SHORT CIRCUIT

Newsletter of the Canberra Mathematical Association INC

VOLUME 11 NUMBER 7 OCTOBER 2020

NEWS AND COMMENT

Thankyou to those who have indicated that they plan to attend the **AGM**. It has not been decided yet whether the event will be virtual or held in a physical location. We will let you know once the decision is made if you have told us that you expect to be there. Please do so as soon as possible.

In this issue of Short Circuit we continue the sequence of articles by Heather Wardrop drawing attention to the need for the parallel development of language in the mathematics classroom.

A second contribution from a reader is Andy Wardrop's piece on Mathematical Black Holes.

We suspect that there are many readers who have stories to tell, musings about pedagogy, ideas about lesson content, and comments about education. Short Circuit would welcome your submission.

CANBERRA MATHEMATICAL ASSOCIATION

Inside:

Coming Events:

CMA conference: CANCELLED.

CMA AGM: 11 November, 2020.

Wednesday Workshop:

MEMBERSHIP

Memberships run from 1 Jan to 31 Dec. each year. Membership forms can be accessed from the CMA website: http://www.canberramaths.org.au

Membership of CMA includes affiliation with the Australian Association of Mathematics Teachers and a subscription to one of two AAMT journals.

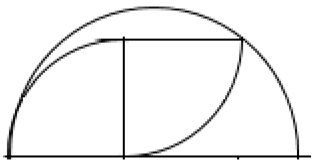
As a member, you are entitled to attractive rates for the CMA annual conference and CMA professional development events.

CMA members may attend conferences of the AAMT affiliates in other states, MAV, MANSW, etc. at member rates.

PUZZLES

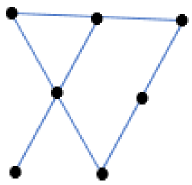
1 Curvy shapes

What proportion of the half-disc is covered by the quarter-discs?



2 Tree planting

In 1821, John Jackson published, in verse form, some puzzles in which given numbers of trees were to be arranged so that a specified number of them formed a row. Such problems are now known as 'orchard' problems. The idea is to maximise the number of rows possible in each case. For example, here are seven trees, three to a row, in four rows.



Can you find an arrangement of seven trees with three trees per row that has six rows?

3 Ramanujan in the Strand

Srinivasa Ramanujan did not invent this puzzle. But he used a remarkable generalisation of it to find infinitely many solutions. Here is a simple version.

There are fewer than 10 houses in a certain street, numbered 1, 2, 3, 4, … consecutively. One house on the street is such that the sum of all the house numbers to the left of it is equal to the sum of all the house numbers to its right. For what number of houses in the street is this possible? Find the number of the house.

As posed by Henry Dudeney in The Strand magazine, the puzzle specified that there were between 50 and 500 houses in the street.

MATHEMATICAL BLACK HOLES

By Andy Wardrop

Introduction

Whenever I started the semester with a new class, I used to give a lesson on an unusual topic that was fun, interesting and encouraged the students to think in different ways. One of my favourite topics was Mathematical Black Holes. What is a mathematical black hole? It is hard to explain in theoretical terms but the idea is easy to convey using examples. Basically, you choose a number, apply a well-defined mathematical process to that number and then apply the process to the answer. The process is repeated until you get to a point where you can't change the answer or you get into an endless loop of answers. As in the astronomical properties of a black hole in space, you cannot escape from a mathematical black hole.

Words to numbers

This is an easy way to start looking at mathematical black holes. It first came to my attention in an article written by Michael W. Ecker (1992). Write down any whole number and then write the number in words. Now count the number of letters that occur in the written number. If there is a hyphen you can count it as a letter. Write down the count that you have obtained in words and repeat the process.

Example

I chose the number 12 889 . In words this is written as *twelve thousand eight hundred and eighty-nine.*

The number of letters in this written number is 40. In words this is *forty.*

The number of letters in this word is 5. In words this is *five.*

The number of letters in this word is 4. This is written as *four.*

The number of letters in this word is 4. This is written as *four.* We are in a mathematical black hole from which we cannot progress. It took just 4 steps.

Check other numbers. Do they all lead to the number 4? What happens in other languages? Does the size of the initial number determine the number of steps to the black hole? Does it work with fractions? Does it matter if we count the blank spaces between the words? What starting number takes the highest number of steps to get to *four*?

It is clear that the black hole of *four* is language dependent and students enjoyed exploring this further.

To be a potential black hole in *Words to Numbers*, a

(Continued next page)

number must have the same number of letters in its name as its numerical value. Italian *tre* (3), German *vier* (4) and Spanish *cinco* (5) satisfy the criterion.

In order to fall into the black hole, the language also needs to have a number name with that number of letters in the name. Italian *uno* (1), *due* (2) and *sei* (6), German *eins* (1), *zwei* (2), *drei* (3), *fünf* (5), *acht* (8), *neun* (9) and *zehn* (10) and Spanish *siete* (7) indicate that we would get a mathematical black hole in Italian, Spanish and German.

Lithuanian *penki* (5) looks promising but Ladd (1993) pointed out that no other number name has five letters in Lithuanian and so it is not possible to get to a single digit black hole. French gets to an endless loop of trois (3), cinq (5), quatre (4) and six (6). It is possible that some languages would have two or more black holes.

Kaprekar's Constant

This mathematical black hole was discovered in 1949 by the Indian mathematician and schoolteacher Dattatreya Ramachandra Kaprekar (1905-1986).

Take any four-digit number except one with four identical digits. Rearrange the digits in your number to form the largest number possible and the smallest number possible. Find the difference between these two numbers. Repeat the process with the difference. Repeat until you reach a black hole.

Example

I choose 8028. The largest arrangement is 8820 and the smallest is 0288. The difference is 8532.

The largest arrangement is 8532 and the smallest is 2358. The difference is 6174.

The largest arrangement is 7641 and the smallest is 1467. The difference is 6174.

We are in a mathematical black hole. The number 6174 is known as Kaprekar's Constant.

The example above got to the black hole in two steps. The maximum number of steps required is 7.

In the next article I will give further examples of mathematical Black Holes

References

Ecker, M. W. (1992). "Caution: Black Holes at Work" <u>*New Scientist*</u> Issue 1853 Dec 19/26 p.38

Kaprekar, D. R. (1949). *"Another Solitaire Game". <u>Scripta Mathematica</u>. **15**: 244–245.*

Ladd, D. R. (1993). "Letters: Names of Numbers" New Scientist Issue 1857 Jan 23 p. 51

LANGUAGE STRATEGIES FOR MATHEMATICS—STRATEGY 3

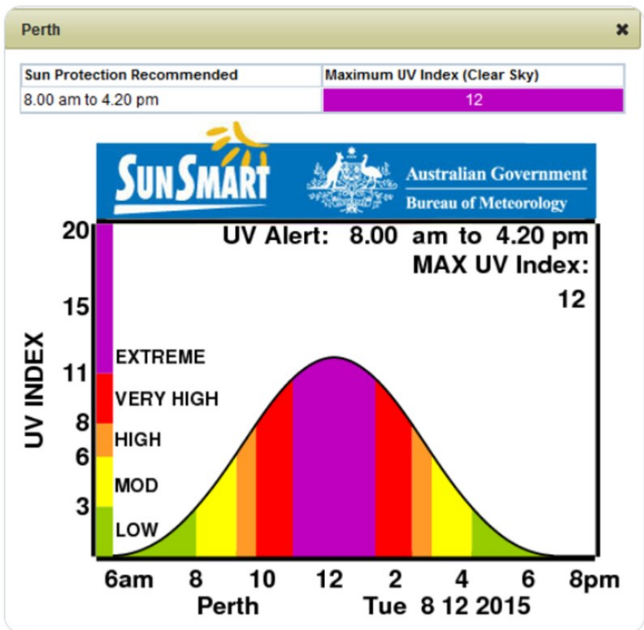
By Heather Wardrop

<u>The Three-level Guide using a diagram or graph</u>

Continuing the questioning strategy from last issue, the following is an example of how a graph can be the basis for a lesson exploring a relevant issue mathematically. The questions are designed to prompt students to look closely at a graph and to discover how to extract the information it contains.

As before, Level 1 questions can be answered directly from the information in the diagram. Level 2 questions need some deductive reasoning. Level 3 questions require a response in written language.

This example is timely with summer coming up and it is suitable for use with all ages.



Each question is to be answered with one of the responses

T / F / unable to determine.

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NEWSLETTER OF THE CANBERRA MATHEMATICAL ASSOCIATION INC

PO Box 3572
Weston ACT 2611
Australia

E-mail: canberramaths@gmail.com

We're on the Web!
http://www.canberramaths.org.au/

ABOUT THE CMA

The Canberra Mathematical Association (Inc.) is the representative body of professional educators of mathematics in Canberra, Australia.
It was established by, among others, the late Professor Bernhard Neumann in 1963. It continues to run - as it began - purely on a volunteer basis.

Its aims include

* the promotion of mathematical education to government through lobbying,
* the development, application and dissemination of mathematical knowledge within Canberra through in-service opportunities, and
* facilitating effective cooperation and collaboration between mathematics teachers and their colleagues in Canberra.

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http://www.facebook.com/pages/Canberra-Mathematical-Association/110629419011275

Level 1 Questions (add many more if you like)

- The UV index is low between 6am and 8 am
- The UV index is dangerous before and after school
- The data set is from Perth
- The maximum index possible is 20

Level 2 Questions

- The U.V. level is extreme for about 2 hours
- Each scale marking on the vertical axis is 3 index points
- One sixth of the total day has an extreme UV index
- The index described as "low" is 20% of the maximum
- At 3pm the sun's rays are more harmful than at 9am
- An alert is issued for 8 hours
- It's a graph for a summer day

Level 3 Questions. Reasons must be given

- Lunch time should be abolished in schools. It needs to be replaced with long morning and afternoon recess breaks to reduce the harmful effects of the sun.
- Parents need to teach their kids about this. It's not the school's job

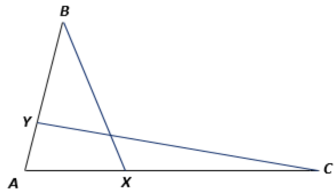
FROM A SCHOOL IN VICTORIA

A reader has alerted us to a Virtual Tour to be conducted by the Lumineer School in Williamstown, Victoria.

The school wishes to advertise its progressive approach to teaching and learning to potential new staff (and students). The next tour will be on October 7. Bookings can be made by clicking the link.

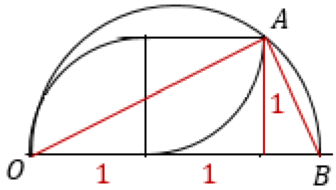
PUZZLE SOLUTIONS

0. In the land of the medians—see last issue.



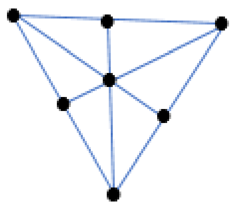
X divides AC in the ratio 1:2, and Y divides AB in the same ratio. Let $AC = \boldsymbol{u}$ and $AB = \boldsymbol{v}$ be a pair of independent vectors. Then, $XB = \boldsymbol{v} - 1/3\ \boldsymbol{u}$ and $YC = \boldsymbol{u} - 1/3\ \boldsymbol{v}$. With the point of intersection I, put $AI = \boldsymbol{x} = 1/3\ \boldsymbol{u} + m\ XB$ and $\boldsymbol{x} = 1/3\ \boldsymbol{v} + n\ YC$. Then, after substitutions and simplification, $(1-m)/3\ \boldsymbol{u} + m\ \boldsymbol{v} = n\ \boldsymbol{u} + (1-n)/3\ \boldsymbol{v}$. Linear combinations of vectors are equal iff the coefficients are equal. So, we deduce $m = n$. Also, $(1-m)/3 = m$, so that $m = 1/4$.

1. Curvy shapes



By Pythagoras, the diagonal OA is $\sqrt{5}$. By similar triangles, $OB/OA = OA/2$. So, the radius of the larger circle is 5/4 times the radius of the smaller. Hence, the quarter-discs cover 16/25 of the half-disc.

2. Tree Planting



3. Ramanujan in the Strand

By trial and error: the 6th house in a row of 8.

If house number n in a street of y houses satisfies the condition, then $(n-1)n/2 = y(y+1)/2 - n(n+1)2$. Ramanujan saw that this was $(2y + 1)^2 - 2(2n)^2 = 1$, a Pell equation. It has the form $Y^2 - 2X^2 = 1$. For large enough values of X and Y it is approximately true that $Y/X = \sqrt{2}$. Integer solutions in X and Y correspond to the convergents of the continued fraction expansion of $\sqrt{2}$, and these lead to the solutions in n and y. E.g. house 35 in a row of 49.