Short Circuit

Newsletter of the Canberra Mathematical Association INC

VOLUME 14 NUMBER 7 JULY 2023

NEWS AND COMMENT

Feedback from teachers contributes to the decisions students make about their learning trajectories and eventual careers and life choices. It is an important and delicate matter.

Feedback from students, if taken seriously, can contribute to decisions made by teachers about their teaching strategies and to improvements in their effectiveness. Such feedback is often non-verbal and may have to be inferred indirectly from the subtle cues that students give.

Feedback from readers of Short Circuit about the publication tends to be sparse and indirect. We think there may be some readers out there, but it is hard to be sure. Comments about particular items or content generally would be useful and welcome. It happens, now and then, that contributors of articles wonder whether what they wrote made sense or provoked any kind of response. It would be good to have something to report in those cases.

Respectful feedback from teachers about industrial or pedagogical matters should certainly find a place in Short Circuit. It could conceivably contribute to decision making in places well beyond the average pay grade.

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re(Solve) is offering online professional learning courses again in Term 3.

The courses are free for Australian teachers, take place entirely online, and because they're accessible 24/7, can be completed at your own pace!

Enrolments for Term 3 courses will open on 10 July and close on 1 October. For details about courses keep an eye on the reSolve website

CMTQ 2023

Details are on the CMA website, and see page 3.

CANBERRA MATHEMATICAL ASSOCIATION

Coming Events:

CMA Annual Conference:

March 16 2024

MEMBERSHIP

Memberships run from 1 Jan to 31 Dec. each year. Membership forms may be downloaded from the CMA website:
http://www.canberramaths.org.au

The several benefits of Membership of CMA may be found on the website.

SHORT CIRCUIT

The CMA newsletter, Short Circuit, is distributed monthly to everyone on our mailing list, free of charge and regardless of membership status.

That you are receiving Short Circuit does not imply that you are a current CMA member.

CMA welcomes all readers.

Inside:

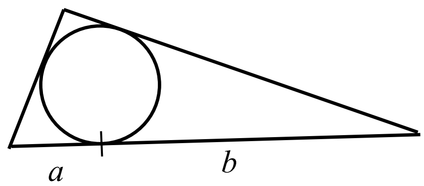
PUZZLES

1. Cryptic

One hundred lots of e on phi,
A week of hours passes by.
Well, not quite, a fraction shy
But only seconds to report.

How many whole seconds are we short?

2. Incircle



A teacher draws the incircle in a right-angled triangle. The hypotenuse is divided by the point of tangency with the incircle in the ratio *a*:*b*. The teacher explains to the class that this puzzle came to him in a dream, and that it will be a test of the students' abilities to reason geometrically, requiring some recently encountered facts. The teacher asks, 'If the letters *a* and *b* were replaced by particular numbers, would there be enough information in the arrangement to determine the area of the triangle?'

3. Who knew?

On a scrap of manuscript, it is written

…A, B, C are the angles of a triangle, …

$\tan A + \tan B + \tan C \equiv \tan A . \tan B . \tan C$

Having studied some mathematics, you have a fair idea of what the symbols mean, but you doubt that the statement could be true. Eventually, by experiment, you decide, to your surprise, that it is correct, at least in the finite number of cases you have checked.

However, there is a class of triangles for which it does not apply. What triangles are they? Can you prove the identity for the remaining cases?

4. Explain this

'To form certain Pythagorean triples, simply cleave the square of any odd number to form two integers of least difference. The odd number and those integers form a Pythagorean triple.'

For example, $7^2 = 49$ and so, (7, 24, 25) is a triple. Why does this work?

NO END TO THE PRIMES

From Ed Staples

Even though the density of primes amongst the natural numbers decreases the further out we go, they never quite disappear altogether. Over the centuries, many proofs of this result have been established, including a well-known one by Euclid often shown to students in senior classes. Less well known is a proof by the Dutch mathematician Thomas Joannes Stieltjes (1856-1894).

Just as in Euclid's proof, Stieltjes shows that any finite list of primes is necessarily incomplete. But before presenting his proof formally, it might be prudent to introduce it with an example.

Suppose (simplistically) we claim that the list of four primes 2, 3, 5 and 7 is a complete list of all primes that exist. That is to say, there are no other primes other than these four. Then the task in front of us is to show that such an assumption is false, not because we know there are other primes out there, but rather because this list itself implies the existence of primes other than the ones on it.

Using Stieltjes' argument, we first form the number $N = 2 \times 3 \times 5 \times 7 = 210$, and choose any factor pair m and n such that $N = mn$, with $m,n > 1$.

For example, we might choose $m = 2$ and $n = 3 \times 5 \times 7 = 105$ so that $2 \times 105 = 210$.

Alternatively, we might choose $m = 2 \times 3 = 6$ and $n = 5 \times 7 = 35$ so that $6 \times 35 = 210$, etc. (There are actually seven different ways to do it, but this is immaterial to the proof.)

Now form the number $S = m + n$. As an example, we might put $S = 6 + 35 = 41$.

Observe that 41 is not divisible by any prime on the list. The reason being that the primes that form 6 are not present in 35 and those in 35 are not present in 6. *

Hence, we have a contradiction and therefore the list we began with must be incomplete. Whichever S we choose, it certainly has one or more prime factors and they are not on the list. Note carefully that we are not claiming that 41 is prime, but rather that there are no primes in the list that divide it. The argument used by Stieltjes can be generalised. Take any list of primes $p_1, p_2, \ldots, p_n$ and form $N = p_1 \times p_2 \times \ldots \times p_n = mn$ and choose $m,n > 1$.

Then for $S = m + n$, no prime in m exists in n (and vice versa). So, S cannot be divided by a prime in the list. There must be at least one prime missing from the list since either S is prime and not in the list, or S is composite with prime factors that are not in the list.

*[A prime divisor of a sum of two numbers must divide both or neither of the numbers in the sum. *Ed.*]

THE CANBERRA MATHEMATICS TALENT QUEST 2023

The National Mathematics Talent Quest has provided a venue to showcase the creative thinking skills of students in Australia for many years. To be eligible to enter the national quest a project has to be successful in a similar quest at the state level. Students throughout the ACT put considerable time and effort into mathematics assignments and projects and now they have a means to get local or even national recognition and encouragement for their work.

Students may participate in one of three categories:

- Submit an **individual** entry
- Be part of a **small group** (up to 6 students)
- Be part of a **whole class** entry (7 **or** more students)

Entry is free.

All students from Kindergarten to Year 12 in the ACT are able to submit an entry.

The project or assignment can be the student's own idea or a teacher's set task with an outstanding student response.

The projects or assignments may be presented in any format including;

- Essays, scripts, stories, poems, diaries, illustrated texts, newspaper format or any other form of writing
- Posters
- Videos
- Models – static or working
- Computer based (coding)
- PowerPoint presentation
- Spreadsheet or database.

Entries that win their category are automatically entered in the national competition. Entries in the National Mathematics Talent Quest **must be submitted electronically**. Schools can submit up to two entries per category (individual, group or class) per year group to be assessed by an ACT judging panel.

Follow these links to see some examples of student work from Victoria, NSW and WA:

You can start anytime but the entry date is Monday 7th August 2023.

Updates will be provided on the CMA webpage:

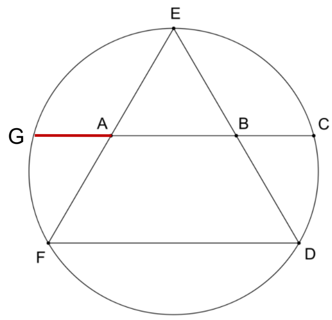
PUZZLE SOLUTIONS from Vol 14 No 6

1. **Narayana's Cows**

See page 6.

2. **Odom's construction**

Let A and B be midpoints of the sides EF and ED of an equilateral triangle DEF. Extend AB to meet the circumcircle (of DEF) at C. Show that B divides AC according to the golden section.



Extend BA to meet the circle at G (as shown). Now, GB.BC = EB.BD by the 'power of a point' theorem. (This is a theorem about lines through a point and a circle. It can be proved using similar triangles.)

We have, GB = GA + AB = AC.

So, AC.BC = EB.BD. But EB = BD = AB.

Therefore, AC.BC = AB^2 .

That is, AC/AB = AB/BC.

If we set BC = 1 in this ratio, we have (1 + AB)/AB = AB or $AB^2 - AB - 1 = 0$.

Hence, AB = $(1 + \sqrt{5})/2 = \varphi$.

So, AC/AB = AB/BC = φ.

ABOUT THE CMA

NEWSLETTER OF THE CANBERRA MATHEMATICAL ASSOCIATION INC

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We're on the Web!
http://www.canberramaths.org.au/

The Canberra Mathematical Association (Inc.) is the representative body of professional educators of mathematics in Canberra, Australia.

It was established by, among others, the late Professor Bernhard Neumann in 1963. It continues to run - as it began - purely on a volunteer basis.

60 years ago

Its aims include

* the promotion of mathematical education to government through lobbying,
* the development, application and dissemination of mathematical knowledge within Canberra through in-service opportunities, and
* facilitating effective cooperation and collaboration between mathematics teachers and their colleagues in Canberra.

THE 2023 CMA COMMITTEE

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Theresa Shellshear is CMA's COACTEA representative.

Sue Wilson is CMA's AAMT representative.

Joe Wilson is the website manager.

Short Circuit is edited by Paul Turner. **ISSN 2207-5755**

Find us on Facebook

http://www.facebook.com/pages/Canberra-Mathematical-Association/110629419011275

CAREERS AND MATHEMATICS

From Frances Moore

Mathematics is in every job - we all know that but do our students? We will explore a different job and the mathematical activities involving this job from the website "On the Job".

Let's have a look at the Food Process Worker. Detailed information about the Food Process Worker can be found here Food Process Worker - Career Advice - Manufacturing Production - On The Job

Context and relevance: At the moment, ordinary Australians are doing it tough. The first activity will help students understand how their families can get value for money with their food while the second activity shows how big businesses can help too.

On the Job Activities for the Classroom: These activities are to interest students and encourage them to look, in our case, at the mathematics involved in each particular job.

Activity 1: Tastes like Chicken: How much chicken do you consume each week?

After watching BTN, students are to create a timeline for the processing of chickens. They are to then investigate at the local supermarket the cost of different chicken products, create an Excel spreadsheet and work out best value for money. The students are to do some research about the amount of chicken consumed by Australian families and survey the class about this consumption.

Activity 2: Thomas Foods International – their business explored!

After an introduction to Thomas Foods, students are to create a Lucid Flow Chart showing the process of the production of meat from Paddock to Plate. This activity can be challenging as the video with the information of the process of production starting at the Plate! Thomas Foods donate 4,300 kilos of sausage meat each month to Foodbank. Students are to work out the donation and respond via a letter to this donation.

Activity 3 : Building a Conveyor Belt (from Try Engineering)

Students work in teams to design and build a conveyor belt system out of everyday materials to transport pieces of candy/sweets 120cm. 2 – 3 lessons of 45 mins are required. Students are asked to reflect on their achievements and interactions with each other – good soft skills activity involving creative thinking skills within the team.

Careers & Mathematics can be found at

https://onthejob.education/teachers_parents/Mathematics_Teachers/Careers_Mathematics_Index.htm

Contact Information

If you are investigating a job or person in that job, please contact me Frances Moore – I would be happy to hear from you.

Frances.Moore@onthejob.education

Mob 0410 540 608

NARAYANA'S COWS

Each term in the sequence is obtained by adding the number of cows in the previous year to the number of cows three years ago. For the first three years, there is only one cow. Then, from year 4 on, we can use the recursive formula $N_n = N_{n-1} + N_{n-3}$, to obtain the sequence 1, 1, 1, 2, 3, 4, 6, 9, 13, 19, 28, 41, 60, … .

This can be continued as far as we like, by brute force or more easily with the help of a spreadsheet or other software. In this way we find that $N_{10} = 19$, $N_{17} = 277$, and $N_{30} = 39865$.

This answers the puzzle question, but there is more. We might ask what kind of a sequence this is, and whether there could be an explicit formula for its terms.

It is clearly not an arithmetic sequence as the gaps between terms are increasing. Nor is it a geometric sequence, as the ratios between successive terms are not constant. However, as the sequence proceeds, it does seem to come closer to a geometric sequence with each new term somewhat less than 1.5 times the one before.

We might postulate the existence of a number k for which it is approximately true that $N_n = kN_{n-1}$ when n is large enough. That is, the ratios between pairs of successive terms seem eventually to approach a value k.

Far out along the sequence, we would have $N_{n+1} = kN_n$, then $N_{n+2} = k^2N_n$, then $N_{n+3} = k^3N_n$, and so on.

But it is also true that $N_{n+3} = N_{n+2} + N_n$. So, after substituting for N_{n+2} and N_{n+3} in this formula, we have $k^3N_n = k^2N_n + N_n$, or just $k^3 = k^2 + 1$. This is the so-called *characteristic equation*. If the number k exists, it must satisfy this condition.

Notice the nice correspondence between the characteristic equation and the recursive formula

$N_{n+3} = N_{n+2} + N_n$

$k^3 = k^2 + 1$ (We recognise 1 as k^0)

(A similar thing happens with the Fibonacci sequence where the recursive formula is $F_{n+2} = F_{n+1} + F_n$, (with $F_1 = F_2 = 1$), and the characteristic equation is $\varphi^2 = \varphi + 1$.)

The characteristic equation in the case of the Narayana numbers is a cubic. One way to solve it (approximately) for the real root is by rearranging the equation and iterating $\sqrt[3]{(k^2 + 1)} \rightarrow k$, using a calculator.

To 9 decimal places we find $k \approx 1.465571232…$ By experiment, we see that $N_{n+1} = kN_n$ is indeed approximately true for this value of k. Sometimes the estimate is over and sometimes it is under, but it is always close. If we start with $N_4 = 2$, and multiply by successive powers of k, we get an underestimate for N_5 but overestimates thereafter. For example, $2k^{16} \approx 906$ but $N_{20} = 872$. It turns out that this and subsequent estimates are too high by a factor of about 906/872 = 1.039.

Thus, N_{30} should be about $2k^{26} / 1.039 \approx 39865$, which we know to be correct. However, subsequent estimates by this method are increasingly imprecise. It remains a challenge to produce a reliable formula.

In the case of the Fibonacci sequence, an explicit formula has long been known, under the name Binet's formula. It is a linear combination of the nth powers of the two roots of the Fibonacci characteristic equation.

$$F_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$$

In the Narayana's Cows sequence, the characteristic equation has three roots because the equation is a cubic. It is possible to follow a similar procedure as is done in the Fibonacci case, looking for a suitable linear combination of powers of the three roots.

However, in the Narayana case, two of the roots and two of the coefficients are complex numbers. The explicit formula certainly exists, but it is cumbersome to use and imprecise because of the necessary decimal approximations.

In comparison, the recursive formula is exact and much easier to implement.

Scan the QR code to load the website

TAX, SUPER +YOU

COMPETITION

29 MAY — 25 AUGUST 2023

Calling all high school students!

Enter our competition for a chance to win a share of $13,200 in prizes and learn more about the value of tax and super.

Want to enter?

Unleash your creativity and tell us about tax and super in a fun and creative way. You can start working on your entry right away, either individually or as part of a team.

Entry topics for 2023

Junior (Year 7–9): How do tax and super help me and my community?

Senior (Year 10–12): What are the most important things you need to know about tax and super to be work ready?

Write it! Make it! Film it!

There are heaps of fun ways to enter, including:
short stories • poems • scripts • case studies
artworks • comic strips • songs • videos • animations
skits • music clips • prototypes of apps or games

Australian Government
Australian Taxation Office

taxsuperandyou.gov.au/competition
Terms and conditions apply.