SHORT CIRCUIT

Newsletter of the Canberra Mathematical Association INC

VOLUME 11 NUMBER 8 NOVEMBER 2020

NEWS AND COMMENT

In November the school year begins to wind up for the senior students although there is, of course, still some way to go for the younger groups. Teachers enter end-of-year assessment and report writing mode and will be planning for 2021. In a few days the CMA will hold its Annual General Meeting after which a new council will consolidate plans for 2021.

The major event run by the CMA, the annual conference, as well as several smaller professional development opportunities, were cancelled in 2020, due to the pandemic and there have been few ways for the current committee to fill the gaps. Through this difficult time, we hope that your professional association and this newsletter have at least been able to disseminate some useful information and have promoted a sense of community among mathematics teachers.

The continuation of these and other roles (see page 4) depends critically on the membership. Your ideas and your subscriptions make CMA possible.

In this edition the story of mathematical black holes by Andy Wardrop is continued, as is the series on language strategies for mathematics by Heather Wardrop. We have crammed as much as possible into the puzzles page.

On page 7, there is an announcement involving a new expression that has entered the pedagogical debate: 'teaching maths for mastery'. There is an opportunity to find out what this is.

As always, Short Circuit welcomes your input.

CANBERRA MATHEMATICAL ASSOCIATION

Inside:

Coming Events:

CMA conference: CANCELLED.

CMA AGM: 11 November, 2020.

Wednesday Workshop:

MEMBERSHIP

Memberships run from 1 Jan to 31 Dec. each year. Membership forms can be accessed from the CMA website: http://www.canberramaths.org.au

Membership of CMA includes affiliation with the Australian Association of Mathematics Teachers and a subscription to one of two AAMT journals.

As a member, you are entitled to attractive rates for the CMA annual conference and CMA professional development events.

CMA members may attend conferences of the AAMT affiliates in other states, MAV, MANSW, etc. at member rates.

PUZZLES

From the yellowing pages of an old book, Classic Brain Puzzlers by Philip J. Carter & Ken A. Russell, we reproduce the following three puzzles:

1 Gruesome

This one originated with Lewis Carroll.
On return from the battlefield, the regiment is badly battle-scarred. If 70% of the soldiers have lost an eye, 75% have lost an ear, 85% have lost a leg and 80% have lost an arm, what percentage at least must have lost all four?
(A much easier question is: What percentage *at most* must have lost all four?)

2 Carpenter

From the 15th century French mathematician Nicolas Chuquet:
A carpenter agrees to work on the condition that he is paid 2 units for every day that he works, while he forfeits 3 units for every day that he does not work. At the end of 30 days he finds that he has paid out exactly as much as he has received. How many days did he work?

3 Diophantus of Alexandria

Three numbers are such that their sum is a perfect square, and the sum of any two of them is a perfect square. Find the three numbers if all of them are less than 500?

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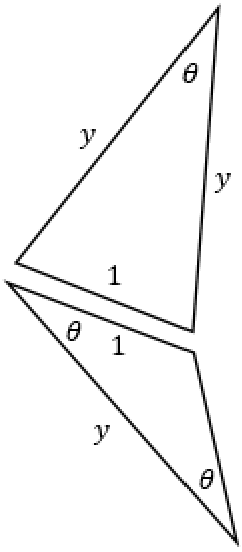
4 A pair of triangles

To the mathematician John Horton Conway, who died in April this year from the corona virus, we owe the game of *sprouts* and the fascinating cellular automata originating with the *game of life*. The following puzzle, we speculate, fits with his thoughts on the subject of tiling to which he contributed.

Conway observed that there exists a pair of non-similar isosceles triangles that nevertheless have a common angle and the same ratio between their different sides. We illustrate this below.



Find the values of the side length y and the shared angle θ, and observe anything special.

5 Sprouts

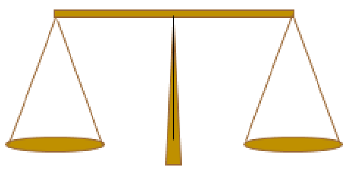
Continuing the Conway theme, consider the game of sprouts. To quote him, *The day after sprouts sprouted, it seemed that everyone was playing it, at coffee or tea times, there were little groups of people peering over ridiculous to fantastic sprout positions.* There is an explanation of the rules of sprouts on the NRICH site here, and a thoughtful article. See also the site of Madras College, St Andrews, Fife, Scotland, which has further links.

3 Tessellation

Triangles, squares or regular hexagons tile the plane but regular pentagons do not because their internal angle of 108° does not divide 360°. However, it is now known that there are exactly 15 kinds of *irregular* convex pentagons that do tile the plane. One might imagine that there could be an irregular heptagon that tiles the plane similarly. Explain why this is impossible.

7 Coin

One coin among eight is lighter than the others. Coins can be compared on a balance. How many weighings are needed, at most, to find the false coin?



MORE MATHEMATICAL BLACK HOLES

By Andy Wardrop

The Sisyphus String

In Greek mythology, King Sisyphus was condemned by Zeus to an eternity of frustration in the underworld. His task was to roll a large boulder to the top of a hill but as he got close to the top it would roll back down to the foot of the hill. He had to repeat this task endlessly even though it was futile. The mathematical process known as the Sisyphus String leads very quickly to a mathematical black hole and it is futile to try to find a number that is an exception.

Pick any whole number (a string of digits). Count the number of even digits, the number of odd digits and the total number of digits. Write these totals down as a new number thus: even/odd/total. Repeat the process until you get to a black hole.

Example
I choose the number 232 115 788 901 100 249 448. There are 12 even digits, 9 odd digits and 21 digits in total. This is now written as 12 921. There are 2 even digits, 3 odd digits and 5 digits in total. This is now written as 235. There is 1 even digit, 2 odd digits and 3 digits in total. This is now written as 123. There is 1 even digit, 2 odd digits and 3 digits in total. We have a black hole.

Kaprekar's other constant

Although 6174 is called Kaprekar's Constant, there is also a number associated with three-digit numbers. Take any three-digit number except one with three identical digits. Rearrange the digits in your number to form the largest number possible and the smallest number possible. Find the difference between these two numbers. Repeat the process with the difference. Repeat until you reach a black hole.

Example
I choose 937. The largest arrangement is 973 and the smallest is 379. The difference is 594. The largest arrangement of this is 954 and the smallest is 459. The difference is 495. The largest arrangement is 954 and the smallest is 459. The difference is 495. We have a black hole.

Interestingly, if you change the process slightly you get an endless loop instead of a single number black hole. Take any three-digit number except one with three identical digits or where the first and third digit are the same. Find the difference between this number and the number created by reversing the order of the digits. Repeat until you reach a black hole.

Example
I choose 937. The reverse number is 739 and the difference is 937 - 739 = 198. The reverse is now 891 and the difference is 891 – 198 = 693. The reverse is now 396 and the difference is 693 – 396 = 297. The reverse is now 792 and the difference is 792 – 297 = 495. The reverse is now 494 and the difference is 594 – 495 = 099. The reverse is now 990 and the difference is 990 – 099 = 891. The reverse is now 198 and the difference is 891 – 198 = 693. There is an endless loop of
693 – 297 – 495 – 099 – 891 – 693 ….

If you go to the wolfram page, https://mathworld.wolfram.com/KaprekarRoutine.html there is a graphic based on the number of iterations it takes each of the 9999 four-digit numbers to get to 6174. They have used leading zeros to make smaller integers into four-digit numbers and have colour coded the number of iterations required. It is very interesting.

Reference

Weisstein, Eric W. "Kaprekar Routine." From *MathWorld*--A Wolfram Web Resource. https://mathworld.wolfram.com/KaprekarRoutine.html

NEWSLETTER OF THE CANBERRA MATHEMATICAL ASSOCIATION INC

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We're on the Web!
http://www.canberramaths.org.au/

ABOUT THE CMA

The Canberra Mathematical Association (Inc.) is the representative body of professional educators of mathematics in Canberra, Australia.
It was established by, among others, the late Professor Bernhard Neumann in 1963. It continues to run - as it began - purely on a volunteer basis.

Its aims include

* the promotion of mathematical education to government through lobbying,
* the development, application and dissemination of mathematical knowledge within Canberra through in-service opportunities, and
* facilitating effective cooperation and collaboration between mathematics teachers and their colleagues in Canberra.

THE 2020 CMA COMMITTEE

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http://www.facebook.com/pages/Canberra-Mathematical-Association/110629419011275

LANGUAGE STRATEGIES FOR MATHEMATICS—STRATEGY 4

By Heather Wardrop

Strategy 4: The use of a structured overview.

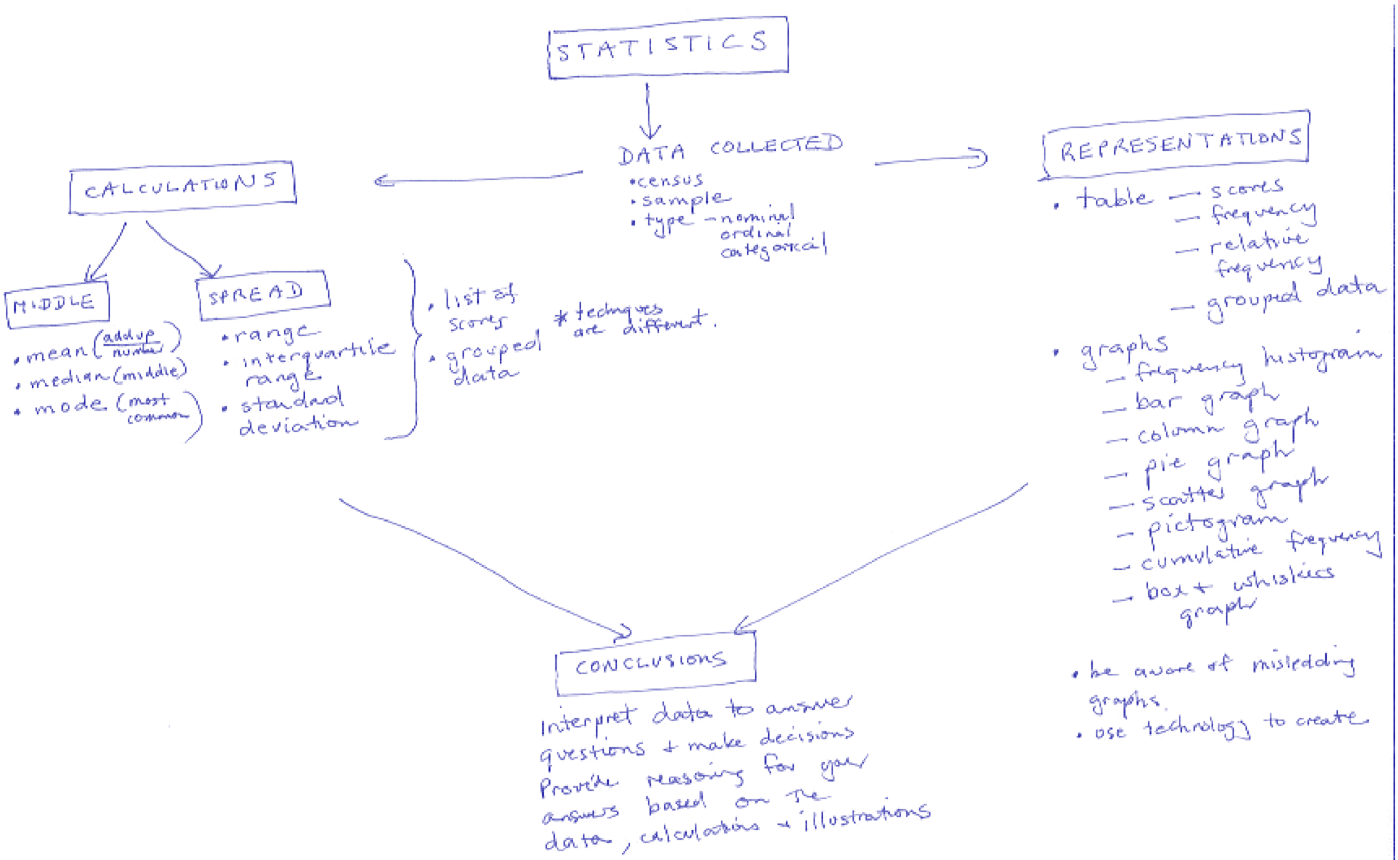
I have used structured overviews extensively. They tie the various topics within a mathematical theme together so that interconnections are made explicit. The diagrams aid learning and provide a summarising technique.

I have included two examples but clearly each diagram depends on what has been taught. All topics in mathematics can be extended or truncated, depending on the group one is working with. The diagrams can be done by the teacher but eventually students should be encouraged to do their own.

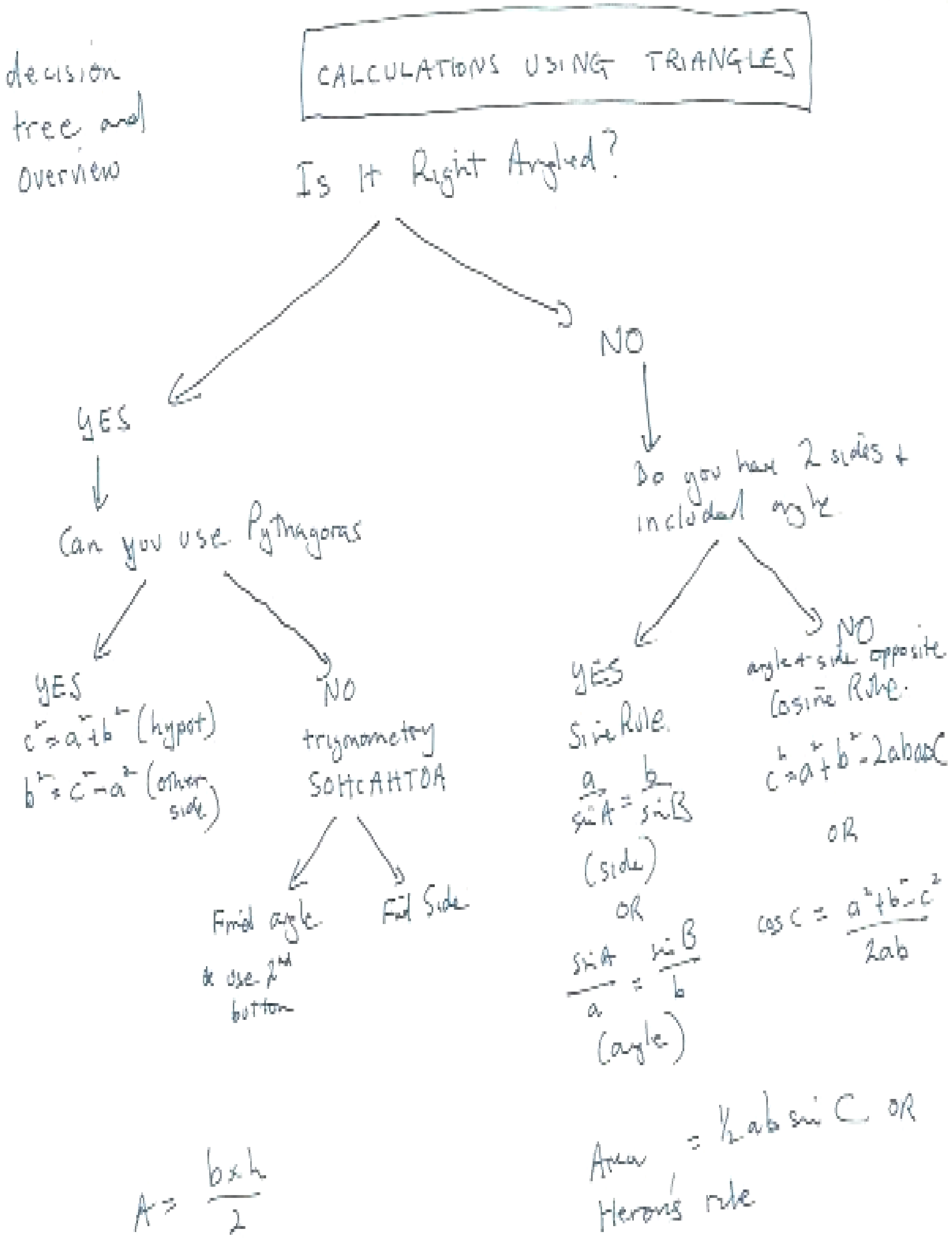
Typically, the diagrams will be hand-drawn, since the students are directly involved in the work.

Although a single word may often be used for an item in a dot-point list, it is important to use enough words and to choose them carefully so that the meaning will still be clear when it is time for revision. This is a connection with language.

The example below comes from Statistics. There is another, concerning triangles and Trigonometry on the next page.



The use of a structured overview—continued from page 5.



PUZZLE SOLUTIONS

1 Gruesome

The *at most* part is 70%, the size of the smallest subset. For the *at least* question, consider the smallest possible overlaps: 45% must have lost both an eye and an ear; then, since 85 + 45 = 130, 30% must have lost an eye, an ear and a leg; and similarly 10% must have received all four injuries.

2 Carpenter

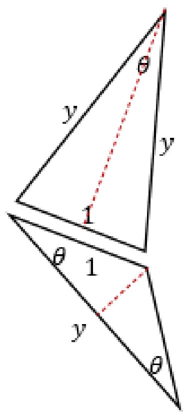
Was Chuquet, in the year 1484, promoting the use of *algebra* perhaps? If w and n are respectively the numbers of worked and non-worked days, we have $2w = 3n$ and also $w + n = 30$. Hence, $w = 18$.

3 Diophantus of Alexandria

The back of the book provides the solution 41, 80, 320. That is, $41 + 80 = 11^2$, $80 + 320 = 20^2$,
$41 + 320 = 19^2$, and $41 + 80 + 320 = 21^2$.
[It remains for readers to reveal an elegant way to find this and any further solutions. Near miss solutions exist. E.g. 1, 24, 120, or if negative integers are allowed, –80, 96, 129.]

4 A pair of triangles

Insert the bisectors as shown. Then,



$$\sin\frac{\theta}{2} = \frac{1}{2y} \text{ and } \cos\theta = \frac{y}{2}$$

From these, deduce $y^3 - 2y^2 + 1 = 0$. By inspection, this has a solution $y = 1$, which would make the triangles equilateral. Factorising, we have $(y - 1)(y^2 - y - 1) = 0$ and so obtain

$$y = \frac{1 + \sqrt{5}}{2}$$

This is the special number 'phi' associated with the Fibonacci sequence and also with the regular pentagon. In fact, these triangles are formed by the diagonals of the pentagon. The angle is

$$\cos^{-1}\frac{1 + \sqrt{5}}{4} = \frac{\pi}{5}$$

6 Tessellation

The internal angles of a heptagon sum to 540° so that the angles are more and less than the average value of 540°/7. The average number of heptagons that fit around a vertex must be 360°/(540°/7), which is less than 3. For the average to be less than 3 there must be a vertex at which fewer than 3 vertices meet, but this is impossible since a vertex ceases to exist under this condition.

7 Coin

Break the coins into two lots of three and one of two. Compare the threes. If the threes are the same, the false coin is among the remaining two and can be found with one further comparison. If one lot of three is lighter, it contains the false coin. Remove one coin from the three and compare the other two. If these are the same, the removed coin is false. Otherwise, the lighter coin of the two is the one. Thus, just two comparisons are needed.

TEACHING MATHS FOR MASTERY

AAMT is pleased to offer a series of 2 webinars in Teaching Maths for Mastery (November 12 and 19). The webinars are given by Mathematics Education Innovation (MEI) and the National Centre for Excellence in the Teaching of Mathematics (NCETM) in the UK.

The first webinar will look at the impact on maths education in England as a result of the widespread promotion and adoption of mastery teaching and learning practices. The second will focus on the detail of the structure and delivery of a mastery maths lesson in multiplicative reasoning at primary level. See here for further information.

The webinars are supported by the Department for Education in South Australia. Government employed educators in South Australia should click here.

The webinars are supported by the Association of Independent Schools of South Australia. Educators who are a member of AISSA should click here.

Educators in other jurisdictions, please click here.

Any schools, sector or departments that wish to provide a bulk registration for their staff, please contact Duncan at drayner@aamt.edu.au