CIRCUIT

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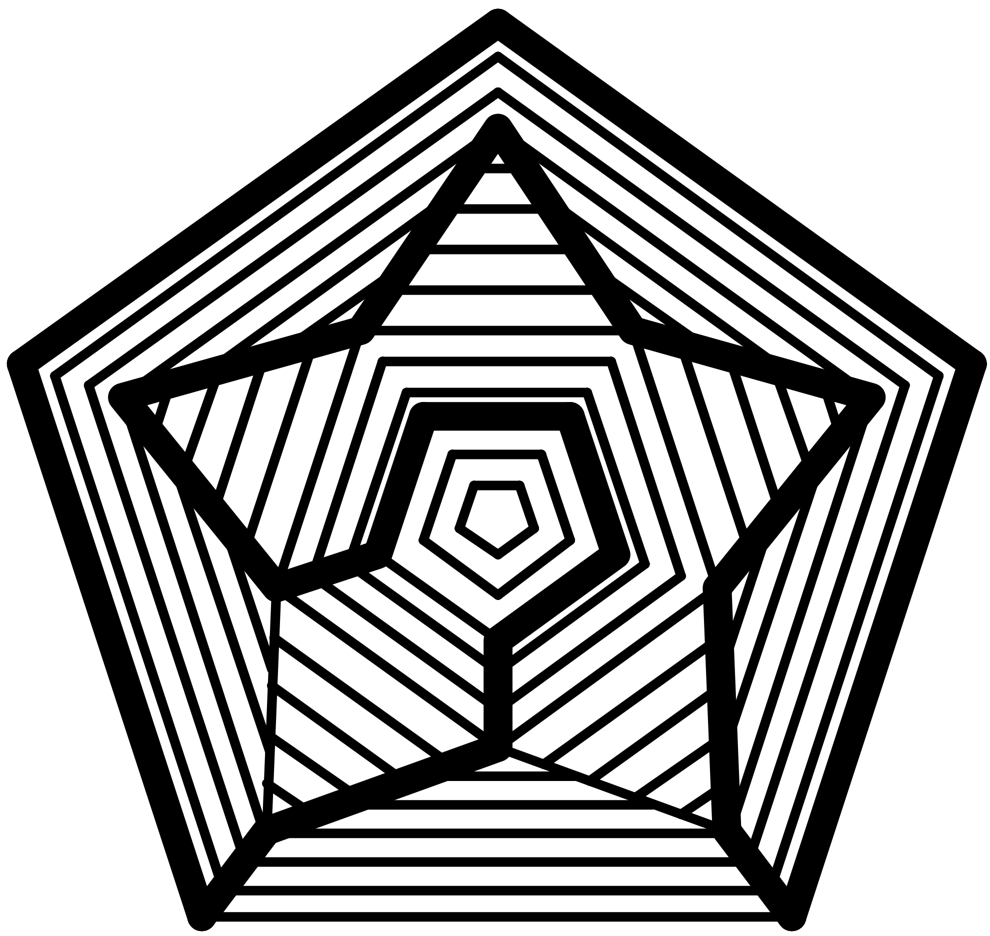

CANBERRA MATHEMATICAL ASSOCIATION

OFFICE BEARERS 1998

EDITOR *CIRCUIT* 1999

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CMA INTERNET HOME PAGE

http://education.canberra.au/projects/cma/home/html

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The objects of the Canberra Mathematical Association are to promote interest in mathematics, to encourage improvements in the teaching of mathematics and its applications, to provide means of communication among teachers and students and to advance the views of the Association on any question affecting the study or teaching of mathematics and its applications.

The Canberra Mathematical Association Logo depicts a Hamiltonian Circuit on a dodecahedron.

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CONTRIBUTIONS

Circuit is always keen to receive articles, notes, problems, letters and information of interest to members of the CMA. Please contact the Editor if you wish to contribute to Circuit. If possible contributions should be submitted as Word 97 documents.

FROM THE PRESIDENT

Welcome to 1999.

AAMT Conference

In January, with several members of our association, I attended the 17th Biennial Conference of the Australian Association of Mathematics Teachers Inc. (AAMT). We found out much about what is happening in mathematics and numeracy development in Australia and overseas and hope to share some of these new understandings with the teachers of mathematics in the ACT through articles in this journal and professional development activities throughout the year.

A particularly inspiring address was given by the President of the National Council of Teachers of Mathematics in USA, Glenda Lappan. She introduced her talk with a quote from Carl Sagan,

"We've arranged a civilisation in which most crucial elements profoundly depend on science and technology. We have also arranged things so that almost no one understands science and technology. This is a prescription for disaster. We might get away with it for a while, but sooner or later this combustible mixture of ignorance and power is going to blow up in our faces."

Glenda focused on the need for mathematics to make sense to our students. The goals of mathematics education need to be for ALL students to engage in mathematical thinking, reasoning, problem solving, connecting, communicating, seeking evidence and constructing arguments to make predictions and support conclusions. Worthwhile mathematical tasks, classroom discourse and classroom environment all contribute to students views of themselves as mathematics learners. Glenda cautioned that mathematics education reformers need to have their beliefs firmly supported by best practice and research to enable them to engage in broad community discourse with those of opposing views.

AAMT 2001 - Mathematics Shaping Australia

We are fortunate to be holding this conference in Canberra so keep the 15-19 January 2001 free to attend the conference. The organising committee is developing an exciting program with much of interest for educators from all sectors.

Professional Development Activities 1999

A draft program for CMA workshops is included in this Circuit. If you have any suggestions for PD activities don't hesitate to contact us with your ideas. Activities will be advertised in more detail in the ACT Education and Community Services Bulletin as well as by broadcast fax.

Primary School System Day 1999 - Numeracy

1998 saw primary schools developing literacy plans and now it's numeracy's turn for a share of the spotlight. For the first time all ACT primary teachers will be brought together for a common professional development program. In the morning session leaders in the mathematics education community will share their wisdom and provide food for thought on what numeracy means for them and what primary schools may do to improve numeracy outcomes for their students. During the afternoon teachers have time to reflect with their colleagues on numeracy. No doubt there will be much debate about what constitutes numeracy and what teachers can do to assist students' numeracy development. Other states and territories have been working on numeracy for some time now and there is much that we can learn from their efforts.

AAMT's Policy on Numeracy Education in Schools makes for worthwhile reading and can be accessed at

http://www.aamt.edu.au

By the way I have been assured that this is not a softener for a requirement that all primary schools have a numeracy plan by the end of the year.

Paulene Kibble

FROM THE EDITOR

Another year is already well under way, and although I don't imagine that too many of you are actually hanging out for this first *Circuit*, I do hope you will find at least some of our material to be of interest. As usual, this issue contains a mix of original and reprinted material, and attempts to provide stimulation and food for thought and action across the whole range of mathematics education from early childhood through to tertiary levels. If you think the coverage is a bit thin or non-existent in your areas of interest, then please let us have your suggestions and contributions, and encourage your colleagues "to think of stuff which could go in *Circuit*."

Across all sectors of educational activity in Australia at the moment the lack of access to professional development activities for teachers, the ever increasing demands on teachers' time, and the limited or non-existent access to up-to-date resources and technology in the classroom bespeak a national crisis. Of course the CMA sets out to provide a forum, a focus, and a lobby group for tackling these and other issues. This year, with your support and contributions, I hope *Circuit* can provide both a forum and a focus for discussion and action on such important matters.

Using the Web To Enhance Teaching - An Example

First Steps

I recently used the Internet to find resources for class-room use. It took me about fifteen minutes to find an activity, and en route I found heaps of ideas I hope to use at another time. Here is my story of venturing out on the web.

The Problem

I teach a bridging mathematics unit. My group last semester comprised 46 international students representing 17 nationalities / languages.

I wanted to modify the lecture format from being solely declarative - me telling the students, to one that allowed for more student participation. I already use a lot of questioning techniques in my lectures but I wanted more. So I decided to implement a workshop program. I allowed one lecture a fortnight for the workshop.

My aims for the workshops were to have activities that allowed the students to use the mathematical concepts and language we had been discussing in lectures, and to encourage them to use English in a mathematical context. The workshop which took me to the web was to be the conclusion to the function / pre calculus topic.

The content for this topic covered the domain and range of functions plus the graphs and features of functions. The functions we explored were linear, parabolic, hyperbolic, exponential, logarithmic and piecewise defined. Except in the case of linear and parabolic functions we did not model situations using the functions but started with a proposed model and used the model to answer questions about a particular situation. When it came to the workshop I had no ideas for a suitable activity.

The Solution

I decided to have a look for ideas on the web. I had heard that the AAMT home page has lots of links, so I started there. Links are set up by the page makers to help you find connected material so you don't have to use search engines. This means you don't find as much but it also means you don't find as much rubbish. I just chose options on each page which looked most related to my query. When I came to a 'dead end' I used the Back button to return to the page I had just come from, then reselected. Amazingly it took only about 15 minutes to find an excellent activity. I had never tried searching like this on the web before and although I had heard it has heaps to offer I was a bit cynical about actually being able to find something useable.

The activity involved:

- students working in pairs;
- giving each pair a function definition;
- asking each pair to sketch their given function;
- requiring each pair to write a description of the graph they had sketched;
- passing each description on to another pair;
- asking the new pair to sketch the graph from the given description;
- asking each pair to try to write a function definition from their sketch;
- checking their solution against the original function definition. Possible outcomes are:

(i) a correct function definition and the pair begin again;

(ii) an incorrect function definition, followed by a helpful hint along the lines of "have you considered this piece of the description ...?", another solution, and further feedback;

(iii) an incorrect function definition and further feedback along the lines of “we forgot to tell you that ...”, leading to another solution attempt and more checking.

In preparation for the workshop I:

- made up instruction sheets for each student;
- made up a set of function cards;
- took along with me graph paper for sketching and small cards for descriptions to be written on.

To start the workshop I demonstrated the process, and briefly reviewed the descriptive words that could be used. The students then spent a busy 100 minutes reviewing function definitions and notations, sketching, writing about, and discussing functions.

The Reflection

I was really pleased with the workshop.

- The topic content was thoroughly reviewed. Most pairs completed the cycle for about 6 different functions. For each function this meant that they reviewed the features of that function type, including definition, picture and discussion.
- Working in pairs encouraged dialogue in English. Because their work was to be passed on to another pair, the students were fairly rigorous in checking work before passing it on.
- Writing the function descriptions and then interpreting others’ descriptions used lots of mathematical language.
- The activity was easily understood by the students.
- Overall there was little off task time, as there was enough variety to maintain interest.

In summary, my aims were achieved, the students enjoyed the workshop and I have gotten started in using the web as a resource for my classroom.

For your interest I have included a copy of the student instruction sheet, a sample of the function definitions I used and some of the student description cards.

General Mathematics

Functions and Graphs

- You have been given a function definition.Sketch the function.
- With your partner write a description of the appearance of the function. DO NOT include the function definition. Neatly copy this description onto two cards.
- Exchange descriptions with another pair of students. From the description you have been given try to work out the function definition.
- When you think you have worked it out check with the writers.

  The writers may tell you 'that's correct',

  or 'that can't be the function because it doesn't satisfy a particular aspect of the description',

  or 'Oh no! our description is incorrect, we need to change it.'

Function Definition #4

$f(x) = 6x^2 - 5x + 12$

Function Definition #9

$H(a) = e^a$

This function has a minimum point.

The axis of symmetry is 5/12.

The x intercept of the function is 1 and there is no y intercept.

The function is very close to the negative section of y axis, but it becomes far from the positive section of y axis.

The function has only one intercept on x axis.

The domain is all values of "a" and the range is the positive values of H, with H intercept of 1. It is nearly parallel to the a axis and as it moves and cuts across +H intercept at 1 it moves up steeply parallel to the H axis.

Yvonne Wisbey
Foundation Studies
University of Canberra

Multicultural mathematics

On 26 October 1998, nearly sixty ACT teachers of all levels enjoyed a taste of Asia, including culture, mathematics and food. We began with a visit to the Embassy of the People's Republic of China, where we given a tour of the gardens by Jing Wei, the education officer of the Embassy. Jing Wei told us about the architecture and history of the Embassy, relating it to what we would find on a visit to China.

The Embassy visit was followed with a workshop at Yarralumla Primary School. Annabelle Cassells presented some practical ideas about how Chinese perspectives on mathematics could help to enrich our teaching in primary schools. Steve Thornton presented an activity called 'The Broken Bamboo', which took an ancient Chinese problem and looked at how it could be solved using Pythagoras' Theorem. Without a doubt the most impressive part of the workshop was a demonstration of the abacus by Hisami Shimba, a visiting Japanese teacher. The speed with which Hisami was able to carry out calculations was astonishing, but more significant from a teaching point of view was the number visualisation that was developed through the use of the abacus.

The workshop was followed by a sumptuous banquet at the Emperor Court Chinese Restaurant in Yarralumla. The opportunity for people from different backgrounds to socialise, and to talk about mathematics teaching, was greatly appreciated by all.

We would like to thank all those who made the afternoon and evening possible, especially Jing Wei, Hisami, and Meg Smith and Frank Howard from Yarrlumla Primary School.

We are planning to follow up on the success of our first venture into the Mathematics of Asia by holding a similar event associated with, perhaps, India or Japan. Watch out for further details.

Extracts from the 'Broken Bamboo' Activity

The Problem


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(a)

An ancient Chinese problem in the Chiu Chang (approximately 150 BC) concerns a bamboo of length 10 chih, which is broken so that the top reaches the ground 3 chih from the foot of the stem. The task is to find the height of the break.

Chinese solution method

The Chiu Chang gives a general solution for this problem. It says:

'Take the square of the distance from the foot of the bamboo to the point at which its top touches the ground, and divide this by the length of the bamboo. Subtract the result from the length of the bamboo, and halve the resulting difference. This gives the height of the break.'

Recommended Resources

Two sets of resources were displayed, courtesy of Jacaranda Educational Supplies, Macquarie:

Maths from many Cultures

Six kits consisting of colourful posters and worksheets, containing interesting mathematical background suitable for early childhood to middle high school.

Posters

Eyecatching depictions of mathematics in a variety of cultures.

Annabelle Cassells, Dept of Education and Community Services, and Steve Thornton, Australian Mathematics Trust.

New Trends in Vocational Mathematics

Lake Ginninderra College has taken some major steps towards meeting the needs of its students with the writing of its new *Accredited Maths* course.

We recognise that students who elect to do this level of mathematics are looking to develop career-oriented skills. Most of these skills are generic, some are specific. We have packaged units that allow students to do the generic and as many of the specific as they wish.

In designing the course, six broad vocational pathways were identified: Business, Auto, Science, Apprenticeship, Sport and Hospitality. Each of the college's related faculties were asked to identify mathematical prerequisites and then short units were written to service the needs of those faculties.

The CIT was consulted to ensure that the groupings of units for each pathway covered all prerequisite knowledge and skills necessary for further training in the given field.

The course consists of eleven half units. Apart from the first unit, "Foundations and Finance", which is compulsory, all other units are available regularly over the two years.

The new structure provides a lot of flexibility. Students can focus their attention and complete the mathematics needed for any one vocational pathway within a year or less. They would then be free to elect another subject or do more work experience. Students who wish to study maths for two years will create a number of pathways for themselves. *Students can now tailor their maths course to suit their needs within the context of their total package.*

Staff at Lake Ginninderra College are excited about the introduction of this course for a number of reasons. It allows us to work cooperatively with the rest of the school in producing effective vocational pathways. We're also confident that it will empower and motivate our students and therefore contribute to a healthier working environment for all parties.

Greg Anderson-Clift

The following talk is reprinted with permission from the Edge Foundation, Inc. This organization was established in the USA in 1988 as an outgrowth of a group known as The Reality Club. The mandate of the Edge Foundation is to promote inquiry into and discussion of intellectual, philosophical, artistic and literary issues, as well as to work for the intellectual and social achievement of society. Edge Foundation is a non-profit foundation.

Since 1981, The Reality Club has invited over 150 individuals to make presentations at Reality Club meetings and more recently Edge Seminars. The motto of the Club is "to arrive at the edge of the world's knowledge, seek out the most complex and sophisticated minds, put them in a room together and have them ask each other the questions they are asking themselves." The Edge Foundation maintains a website at **www.edge.org** *which contains transcripts of more recent presentations and is well worth a browse by educators, whatever their field.*

What Kind of Thing is a Number?

A Talk with Reuben Hersh

Throughout the interview the interviewer John Brockman is referred to by the initials JB.

JB: Reuben, got an interesting question?

HERSH: What is a number? Like, what is two? Or even three? This is sort of a kindergarten question, and of course a kindergarten kid would answer like this: (raising three fingers). Or two (raising two fingers). That's a good answer and a bad answer.

It's good enough for most purposes, actually. But if you get way beyond kindergarten, far enough to risk asking really deep questions, it becomes: what kind of a thing is a number?

Now, when you ask "What kind of a thing is a number?" you can think of two basic answers--either it's out there some place, like a rock or a ghost; or it's inside, a thought in somebody's mind. Philosophers have defended one or the other of those two answers. It's really pathetic, because anybody who pays any attention can see right away that they're both completely wrong.

A number isn't a thing out there; there isn't any place that it is, or any thing that it is. Neither is it just a thought, because after all, two and two is four, whether you know it or not.

Then you realize that the question is not so easy, so trivial as it sounds at first. One of the great philosophers of mathematics, Gottlob Frege, made quite an issue of the fact that mathematicians didn't know the meaning of One. What is One? Nobody could answer coherently. Of course Frege answered, but his answer was no better, or even worse, than the previous ones. And so it has continued to this very day, strange and incredible as it is. We know all about so much mathematics, but we don't know what it really is.

Of course when I say, "What is a number?" it applies just as well to a triangle, or a circle, or a differentiable function, or a self-adjoint operator. You know a lot about it, but what is it? What kind of a thing is it? Anyhow, that's my question. A long answer to your short question.

JB: And what's the answer to your question

HERSH: Oh, you want the answer so quick? You have to work for the answer! I'll approach the answer by gradual degrees.

When you say that a mathematical thing, object, entity, is either completely external, independent of human thought or action, or else internal, a thought in your mind--

you're not just saying something about numbers, but about existence--that there are only two kinds of existence. Everything is either internal or external. And given that choice, that polarity or dichotomy, numbers don't fit - that's why it's a puzzle. The question is made difficult by a false presupposition, that there are only two kinds of things around.

But if you pretend you're not being philosophical, just being real, and ask what there is around, well for instance there's the traffic ticket you have to pay, there's the news on the TV, there's a wedding you have to go to, there's a bill you have to pay--none of these things are just thoughts in your mind, and none of them is external to human thought or activity. They are a different kind of reality, That's the trouble. This kind of reality has been excluded from metaphysics and ontology, even though it's well-known--the sciences of anthropology and sociology deal with it. But when you become philosophical, somehow this third answer is overlooked or rejected.

Now that I've set it up for you, you know what the answer is. Mathematics is neither physical nor mental, it's social. It's part of culture, it's part of history, it's like law, like religion, like money, like all those very real things which are real only as part of collective human consciousness. Being part of society and culture, it's both internal and external. Internal to society and culture as a whole, external to the individual, who has to learn it from books and in school. That's what math is.

But for some Platonic mathematicians, that proposition is so outrageous that it takes a lot of effort even to begin to consider it.

JB: Reuben, sounds like you're about to push some political agenda here, and it's not the Republican platform.

HERSH: You're saying my philosophy may be biased by my politics. Well, it's true! This is one of the many novel things in my book-- looking into the correlation between political belief and belief about the nature of mathematics.

JB: Do you have a name for this solution?

HERSH: I call it humanistic philosophy of mathematics. It's not really a school; no one else has jumped on the bandwagon with that name, but there are other people who think in a similar way, who gave it different names. I'm not completely a lone wolf here, I'm one of the mavericks, as we call them. The wolves baying outside the corral of philosophy.

Anyhow, back to your other question. The second half of my book is about the history of the philosophy of mathematics. I found that this was best explained by separating philosophers of mathematics into two groups. One group I call Mainstream and the other I call Humanists and Mavericks. The Humanists and Mavericks see mathematics as a human activity, and the Mainstream see it as inhuman or superhuman. By the way, there have been humanists way back; Aristotle was one. I wondered whether there was any connection with politics. So I tried to classify each of these guys as either right-wing or left-wing, in relation to their own times. Plato was far right; Aristotle was somewhat liberal. Spinoza was a revolutionary; Descartes was a royalist, and so on. These are well known facts. There are some guys that you can't classify. It came out just as you are intimating, The humanists are predominantly left-wing and the mainstream predominantly right wing. Any explanation would be speculative, but intuitively it makes sense. For instance, one main version of mainstream philosophy of mathematics is Platonism. It says that all mathematical objects, entities, or whatever, including the ones we haven't discovered yet and the ones we never will discover-all of have always existed. There's no change in the realm of mathematics. We discover things, so our knowledge increases, but the actual mathematical universe is completely static. Always was, always will be. Well that's kind of conservative, you know? Fits in with someone who thinks that social institutions mustn't change.

So this parallel exists. But there are exceptions. For instance, Bertrand Russell was a Platonist and a socialist. One of my favorite philosophers, Imre Lakatos, was a right-winger politically, but very radical philosophically. These correlations are loose and statistical, not binding. You can't tell somebody's philosophy from his politics, or vice versa.

I searched for a suitable label for my ideas. There were several others that had been used for similar points of view--social constructivism, fallibilism, quasi-empiricism, naturalism. I didn't want to take anybody else's label, because I was blazing my own trail, and I didn't want to label myself with someone else's school. The name that would have been most accurate was social conceptualism. Mathematics consists of concepts, but not individually held concepts; socially held concepts. Maybe I thought of humanism because I belong to a group called the Humanistic Mathematics Network. Humanism is appropriate, because it's saying that math is something human. There's no math without people. Many people think that ellipses and numbers and so on are there whether or not any people know about them; I think that's a confusion.

JB: Sounds like we're talking about an anthropic principle of mathematics here

HERSH: Maybe so; I never thought of that. I had a serious argument with a friend of mine at the University of New Mexico, a philosopher of science. She said "There are nine planets; there were nine planets before there were any people. That means there was the number nine, before we had any people."

There is a difficulty that has to be clarified. We do see mathematical things, like small numbers, in physical reality. And that seems to contradict the idea that numbers are social entities. The way to straighten this out has been pointed out by others also. We use number words in two different ways: as nouns and adjectives. This is an important observation. We say nine apples, nine is an adjective. If it's an objective fact that there are nine apples on the table, that's just as objective as the fact that the apples are red, or that they're ripe, or anything else about them, that's a fact. And there's really no special difficulty about that. Things become difficult when we switch unconsciously, and carelessly, between this real-world adjective interpretation of math words like nine, and the pure abstraction that we talk about in math class.

That's not really the same nine. Although there's of course a correlation and a connection. But the number nine as an abstract object, as part of a number system, is a human possession, a human creation, it doesn't exist without us. The possible existence of collections of nine objects is a physical thing, which certainly exists without us. The two kinds of nine are different.

Like I can say a plate is round, an objective fact, but the conception of roundness, mathematical roundness, is something else.

Sad to say, philosophy is definitely an optional activity; most people, including mathematicians, don't even know if they have a philosophy, or what their philosophy is. Certainly what they do would not be affected by a philosophical controversy. This is true in many other fields. To be a practitioner is one thing; to be a philosopher is another. To justify philosophical activity one must go to a deeper level, for instance as in Socrates' remark about the unexamined life. It's pathetic to be a mathematician all your life and never worry, or think, or care, what that means. Many people do it. I compare this to a salmon swimming upstream. He knows how to swim upstream, but he doesn't know what he's doing or why.

JB: How does having a philosophy of mathematics affect its teaching?

HERSH: The philosophy of mathematics is very pertinent to the teaching of mathematics. What's wrong with mathematics teaching is not particular to this country. People are very critical about math teaching in the United States nowadays, as if it was just an American problem. But even though some other countries get higher test scores, the fundamental mis-teaching and bad teaching of mathematics is international, it's standard. In some ways we're not as bad as some other countries. But I don't want to get into that right now.

Let me state three possible philosophical attitudes towards mathematics.

Platonism says mathematics is about some abstract entities which are independent of

humanity.

Formalism says mathematics is nothing but calculations. There's no meaning to it at all. You just come out with the right answer by following the rules.

Humanism sees mathematics as part of human culture and human history.

It's hard to come to rigorous conclusions about this kind of thing, but I feel it's almost obvious that Platonism and Formalism are anti-educational, and interfere with understanding, and Humanism at least doesn't hurt and could be beneficial.

Formalism is connected with rote, the traditional method which is still common in many parts of the world. Here's an algorithm; practice it for a while; now here's another one. That's certainly what makes a lot of people hate mathematics. (I don't mean that mathematicians who are formalists advocate teaching by rote. But the formalist conception of mathematics fits naturally with the rote method of instruction.)

There are various kinds of Platonists. Some are good teachers, some are bad. But the Platonist idea, that, as my friend Phil Davis puts it, Pi is in the sky, helps to make mathematics intimidating and remote. It can be an excuse for a pupil's failure to learn, or for a teacher's saying, "Some people just don't get it."

The humanistic philosophy brings mathematics down to earth, makes it accessible psychologically, and increases the likelihood that someone can learn it, because it's just one of the things that people do. This is a matter of opinion; there's no data, no tests. But I'm convinced it is the case.

JB: How do you teach humanistic math?

HERSH: I'm going to sidestep that slightly, I'll tell you my conception of good math teaching. How this connects with the philosophy may be more tenuous.

The essential thing is interaction, communication. Only in math do you have this typical figure who was supposedly exemplified by Norbert Wiener. He walks into the classroom, doesn't look at the class, starts writing on the board, keeps writing until the hour is over and then departs, still without looking at the class.

A good math teacher starts with examples. He first asks the question and then gives the answer, instead of giving the answer without mentioning what the question was. He is alert to the body language and eye movements of the class. If they start rolling their eyes or leaning back, he will stop his proof or his calculation and force them somehow to respond, even to say "I don't get it." No math class is totally bad if the students are speaking up. And no math lecture is really good, no matter how beautiful, if it lets the audience become simply passive. Some of this applies to any kind of teaching, but math unfortunately is conducive to bad teaching.

It's so strange. Mathematical theorems may really be very useful. But nobody knows it. The teacher doesn't mention it, the students don't know it. All they know is it's part of the course. That's inhuman, isn't it?

Here's an anecdote. I teach a class, which I invented myself, called Problem Solving for High School and Junior High School Teachers and Future Teachers. The idea is to get them into problem solving, having fun at it, feeling confident at it, in the hope that when they become teachers they will impart some of that to their class The students had assignments; they were supposed to work on something and then come talk about it in class. One day I called for volunteers. No volunteers. I waited. Waited. Then, feeling very brave, I went to the back of the room and sat down and said nothing. For a while. And another while. Then a student went to the blackboard, and then another one.

It turned out to be a very good class. The key was that I was willing to shut up. The easy thing, which I had done hundreds of times, would have been to say, "Okay, I'll show it to you." That's perhaps the biggest difficulty for most, nearly all, teachers-not to talk so much. Be quiet. Don't think the world's coming to an end if there's silence for two or three minutes.

JB: Earlier you mentioned the word beauty. What's with beauty?

HERSH: Fortunately, I have an answer to that. My friend, Gian-Carlo Rota, dealt with that issue in his new book, "Indiscrete Thoughts." He said the desire to say "How beautiful!" is associated with an insight. When something unclear or confusing suddenly fits together, that's beautiful. Maybe there are other situations that you would say are beautiful besides that, but I felt when I read that that he really had something. Because we talk about beauty all the time without being clear what we mean by it; it's purely subjective. But Rota came very close to it. Order out of confusion; simplicity out of complexity; understanding out of misunderstanding; that's mathematical beauty.

Reuben Hersh is professor emeritus at the University of New Mexico, Albuquerque. He is the recipient (with Martin Davis) of the Chauvenet Prize and (with Edgar Lorch) the Ford Prize. Hersh is the author (with Philip J. Davis) of *The Mathematical Experience and Descartes' Dream*, which won the National Book Award in 1983. His book, *What is Mathematics, Really?*, which enlarges on many of the matters covered here, was published in 1997.

QUOTABLE NOTES AND NOTABLE QUOTES

The Elephant of Mathematics

You've heard of the proverbial three blind persons describing an elephant. Upon touching a leg, one says, "It's like a tree." Another, touching the trunk reports, "It's like a snake." The third, touching an ear, says, "It's like a bat." **Now how would you describe/define mathematics**?

Missing picture

To aid your thinking, contemplate the legs on the elephant pictured. While you are contemplating, change the focus of your eyes from elephant tummy to elephant feet. Now you see some legs, now you see others. Are you still comfortable with your description/definition of mathematics?

> Based on a passage from *Strength in Numbers: discovering the joy and power of mathematics in everyday life* by Sherman Stein. The elephant picture comes from an Optical Illusions site on the Web.

Two Anagrams with Mathematical Content

eleven plus two ↔ twelve plus one

a decimal point ↔ I'm a dot in place

From the Web

What Is the Point of Theorems?

A British mathematician, Christopher Zeeman, defined theorems as 'intellectual resting points'.

What Are Proofs Then?

Stories which convince mathematicians.

Gender Differences in Maths Tests

Barbara Fredrickson reported in the *Journal of Personality and Social Psychology* on experiments which investigated whether the amount of clothing worn by mathematics exam candidates affected their results. Volunteers at the University of Michigan were the guinea pigs.

Women wearing a swimsuit did worse in the maths exam than those who were fully clothed. Were they too self-conscious to concentrate? Fredrickson suggests that women are so concerned about their body that it "disrupts their mental performance".

Not so the male test subjects. "Some were even heard to laugh through the closed dressing room door, "writes Fredrickson. What's more, she says that men tended to perform better on the maths test when dressed in less.

Over to you for classroom implications and suggestions for follow-up experiments.

Based on an item from New Scientist, Feedback, 3 October 1998

Mathematics As Monkey Business

Monkeys are surprisingly numerate say two psychologists at Columbia University New York. Elizabeth Brannon and Herbert Terrace gave a pair of rhesus monkeys a touch sensitive video monitor and, using banana flavoured rewards, trained them over about six weeks to the point where they were able to reliably rank groups of up to four items in ascending order. Later the researchers put the monkeys through 150 new trials over five days, with new items - just to make sure they hadn't been using memory tricks all along. "Now we were sure they could differentiate 1, 2, 3 and 4", says Brannon.

The next surprise came when the psychologists tested the monkeys with larger quantities. When they were shown new images containing between five and nine items the monkeys put them in ascending order. They saw each pair of images only once and were not rewarded for getting it right. Yet they continued to respond with 75% accuracy, Brannon and Terrace report in *Science* (vol 282, p 746). "They didn't need training", says Brannon. "They understood the ordinal relationship."

Marc Hauser, a psychologist at Harvard University in Boston, says the study clearly shows that animals can have number skills without language abilities. Brannon and Terrace now hope to discover what kind of mechanisms underlie the monkeys' abilities. "Are they counting?" Brannon asks. "How are they representing the numbers?"

Based on **Natural Order** by Alison Motluk,
in **New Scientist**, This Week,
31 October 1998.

Let Us Now Praise Prime Numbers

Let us now praise prime numbers
With our fathers that begat us:
The power, the peculiar glory of prime numbers
Is that nothing begat them.
No ancestors, no factors
Adams among the multiplied generations.

None can foretell their coming.
Among the ordinal numbers
They do not reserve their seats, arrive unexpected.
Along the lines of cardinals
They rise like surprising pontiffs.
Each absolute, inscrutable, self-elected.

In the beginning where chaos
Ends and zero resolves,
They crowd the foreground prodigal as forest.
But middle distance thins them.
Far distance to infinity
Yields them rarely as unreturning comets.

O prime improbable numbers,
Long may formula-hunters
Steam in abstraction, waste to skeleton patience:
Stay non-conformist, nuisance.
Phenomena irreducible
To system, sequence, pattern or explanation.

Helen Spalding
with thanks to Dianne Barney

PROBLEMS AND ACTIVITIES

Remember that we include a coding system which attempts to indicate in terms of Year levels the suitability range for each item. Thus 6 - 8 suggests an item accessible to students from Year 6 to Year 8.

(1) A Perennial Application of Mathematics

7 - 10

(a) Solve the following problem which appears in Fibonacci's *Liber abaci*, circa 1202.

> A certain man puts one denarius at [compound] interest at such a rate that in five years he has two denarii, and in every five years thereafter the money doubles. I ask how many denarii he would gain from this one denarius in one hundred years?

(b) What annual rate of compound interest doubles money after five years?

(c) See what you can find out about the history and origins of interest charges and the derivation of the word "interest" itself.

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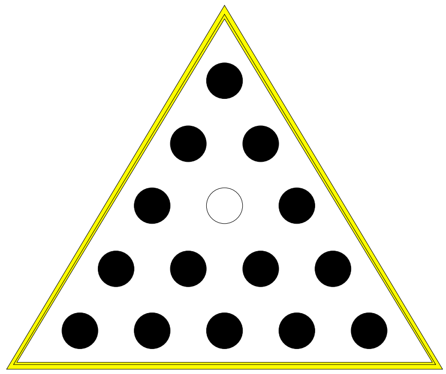
(2) Triangular Peg Solitaire

3 - 12

According to one version, the game of peg solitaire, often called simply solitaire, was invented in the 18th century by a French nobleman imprisoned in the Bastille. It was a modification of a game called Fox and Geese and used a board in which holes were bored as resting places for pegs or marbles. A variety of geometric arrangements for the layout and number of holes can be used, resulting in different levels of difficulty and complexity for solitaire puzzles. There is also some interesting mathematics associated with peg solitaire. See for example the book *Peg Solitaire* by John Beasley.

Now for the activity.

Use a photocopier to enlarge this equilateral triangle array of fourteen black spots and one white spot, so that the black spots can be covered by counters (or buttons, pieces of paper etc.) without overlapping.



The aim is to remove all but one counter by jumping counters over adjacent counters to unoccupied spots. Each counter which is jumped over is removed.

The next three problems come from an interesting British website, the NRICH Online Maths Club at

www.nrich.maths.org.uk

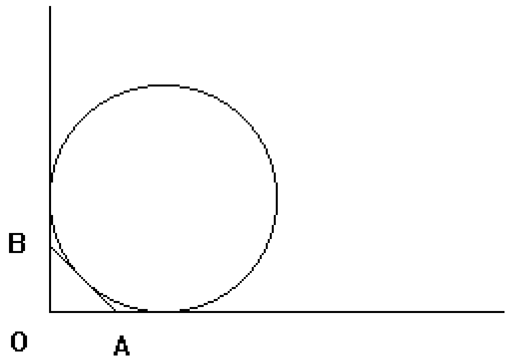
"Besides publishing children's own work, NRICH offers new challenges and resources for maths clubs and up to date educational material designed to take youngsters beyond the confines of the national curriculum and to help them to enjoy problem solving and to appreciate the significance and range of applications of mathematics."

This website overflows with interesting mathematics for students and anyone interested in mathematics education.

(3) Sum Diameter

9 - 12

A circle touches the lines OA extended, OB extended and AB, where OA and OB are perpendicular.



Show that the diameter of the circle is equal to the perimeter of the triangle.

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(4) Square Away

9 - 12

What is the smallest perfect square that ends with the four digits 9009?

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(5) Really the Largest Number

10 - 12

If x, y and z are real numbers such that

$$x + y + z = 5$$

and $$xy + yz + zx = 3$$

what is the largest value that any one of these numbers can have?

THUMBNAIL
(and longer) REVIEWS

Readers are welcome to contribute to this section. Reviews can cover books, periodicals, videos, software, CD ROMs, calculators, mathematical models and equipment, posters, etc.

"Any book that you can learn from is a good book."

T. W. Körner
in *The Pleasures of Counting*

Essential Texas TI-83 Graphics Calculator Companion

by David Main

Published by Cambridge University Press, Melbourne, 1999, $7.95

I should start by declaring my own biases. I have a strong dislike for resources that instruct students or teachers in how to use technology. Technology is not an end in itself. It can be useful as a tool for developing mathematical understanding, and as a means of solving problems which may be tedious or intractable otherwise. So I believe that our time and resources should be directed towards appropriate, sensible and effective use of technology in context.

The TI-83 Graphics Calculator Companion is intended to be, in the words of the back cover, 'an invaluable resource for all mathematics students from years 9 – 12'. It is certainly a very comprehensive publication, ranging from topics such as how to graph straight lines and how to carry out arithmetic computations to financial calculations, calculus and complex numbers. It is well laid out, and printed in a very useable format. It is designed to be kept in a ring-back folder, and used for student reference throughout their secondary years.

Each section contains instructions, illustrated by an example, with screen dumps that clearly indicate which keys to press. Exercises at the end of each section are a useful test of whether or not students can use the calculator effectively, and some investigations are included which encourage students to use the calculator in a more practical context.

I found it very easy to work through most sections of the booklet. As I did so I discovered that the TI-83 could do some things of which I had not previously been aware. However the booklet also omits some things that may be of use to students, particularly in the senior years. It does not deal with such powerful features as data transfer between calculators, programming, nor inferential statistics such as hypothesis testing or confidence intervals.

It concerns me that there is little or no attempt to explain why the various calculator keys do what they do. I found this particularly disconcerting in the Finance section, as my knowledge of terms such as present and future value and reducing balance is limited. I am still no wiser about when N should be the number of years or the number of payments, nor about when the payment should be negative or positive. The assumption is, of course, that students will be familiar with the mathematical ideas before they use the manual, but I still think that the booklet contains too much of the 'how' and not enough of the 'why'.

The TI-83 Graphics Calculator Companion is certainly a useful reference for students, particularly those who own a TI-83 and intend to use it throughout school. However, it is strictly that – a student reference manual. It is certainly not a publication that prompts teachers to reflect upon the effective use of this technology as a way of promoting their students' understanding of mathematics, nor does it provide good ideas or problems for the classroom. As a reference manual it is clear, convenient and comprehensive. However, you have to ask the question: Is it better than the Guidebook that comes with the calculator?

Steve Thornton

The Knots Puzzle Book

by Heather McLeay

Published by Tarquin Publications, Norfolk, England, 48 pages.

Knots have been important over thousands of years of human history, from their early uses in hunting, making shelters, clothing and other artefacts, through the seafarers dependence on knots in the days of sail, to the challenges of tying secure knots in ropes or lines made from modern materials. However, this is NOT a book about practical knot tying. Instead it is an open ended excursion into an area of mathematics which has been around for over a century, yet continues to spark interest because of ongoing discoveries.

McLeay's approach would appeal to anyone with a liking for puzzles or an interest in ideas of a mathematical nature. The format of this introduction to the mathematics of knots is based on single or double page spreads which mostly consist of knot diagrams and puzzles. The bonus is being able to try things out yourself with a length of real string or cord. Necessary terminology is gently introduced along the way, and the reader's grey matter is pleasantly massaged by the combination of brief declarative text, clear diagrams, questions and puzzles. My guess is that the material would be accessible to an interested 10 year old, yet suitable also for project or investigative work by high school and college level students. There is a rich set of ideas and surprises here, and the author does not talk down to the reader. Instead she unassumingly encourages looking, thinking and trying things out - the essence of mathematical activity. The rewards are understanding, insight and pure pleasure, and you may even end up being able to distinguish a reef from a granny knot. The solutions to the puzzles and references to some other books about the mathematics of knots are included.

This delightful publication should be in every school library and every mathematics staff room. It is an admirable example of a self-contained introduction to an important area of mathematical activity and ideas.

Peter Enge

The French Mathematician

by Tom Petsinis

Published by Penguin Australia, 1997.

This novel is an unusually creative feat of imagination. Narrated in the first person, it is Evariste Galois's tale of his multiple obsessions. Among these obsessions are mathematics, French 19th century post-Napoleonic Republicanism, anti-Jesuitical/anti-maternal religiosity, anti-establishment activism, and fiercely independent thought and action. Galois' youthful genius drives him ceaselessly to push the boundaries of mathematics and politics. The stream of consciousness narrative, set against the background of the political turmoil in the Paris of the time, seethes with spiritual energy, passionate certainty, impatience, and the constant frustration of the young subject at the seeming arbitrariness of intellectual, academic, social and political conventions. From the start, Petsinis conveys a sense that Galois is destined somehow to be denied success, yet paradoxically this seems to be related to his irrepressible intellectual and spiritual drive.

Petsinis's sustained imaginative portrayal of creative genius bears comparison with Thomas Mann's portrayal of
Leverkühn, the composer (based on Arnold Schönberg) in *Doctor Faustus*.

Undeliberate aloofness seems to be a common hallmark of the geniuses in these two books. Somehow Petsinis sustains a rhythm in the story, cycling around from Galois's inner life, through his living situation, his relation with his parents, his interaction with republican contacts, and his frustration with the do-nothing Academie Francaise, all against the background of Parisian life and events. Strangely, after over 400 pages of interior happenings, we are no clearer about the sources of Galois's genius. Genius seems to be a congenital affliction, but in this case thanks to Petsinis's creative achievement the reader gets a taste of it.

Peter Enge

SOLUTIONS TO PROBLEMS AND ACTIVITIES

(1)

(a) $2^{20} - 1$ denarii.

(b) If r% is the annual compound interest rate required to double money every 5 years then

$$\left(1 + \frac{r}{100}\right)^5 = 2$$

so $r = 100\,(\sqrt[5]{2} - 1) \approx 14.87$.

(c) The following material was provided by Beth Lee.

The custom of charging interest is found as early as 2000 B.C., as recorded on ancient Babylonian clay tablets. One example given by D.E. Smith is:

> Twenty manehs of silver, the price of wool, the property of Belshazzar, the son of the king ... All the property of Nadin-Merodach in town and country shall be the security of Belshazzar, the son of the king, until Belshazzar shall receive in full the money as well as the interest upon it.

Interest rates in Babylonia ran as high as 33 percent. In Rome during Cicero's day 48 percent was allowed; Justinian later set the maximum allowable rate at 0.5 percent per month, which gave rise to the common rate of 6 percent a year. In India, however, during the twelfth century, rates as high as 60 percent were recorded.

The origin of the word "interest" is related to church policy, which forbade usury (a payment for the use of money). The moneylender got around this church restriction by collecting a fee only if the money was late being repaid (which happened often even in those days). The lender argued that the fee compensated him for the monetary difference between his reduced financial standing because of the late repayment, and what his financial standing would have been had the money been repaid on time. This difference was referred to as "id quoi interest" (that which is between).

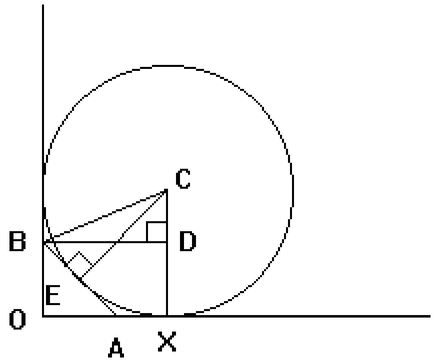
Sources

Historical Topics for the Mathematics Classroom, National Council of Teachers of Mathematics, USA, 1993, pp 325-326.

History of Mathematics, Smith, D. E., 2 Vols 1923, 1925. New York, Dover Publications, 1958.

(2) We only give a hint: try working backwards from a single counter to find configurations of not too many counters which leave only a single counter at the end.

(3)



Let C be the centre of the circle, let r be its radius and let X be the point of contact of the tangent through OA extended. Let E be the point of contact of tangent AB with the given circle.

Through B draw BD parallel to OA with D on CX.

Then ΔBCD, ΔCBE and ΔCAE are congruent.

So CD = BE = EA = ½AB.

Hence OB + BE = OB + CD

= CD + DX = r

and similarly EA + OA = r.

So the perimeter of ΔOBA

= OB + BE + EA + AO = r + r = 2r

= diameter of circle.

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(4) Since 9009 is not itself a perfect square and all perfect squares which end in 9 must have square roots ending in 3 or 7, we seek if possible the smallest values of a and/or b such that

$(a + 3)^2 = a^2 + 6a + 9 = a^2 + 9009$ (1),

or

$(b + 7)^2 = b^2 + 14b + 49 = b^2 + 9009$ (2).

We also require a and b to be multiples of 100 because if they are any smaller $a^2 + 9009$ and $b^2 + 9009$ will not end in 9009. (Why?)

(1) gives $6a = 9000$, so $a = 1500$.

(2) gives $14b = 8960$, so $b = 640$, not a multiple of 100.

Hence our smallest perfect square ending in 9009 is $1503^2 = 2\,250\,000 + 9009$

$= 2\,259\,009$, and other terms in this sequence of squares ending in 9009 are $11\,503^2$, $21\,503^2$, …

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(5) Since the given conditions are symmetric in x, y and z, we try to maximise x. Any solutions we find for this case will lead to corresponding solutions for y and z.

Because $x = 5 - (y + z)$, we see that we can maximise x if we can minimise $(y + z)$.

Now $xy + yz + zx = 3$ gives

$x(y + z) + yz = 3$, or

$(5 - (y + z))(y + z) + yz = 3$ (1)

Since this is an extreme value problem it might be useful to exploit the relation

$4yz = (y + z)^2 - (y-z)^2$ by substituting it in (1), giving

$(5-(y+z))(y+z) + \frac{1}{4}((y+z)^2-(y-z)^2) = 3$ (2)

Put $a = y + z$, so (2) becomes

$4a(5-a) + a^2 - (y-z)^2 = 12$,

so $3a^2 - 20a + 12 = -(y-z)^2$.

Since y and z are real, we conclude that

$3a^2 - 20a + 12 \le 0$, with equality occurring when $y = z$.

Hence $(3a - 2)(a - 6) \le 0$,

so that $\frac{2}{3} \le a \le 6$.

Thus the minimum possible value for a is $\frac{2}{3}$, making $5 - a = 4\frac{1}{3}$, the largest value x (or any of the other numbers) can take.

The corresponding solution of the original equations is

$(x, y, z) = (4\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$.