SHORT CIRCUIT

Canberra Mathematical Association Inc.

VOLUME 15 NUMBER 2 FEBRUARY 2024

NEWS AND COMMENT

Welcome back!

In this edition we include a contribution from Ed Staples about a historical method of solving quadratic equations that was devised by Karl Georg Christian von Staudt (1798-1867). Why and how von Staudt achieved his method are questions that impinge on current educational practice. They call up notions of culture and creativity, and what it means to be numerate.

Von Staudt was a student of Gauss and assisted him in his astronomical work. Perhaps a quick way of solving quadratics would have been useful in his calculations of the orbits of comets. But, whether this really was his motivation is not known. Perhaps he was simply irrepressibly inventive.

In education, the merits of understanding quadratic equations have long been debated, including in the parliament of the UK.

While the solutions to any particular quadratic equation may be of little consequence, the logical thought process behind a general solution certainly is consequential.

Paradoxically, geometrical methods like those of Carlyle or von Staudt are practical ‘black box’ methods needing procedural knowledge but little real understanding of the hidden workings.

Teachers tend to question the relative merits of procedural knowledge versus a fuller understanding. (Both are needed.)

These days, we would probably not choose to teach von Staudt’s method as a useful tool, and it has to be said that understanding why it works is not easy. Yet the story is part of our heritage. It tells us something about ourselves.

Von Staudt is remembered for his work in projective geometry, a geometry without number or measure. Likewise, in hindsight, his episode on quadratic equations may remind us that mathematics must be more than just number and procedure.

MEMBERSHIP

Memberships run from 1 Jan to 31 Dec. each year. Membership forms may be downloaded from the CMA website:
http://www.canberramaths.org.au

The several benefits of Membership of CMA may be found on the website.

NEWSLETTER

The CMA newsletter, Short Circuit, is distributed monthly to everyone on our mailing list, free of charge and regardless of membership status.

That you are receiving Short Circuit does not imply that you are a current CMA member but we do encourage you to join.

Short Circuit welcomes all readers.

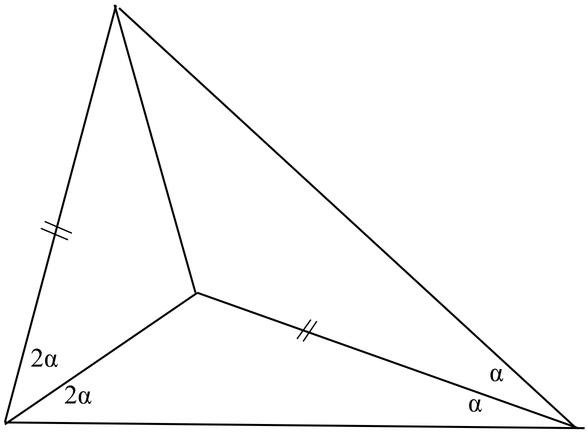
CANBERRA MATHEMATICAL ASSOCIATION

Inside:

PUZZLES

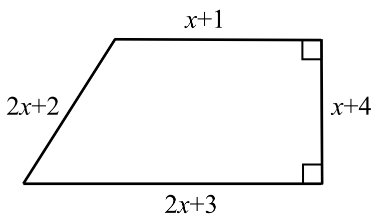
1. Trying triangle

Find the value of angle α.



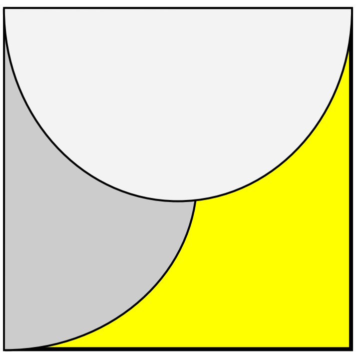
2 Enough information

You are asked to find the precise area of the following trapezium with the side measurements given in terms of an unspecified real number x.



3 Overlaps

Two semi-circular disks are drawn inside a square, as shown. What proportion of the square's area is coloured yellow?



4. Amazing number

In what order should we put the digits 1 to 9 so that by taking the first n of them, making an n-digit number D, we will always find D divisible by n as n ranges from 1 to 9?

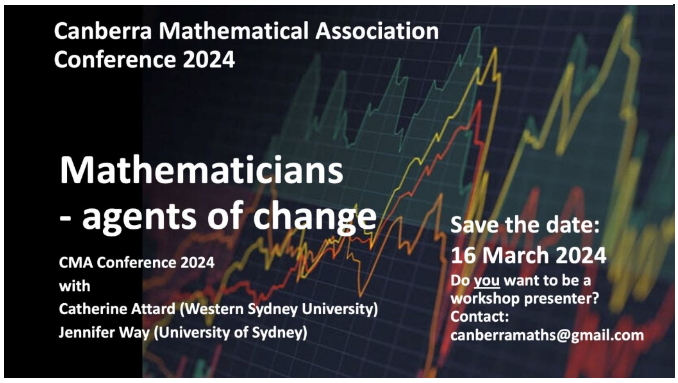
IM^{2}C 2024

Registrations for the International Mathematical Modeling Challenge 2024 are open.

The IM2C 2024 will occur from 13 February – 26 March 2024.

For more information, visit the **IM2C website**.

CMA 2024



TryBooking is now set up to take bookings for the CMA conference.

MAWA VIRTUAL

Join us for the 2024 Virtual Maths Conference on Friday, 3rd May.

ICME-15

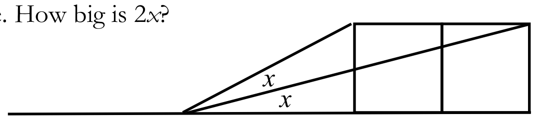
The International Congress on Mathematical Education is the largest international conference on mathematics education in the world.

The 15th International Congress on Mathematical Education (ICME-15) will take place 7-14 July 2024 at International Convention Centre in Sydney, Australia. ICME-15 promises to be an innovative congress that builds on the well-established ICME program, showcasing established and emerging thought leaders from around the world.

PUZZLE SOLUTIONS from Vol 15 No 1

1. Hard way, easy way

In the diagram below there are two adjacent identical squares and some oblique lines that form angles x and $2x$ with the base line. How big is $2x$?



This can be solved with a double angle formula and some algebra but notice that the upper large triangle is isosceles (because of the parallel lines). Then the steeply sloping line has the length of two squares. This makes $\sin 2x = 1/2$, and so, $2x = 30°$.

2. Ramanujan's Christmas tree

$$
\begin{aligned}
&3\\
&= 1+\sqrt{4}\\
&= 1+\sqrt{1+\sqrt{9}}\\
&= 1+\sqrt{1+\sqrt{1+2\sqrt{16}}}\\
&= 1+\sqrt{1+\sqrt{1+2\sqrt{1+3\sqrt{25}}}}\\
&= 1+\sqrt{1+\sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{1+\cdots}}}}}
\end{aligned}
$$

The pattern continues without end. The last term we can write has the form $n\sqrt{(n+2)^2}$. This is

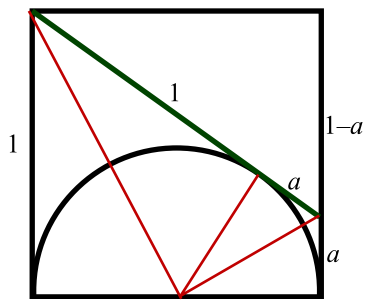
$n\sqrt{(1+(n+2)^2-1)}$

$= n\sqrt{(1+(n+2-1)(n+2+1))}$

$= n\sqrt{(1+(n+1)\sqrt{(n+3)^2})}$, which continues the pattern.

3. A bit less than $\sqrt{2}$

Find the length of the line segment tangent to the semicircle reaching from a corner of the square to a side.



By Pythagoras,

$$(1+a)^2 = (1-a)^2 + 1^2$$

So, $a = 1/4$

And $1 + a = 1.25$

ACT GOVERNMENT REVIEW OF LITERACY AND NUMERACY

The Committee of the CMA is preparing a submission to the ACT Government Review of Literacy and Numeracy, which is required by 14 February.

If you wish to suggest an idea for inclusion in the CMA submission, please let us know.

SIMPLE RULES COMPLEX LIFE

Recent pieces in Quanta magazine call to mind the mathematician John Horton Conway, who died in April 2020 of COVID-19. He is remembered for, among other things, his invention of the games Sprouts, and Game of Life.

Conway's Game of Life contributed to the study of what are called cellular automata, with particular applications in fun and computing. In Game of Life there is an infinite rectangular grid of cells with rules governing how the state of each cell evolves. At each stage, the number of live cells surrounding a given cell is counted. What happens to the cell is then determined by the following rules:

1. Birth: if the current cell is off and the count is exactly 3, the cell is switched on.
2. Survival: if the current cell is on and the count is 2 or 3, the cell is left unchanged.
3. Death: if the count is less than 2 (loneliness) or greater than 3 (overcrowding), the current cell is switched off.

Interest in the game centres on the patterns that emerge given particular initial conditions. Isolated live cells or groups of two blink off immediately, but arrangements of three or more can move across the grid or oscillate in repeating sequences of any length, or eventually die out, or go on growing and evolving for an unpredictable number of generations.

The link in this paragraph leads to a Numberphile YouTube video in which Conway talks about his discovery. Other resources exist on the same platform that provide further insights.

NEWSLETTER OF THE CANBERRA MATHEMATICAL ASSOCIATION INC. INC

PO Box 3572
Weston ACT 2611
Australia

E-mail: canberramaths@gmail.com

We're on the Web!
http://www.canberramaths.org.au/

ABOUT THE CMA

The Canberra Mathematical Association (Inc.) is the representative body of professional educators of mathematics in Canberra, Australia.

It was established by, among others, the late Professor Bernhard Neumann in 1963. It continues to run - as it began - purely on a volunteer basis.

Its aims include

* the promotion of mathematical education to government through lobbying,
* the development, application and dissemination of mathematical knowledge within Canberra through in-service opportunities, and
* facilitating effective cooperation and collaboration between mathematics teachers and their colleagues in Canberra.

THE 2024 CMA COMMITTEE

President	Bruce Ferrington	Radford College
Vice President	Aruna Williams	Erindale College
Secretary	Valerie Barker	
Treasurer	Jane Crawford	Brindabella Christian College
Membership Sec.	Paul Turner	
Councillors	Peter McIntyre	University of NSW Canberra
	Theresa Shellshear	Australian Catholic University
	Heather Wardrop	
	Andrew Wardrop	
	Yuka Saponaro	Melba Copland Secondary School
	Matthew Millikin	Marist College
	Jo McKenzie	ACT Education Directorate
	Jenny Missen	St Francis Xavier College
	Morgan Murray	St Thomas the Apostle Primary
	Bernadette Matthew	

Theresa Shellshear is CMA's COACTEA representative.

Bruce Ferrington is CMA's AAMT representative.

Joe Williams is the website manager.

Short Circuit is edited by Paul Turner. **ISSN 2207-5755**

Find us on Facebook

http://www.facebook.com/pages/Canberra-Mathematical-Association/110629419011275

CHRISTIAN VON STAUDT'S METHOD

From Ed Staples.

The renowned German mathematician Karl Georg Christian von Staudt (1798-1867) developed an ingenious geometric method for solving quadratic equations. We could well imagine the practical advantages of using geometric techniques in an age lacking electronic calculating devices.

The method looks easy, but under the hood there is cleverness. Here are the steps involved in finding the solutions to the quadratic equation $x^2 + bx + c = 0$.

Step **1**: Construct a unit circle centred on (0,1) and label the point (0,2) as *B*. (See the diagram below.) This could be used as a template for any quadratic equation.

Step **2**: Mark in the points $F\left(-\frac{c}{b},0\right)$ and $G\left(-\frac{4}{b},2\right)$ and connect them with a straight line. You can think of *F* and *G* as the unique fingerprints of the quadratic equation.

Step **3**: If the line $\overline{FG}$ intersects the circle in two places, call those intersection points *P* and *Q*. If the line is tangent to the circle, call the point of tangency *T*. If the line misses the circle, then there are no real solutions to find.

Step **4**: From *B* draw a line passing through either *P* and *Q* , or else *T*, and extend these lines to the *x*-axis. Remarkably, the *x*-intercepts reveal the solutions.

The hardest part of the procedure is working out *F* and *G*. The rest is simple line drawing. We could set up a template consisting of a unit circle and an *x*-axis on a piece of graph paper and begin solving lots of quadratic equations.

Let's look at an example.

Suppose we wish to solve the quadratic equation $x^2 + x - 6 = 0$. We first determine *F* and *G* as $F\left(-\frac{c}{b},0\right) = (6{,}0)$ and $G\left(-\frac{4}{b},2\right) = (-4{,}2)$ and mark them on our template's parallel lines. Then with a ruler, draw the line $\overline{FG}$ to cut the circle at *P* and *Q*. Finally, draw in $\overline{BP}$ and $\overline{BQ}$ and extend each to the lower line to reveal the two solutions *x* = 2 and *x* = –3.

But why does it work? And how did Christian von Staudt discover it? While more research is needed on that question, we can make the following observation.

If we use a little coordinate geometry, we find the general line $\overline{FG}$ has the equation given by $y = \frac{2b}{c-4}\left(x + \frac{c}{b}\right)$ and this line intersects the circle when:

$$[(c-4)^2 + 4b^2]x^2 + [4b(c+4)]x + 16c = 0$$

From the two solutions of this quadratic equation, we obtain the coordinates of *P* and *Q* and so, with *B*, *P* and *Q* we can determine line equations $\overline{BP}$ and $\overline{BQ}$ and hence find the solutions to the original quadratic equation.

In the example, with *b* = 1 and *c* = –6, this formidable quadratic equation reduces to $13x^2 - x - 12 = 0$ and, after factorising to $(13x + 12)(x - 1) = 0$ the coordinates of *P* and *Q* are easily worked out as (1,1) and $\left(-\frac{12}{13}, \frac{18}{13}\right)$.

Finally, with two points on each line, it's not difficult to show that the line $\overline{BP}$ has equation $y = -x + 2$ and the line $\overline{BQ}$ has equation $y = \frac{2}{3}x + 2$.

Setting *y* = 0 in both we find that the solutions to the original quadratic equation $x^2 + x - 6 = 0$ are *x* = 2 and *x* = –3.