Beautiful mathematics problems can help students move from counting dots to connecting them.

The majority of textbook questions focus on repetition. There is a time and place for repetition, but it should not comprise the majority of a student's mathematical diet. To quote one of my former students as she arrived in class after lunch break on a warm day: "Can we do textbook questions today? The stuff you do makes me think too much and I really don't want to have to think today."

So here is an example of the type of question to which the student was referring

Find the equation to the locus formed by the intersection of the altitudes of a triangle with vertices on the function $y = 1/x$.

Students generally avoid this type of problem due to the wording. When I pose this type of question to students I ask them to first underline all the words they do not understand, the barriers that are preventing them from moving forward.

[See: Short Circuit Volume 12 Number 4 – Language Strategies for Mathematics by Heather Wardrop. Strategy 7]

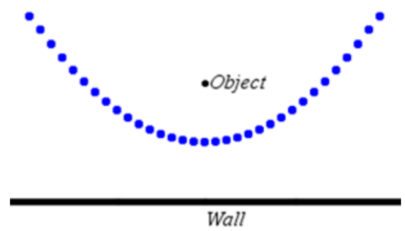
For this particular question, some students highlight the entire question, reflective of their perception pertaining to the level of difficulty. When students are challenged, the most commonly highlighted words are:

- Altitudes
- Vertices
- Locus

Altitude: This word is familiar; the context is the issue. Students identify where they have heard the word used, 'height of a mountain' or 'height of a plane'. A couple of diagrams help to clarify the definition. Draw a simple 'triangular' shaped mountain with a line passing through the 'base' and the 'summit'. Do not draw this line at right angles to the base. Students quickly correct this error and note that the altitude must be perpendicular to the base and eventually arrive at a more formal definition: "the altitude is a line perpendicular to the base passing through the opposite vertex".

Vertices: This word is less familiar in every day language, but can be compared with a more common term: "corner". Associating a more familiar word helps build upon existing connections, with an understanding that mathematical terms may be more rigorous in their definition. A perfect example here is the common term 'average' which is often (mistakenly) taken to be equivalent to the mathematical term, mean.

Locus: This may be treated as a completely new term for students to learn. The formal definition is not initially helpful to most students: "A point, line, or surface moving according to mathematically defined conditions". Getting students to 'model' a locus works well. Students line up across the front of the classroom. Place an object in the middle of the room. Have students walk away from the front of the classroom towards the rear (perpendicular to the front of the room) and instruct them to stop when they are the same distance from the front of the room as they are from the object. [See diagram]



Students should now be standing on a curve. Each student's location has been defined by the condition: 'equidistant from a line (wall) and a point (object)'.

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NEWSLETTER OF THE CANBERRA MATHEMATICAL ASSOCIATION INC

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We're on the Web!
http://www.canberramaths.org.au/

ABOUT THE CMA

The Canberra Mathematical Association (Inc.) is the representative body of professional educators of mathematics in Canberra, Australia.
It was established by, among others, the late Professor Bernhard Neumann in 1963. It continues to run - as it began - purely on a volunteer basis.

Its aims include

* the promotion of mathematical education to government through lobbying,
* the development, application and dissemination of mathematical knowledge within Canberra through in-service opportunities, and
* facilitating effective cooperation and collaboration between mathematics teachers and their colleagues in Canberra.

THE 2021 CMA COMMITTEE

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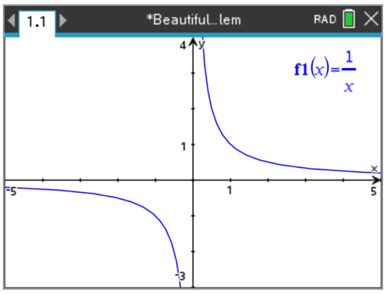
Continued from page 3.

The curve is a locus. This particular locus, a parabola, has a familiar equation.

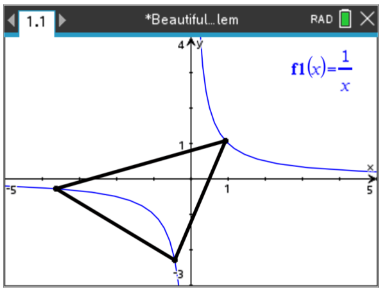
With an understanding of all the words in the question, students can now start solving the problem.

What parts can you do?

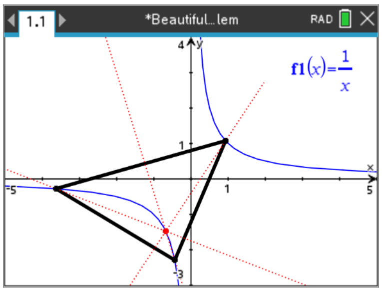
Draw a graph of the function:



Draw a triangle so the vertices (corners) are on the function:

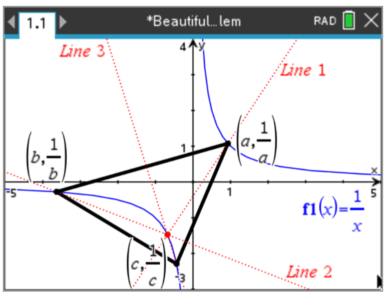


Draw the three altitudes of the triangle.



When this diagram is produced dynamically, the vertices of the triangle can be moved, the corresponding altitudes move and the point of intersection traces out the locus. It appears that the locus sits on the original curve: $y = 1/x$. Can we prove it?

Solving the Problem



Line 1: [BC is the line between points with x coordinates b and c.]
The gradient of the line BC is $(1/b - 1/c)/(b - c)$.

Since Line 1 is perpendicular to BC, its gradient simplifies to bc.

Using the translational form of a straight line, the equation to Line 1 is:
$y = bc(x - a) + 1/a$.

It follows that Line 2 is:
$y = ac(x - b) + 1/b$ and Line 3 is:
$y = ab(x - c) + 1/c$.

Solving these equations simultaneously: $x = -1/abc$ and $y = -abc$. In other words: $x = 1/y$ or $y = 1/x$.

The equation to the locus is that of the original function!

We can dig deeper into this problem to help understand the negative sign in our answer. This is just part of what makes this a beautiful problem. We might also ask about other triangle centres and what path they might follow or explore the orthocentre of a triangle with vertices on other functions such as $y = 1/x^2$.

This problem connects dots between several mathematical skills, the Cartesian plane and Geometry (Coordinate Geometry) and also the notion of mathematical explorations. There are so many wonderful problems like this, and with the exception of a period 6 class on a warm day, students generally enjoy the challenge.

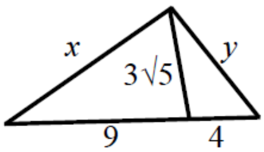
PUZZLE SOLUTIONS

A reader, Peter Fox, sent in corrections to two of the solutions in Short Circuit Vol 12 No 3. Clearly, mathematics is a team enterprise.

1 Some product

Four parts is best: $(10/4)^4 = 39\ 1/16$.

4 Maximum difficulty



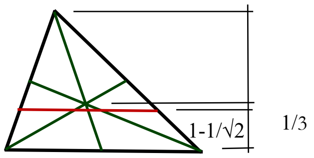
Using the cosine rule, some calculus and attention to detail, we deduce $x + y$ is maximised when $x = 13.5$ and $y = 6$ so that the sum is 19.5. It remains true that the ratio $x/y = 2.25$ is the same as the ratio 9/4 of the segments forming the base of the triangle.

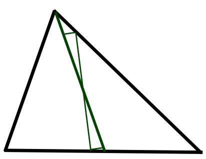
Solutions to puzzles in Short Circuit Vol 12 No 4.

1. Slice

This question turned out to be a lengthy and challenging investigation rather than a simple puzzle. This solution leaves room for refinement.

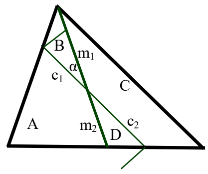
(a) A median bisects a triangle. It is true that the three medians intersect at a common point (at 1/3 of their lengths), but other bisecting lines need not pass through that point of intersection. For example, a bisector parallel to a side cuts the median to that side at a height $1 - 1/\sqrt{2}$ of the median.





(b) & (c) A plausible candidate for the longest cut is a median. In the second diagram above, there is a median and a bisecting cut that is a clockwise rotation of the median. Two right angled triangles are formed. The hypotenuses are longer than the sides forming the cut and the median is longer than the combined hypotenuses. So, in this case the median is longer than the rotated cut.

However, this argument fails if the cut is rotated anticlockwise, in the direction opposite to the skew of the triangle.

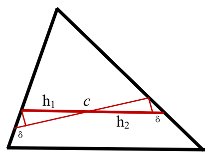


We try another approach. In the diagram above there is a median m with parts m_1 and m_2, and another bisecting cut c, with parts c_1 and c_2. By the previous argument, it is clear that $m_1 > c_1$. For a rotation through a small enough angle α, it is reasonable to assume $c_1/c_2 \approx m_1/m_2 \approx 1$, so that we can take $m_2 = m_1c_2/c_1$.

Now, $m_1 > c_1$ implies $m_1 + m_2 > c_1 + m_1c_2/c_1$. But, $c_1 + m_1c_2/c_1 > c_1 + c_1c_2/c_1 = c_1 + c_2$. Hence, a median is longer than a cut rotated through a small angle whether clockwise or anticlockwise.

Clearly, if the assumption holds, the medians represent cuts that are local maxima.

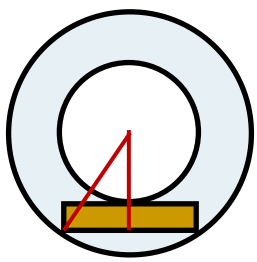
Consider a bisecting cut $h = h_1 + h_2$ as in the following diagram, parallel to a side of the triangle. A very small rotation leads to the cut c, which has length approximately $h_1 + \delta + h_2 - \delta$. So, $c = h$.



That a small rotation in either direction produces no change in the cut length suggests that the cuts h have lengths that are local minima.

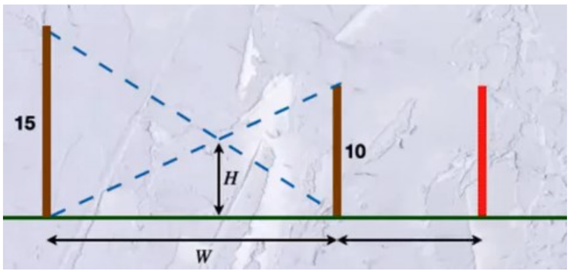
The evidence suggests that the longest median gives the longest cut and the bisector parallel to the shortest side is the shortest.

2. Awkward fit



The outer radius is $\sqrt{(28^2 + (37 + x/2)^2)} = 37 + x$. So, $x = 16$.

3. Poles apart

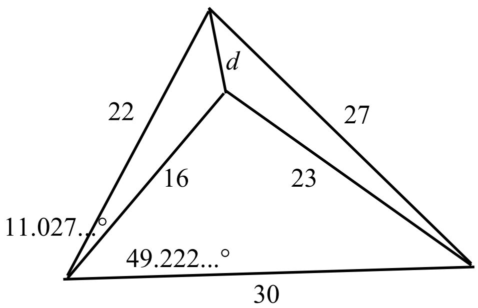


Let $w = w_1 + w_2$, where the two parts join below the point of intersection of the dotted lines. Then, by similar triangles, $w_1/H = w/15$ and $w_2/H = w/10$. Thus, $w = Hw/15 + Hw/10$. Clearly, H does not depend on w and will stay the same however widely the poles are separated. In this case $H = 6$, but H is constant for any pair of poles.

4. Uncertainty

The question rests on whether or not you selected the double headed coin. The probability that the second side is a head given that the side you see is a head is 0.5.

5. Near enough ...?



Two angles are found by applications of the cosine rule. By a third application we obtain $d = 7.000000086$.

For d to be exactly an integer would require an expression in integers like $a^2+m^2 - 2am[(a^2+c^2 - b^2)/(2ac) - (m^2+c^2 - n^2)/(2mc)]$ to be the square of an integer. One wonders how the numbers in the puzzle were discovered.

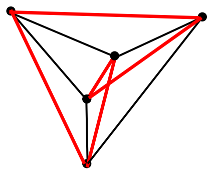
6. Long story short

The probability that your seat is empty is 0.5.

The VIP arrives and does not sit in his assigned seat—let's call it seat 1—but instead sits in seat k. It turns out that throughout the ensuing seating process, the only seats that need concern us are seat 1 and your seat.

There is no problem until the person assigned seat k arrives. If that person sits in seat 1 or in your seat, every other arrival will be able to go to their correct seat, and the outcome concerning your seat depends on the 50-50 choice made by the kth person. If person k instead sits somewhere else, the situation is unchanged and the crucial decision is merely delayed to the next arrival. As soon as someone sits in seat 1 or in your seat the game is over and that person's choice has determined the outcome. Eventually, it may be that 98 people have arrived and the only empty seats are yours and that of the VIP. Person 98 then makes the 50-50 choice.

7. Enigma



This colouring of the complete graph on five vertices avoids cliques of order 3. However, in the complete graph on six vertices it is impossible to avoid triangles with the same coloured edges.

Cliques that are complete subgraphs of order 4 are obviously avoidable for the complete graph on five vertices, as they are for all complete graphs up to order 17, but they are unavoidable for 18 or more vertices. However, there is uncertainty about the smallest complete graph that unavoidably contains a clique of order 5. These ideas are part of 'Ramsey theory'.

A consequence is that among any group of six or more people there is a group of three who know each other or a group of three who do not know each other.