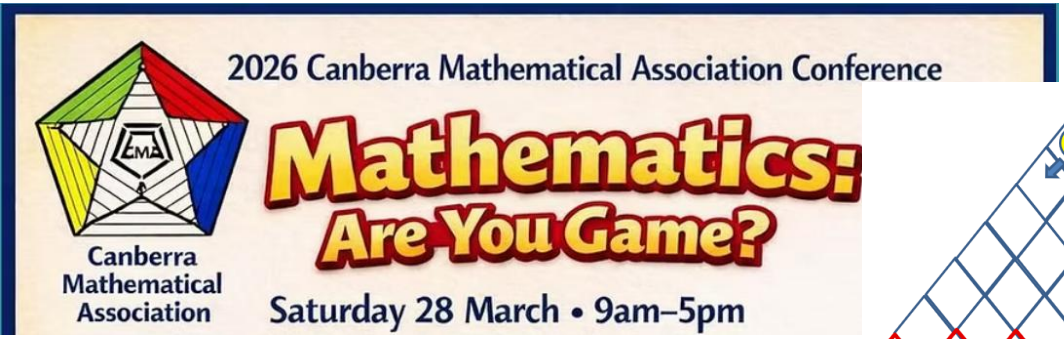
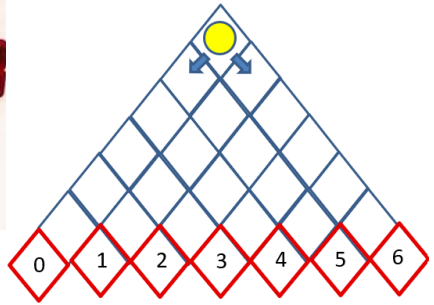
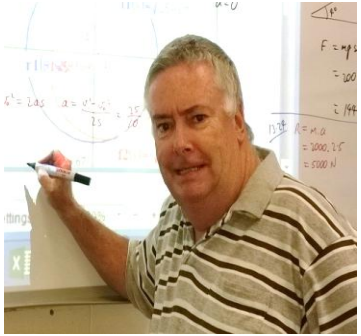




28 March 2026

From a simple game to powerful patterns

Brian Lannen



Brian Lannen

Murray Mathematics Curriculum Services

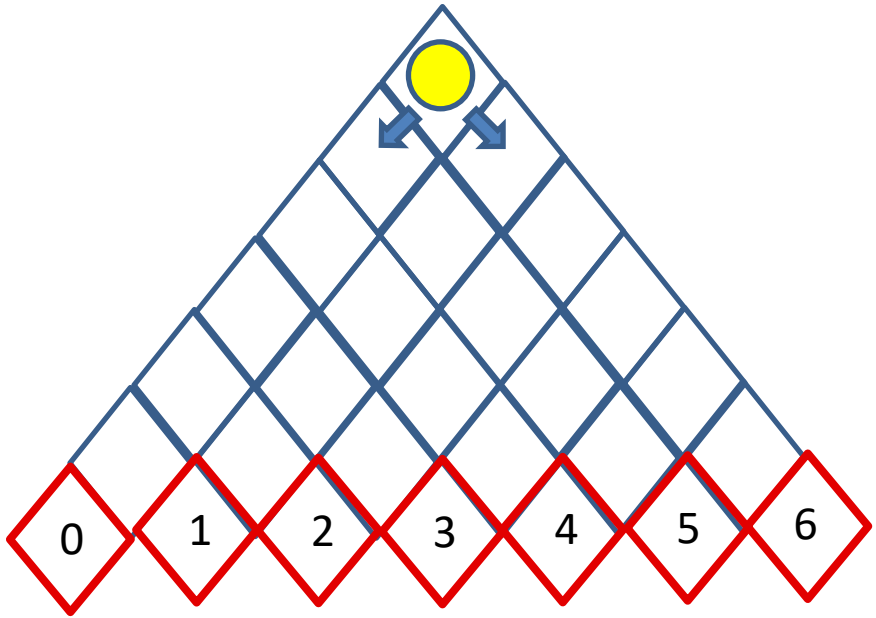
T³ Principal Instructor

From a simple game to powerful patterns

We will start with a simple game, generate some data and see how our empirical results compare to theoretical prediction. Our simple game triangle will reveal a very special triangle full of magical patterns and sequences.

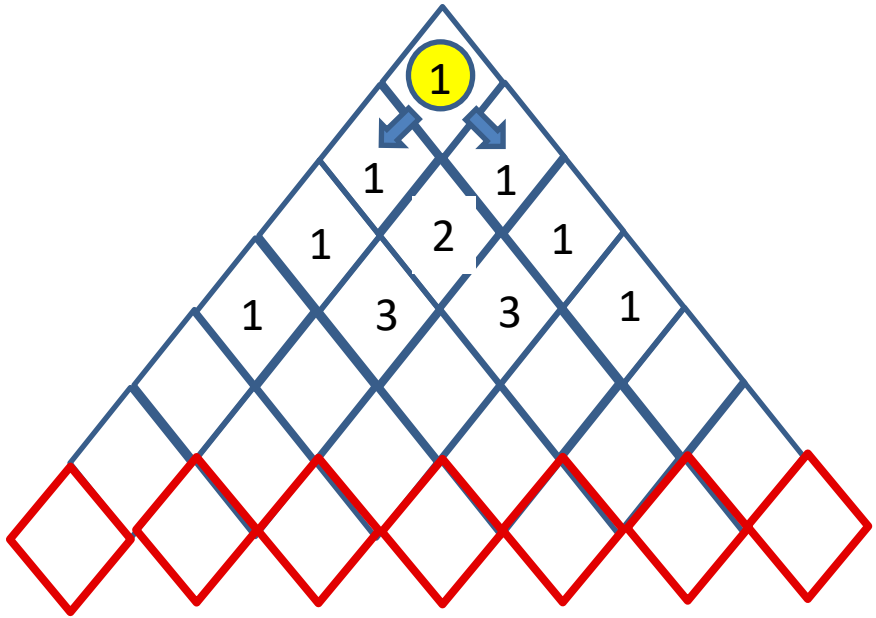
- Triangular numbers
- Tetrahedral numbers
- Fibonacci Sequence
- STEM considerations

Boardgame



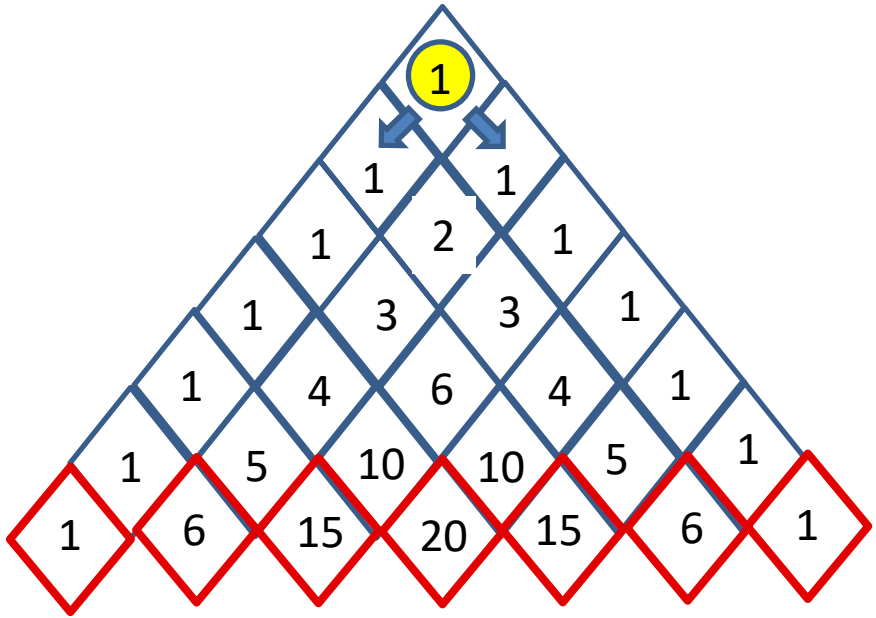
- The game is played by one person.
- Start by placing a moveable counter in the top space.
- Toss a coin and observe Heads or Tails.
- If Heads, move the counter down the board to the left.
- If Tails, move the counter down the board to the right.
- After 6 coin tosses the counter will be in one of the red spaces at base of the gameboard.
- Take note of the space number 0-6.

How many possible pathways are there to each place on the board?



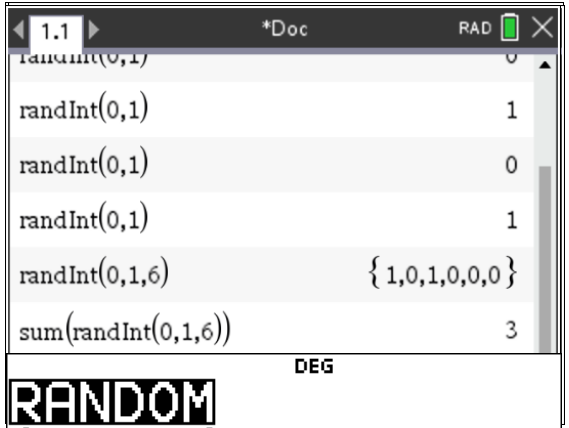
- Take note of the symmetry.
- You complete the next row.

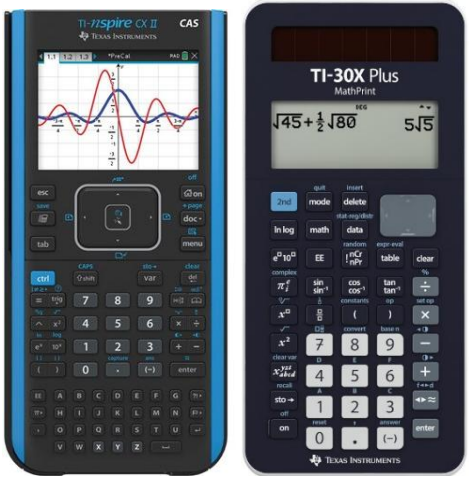
How many possible pathways are there to each place on the board?

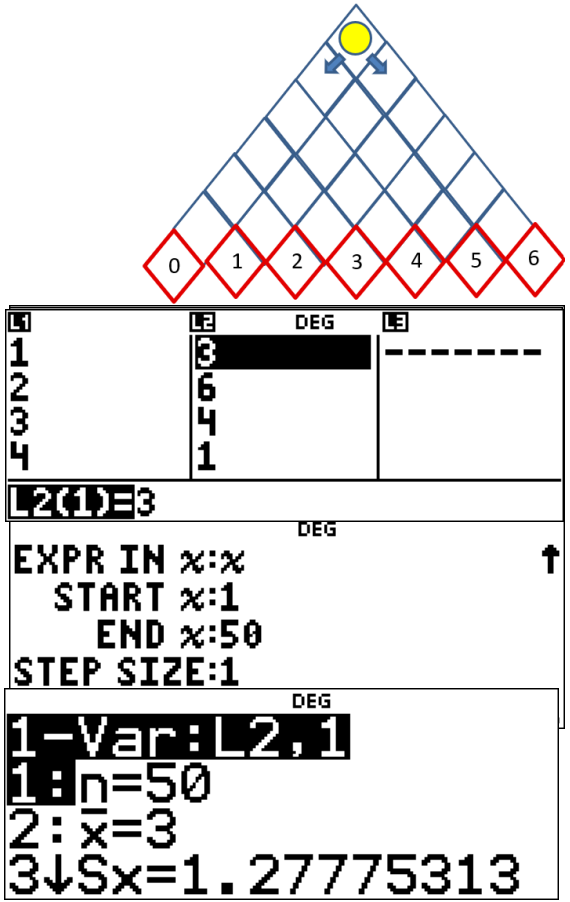


- Take note of the symmetry.
- You complete the next row.
- Take note how each new value is generated from preceding values in the row above.
- You complete the rest.
- Yes, this is the famous **Pascal's Triangle.**
- It contains many magical number sequences within it.
- - including the **Fibonnaci Sequence**.

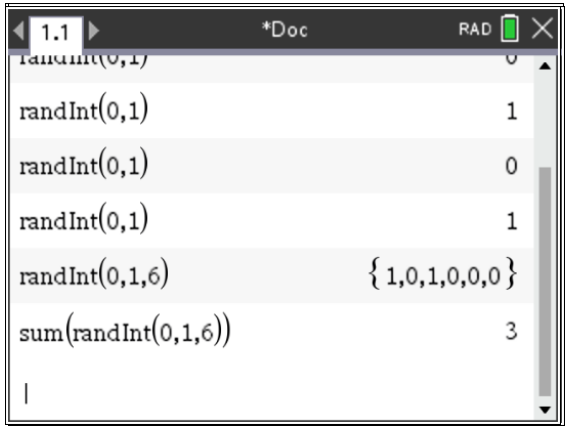
Use of technology to simulate the data

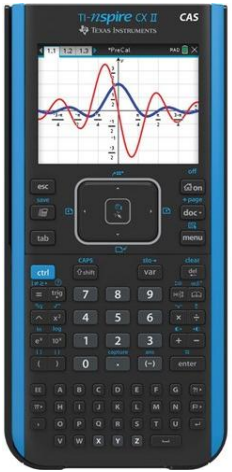


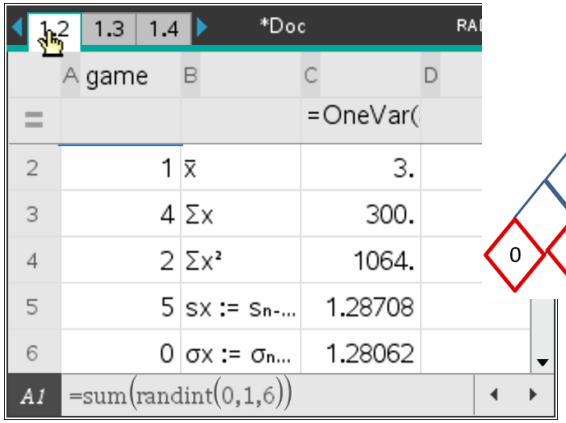


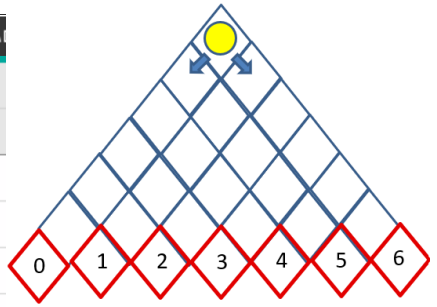


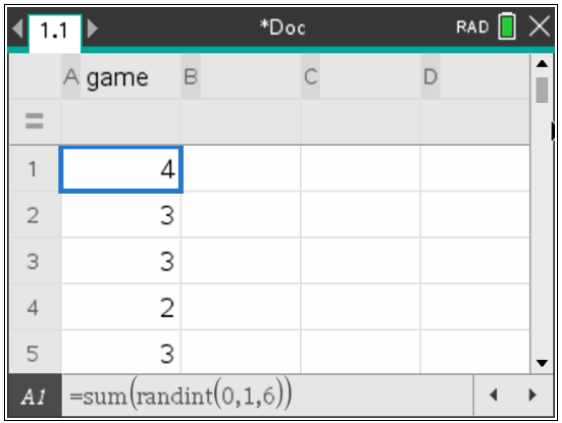
Use of technology to simulate the data

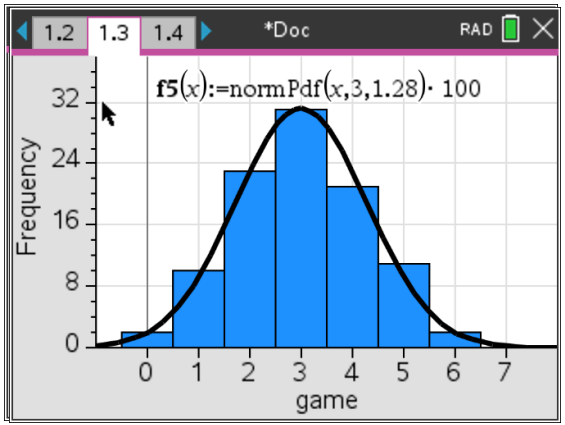




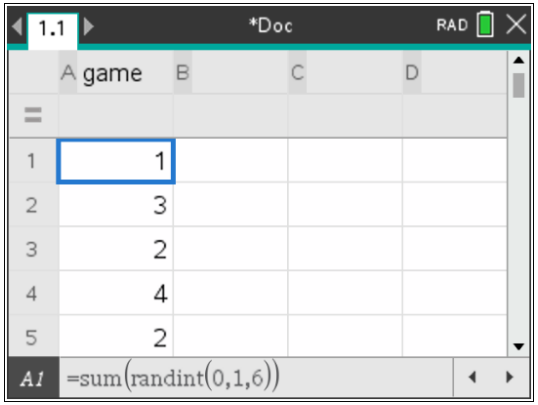


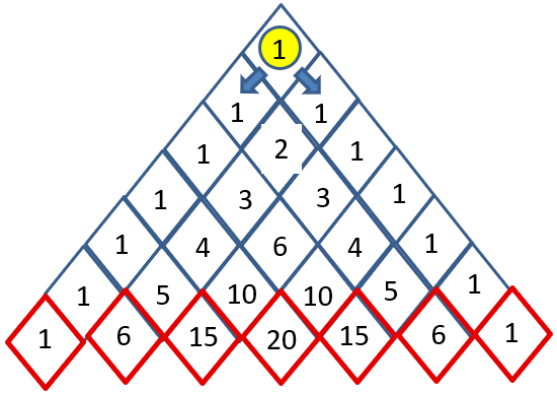






Empirical results V theoretical prediction





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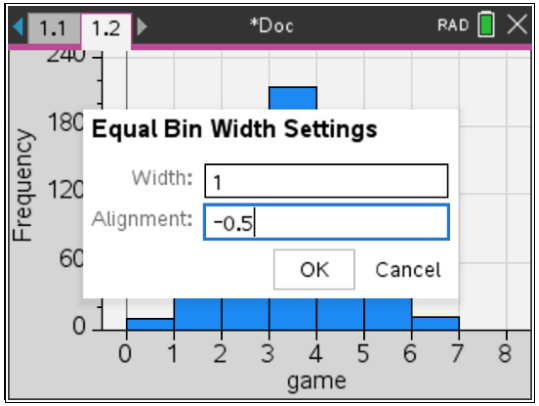
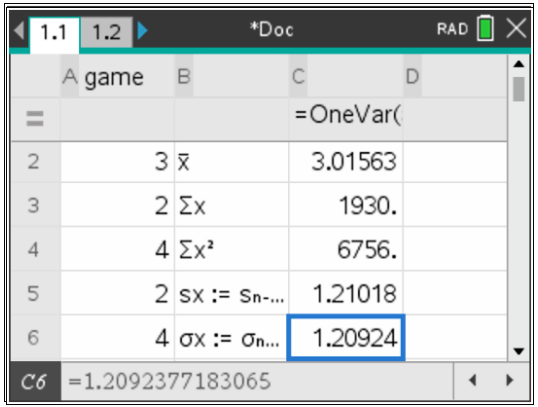
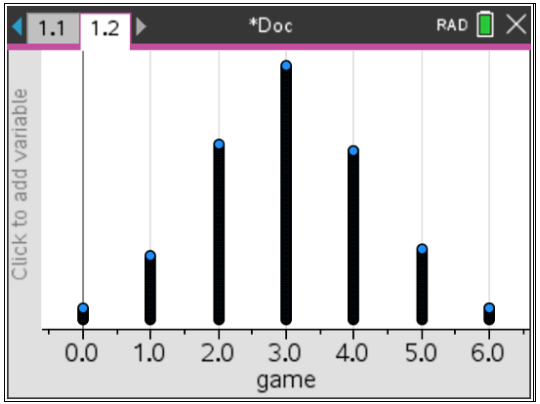
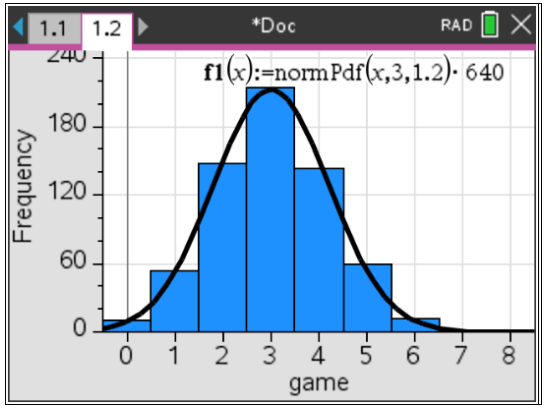
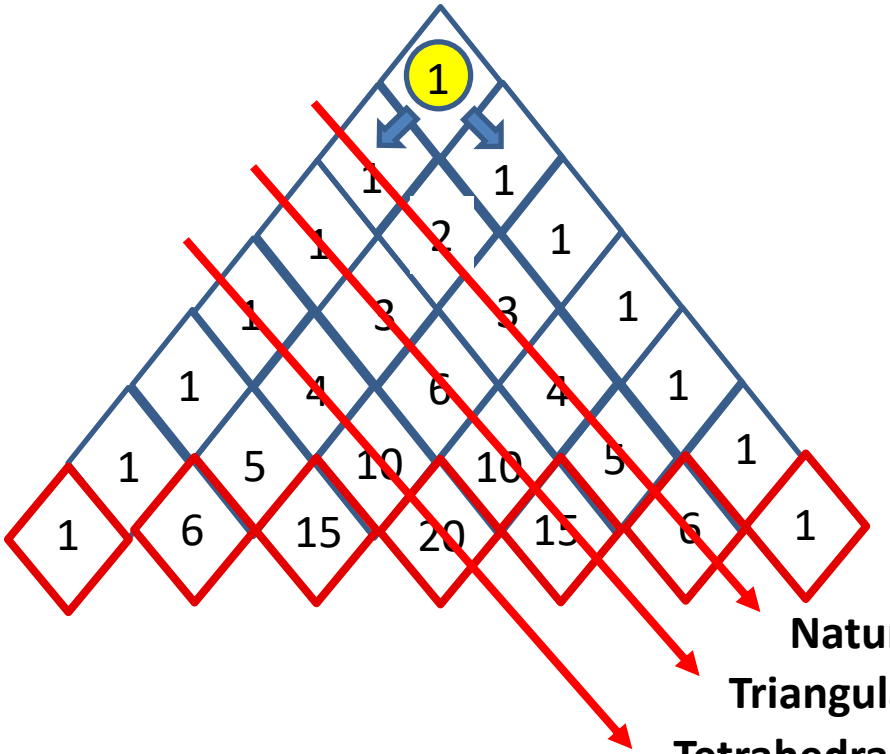


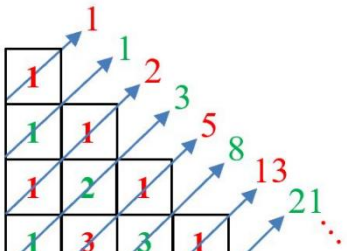






Patterns in the Pascal





Fibinacci Sequence

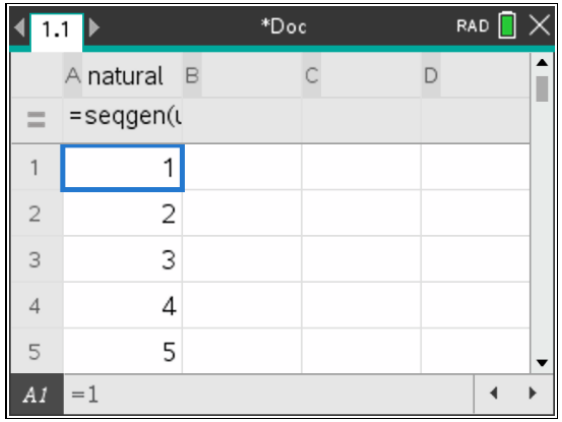
1	1										
1	2	1									
1	3	3	1								
1	4	6	4	1							
1	5	10	10	5	1						
1	6	15	20	15	6	1					
1	7	21	35	35	21	7	1				
1	8	28	56	70	56	28	8	1			
1	9	36	84	126	126	84	36	9	1		
1	10	45	120	210	252	210	120	45	10	1	
1	11	55	165	330	462	462	330	165	55	11	1

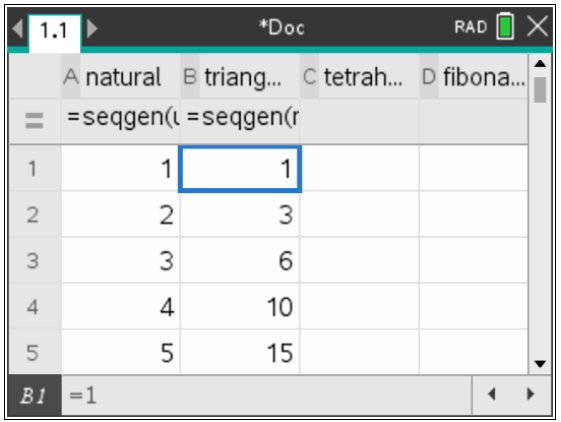
Natural Numbers

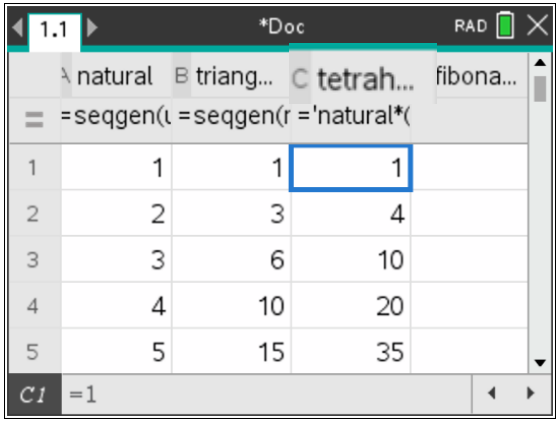
Triangular Numbers

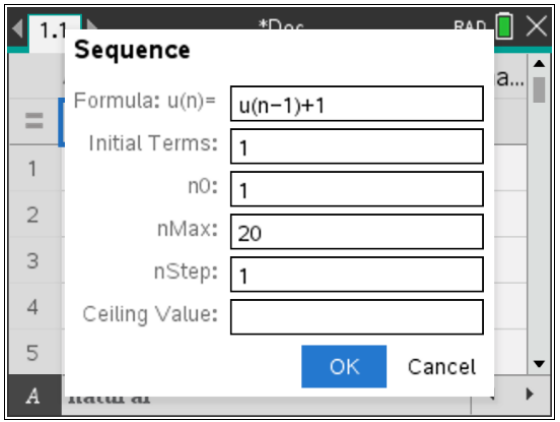
Tetrahedral Numbers

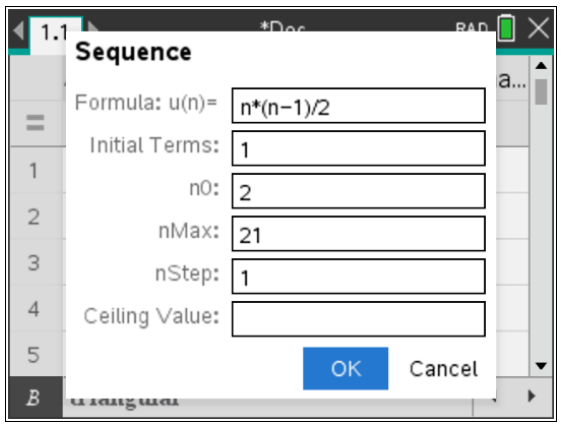
Use of technology to generate these sequences

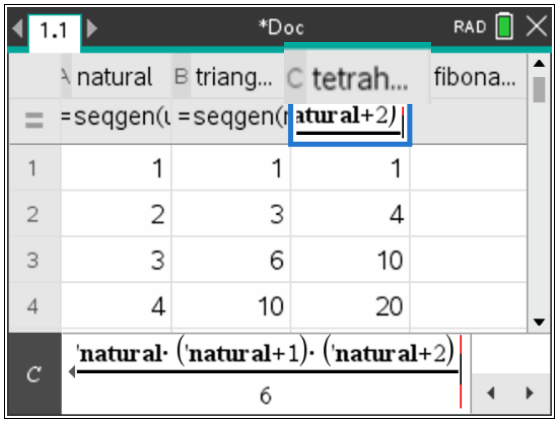




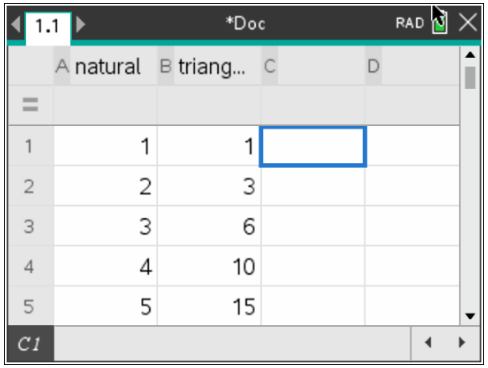


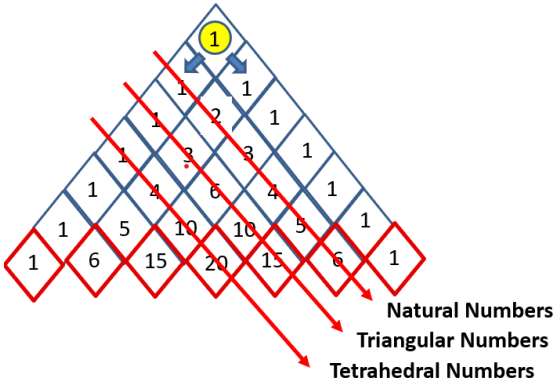


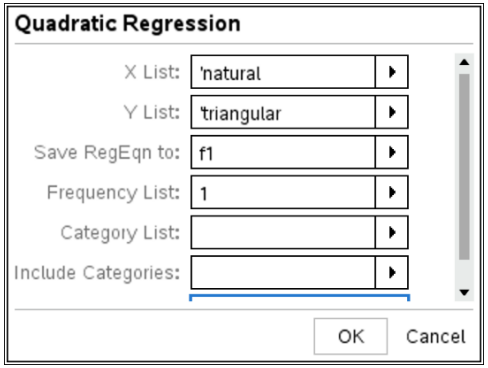


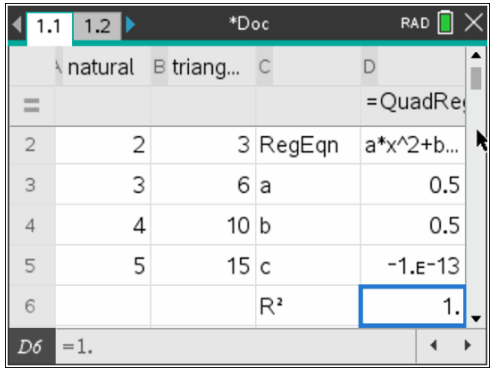


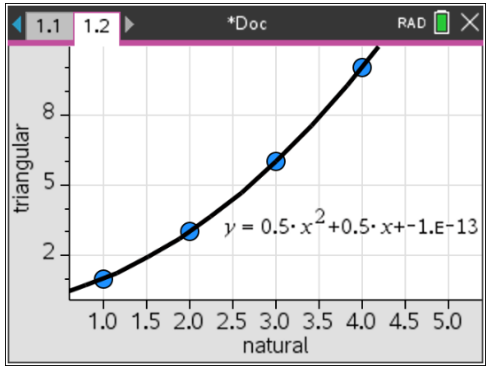
Use of technology to generate these sequences



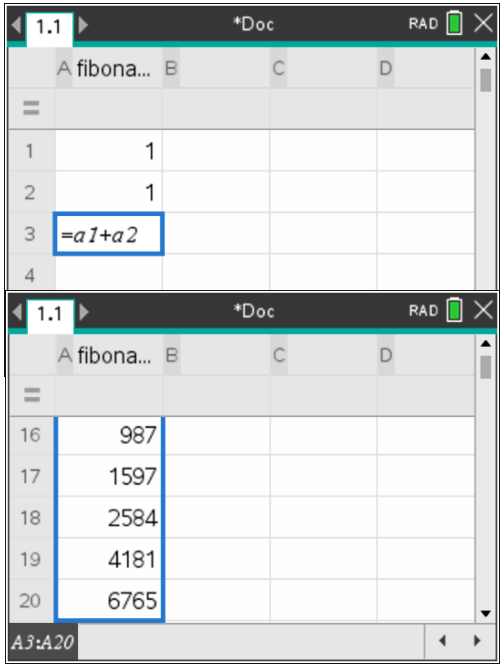


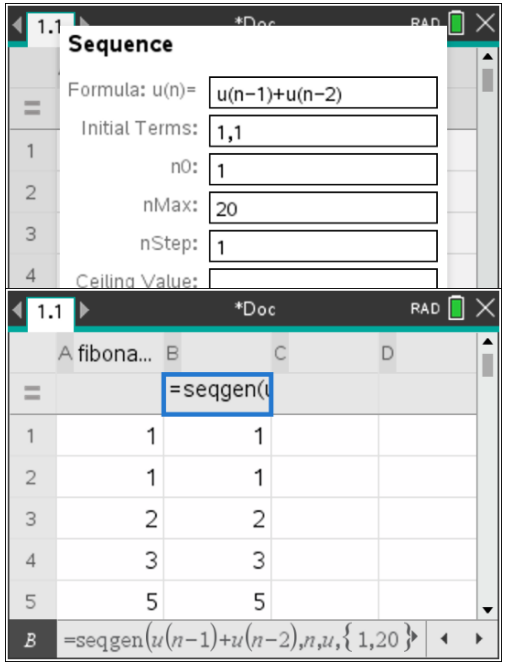


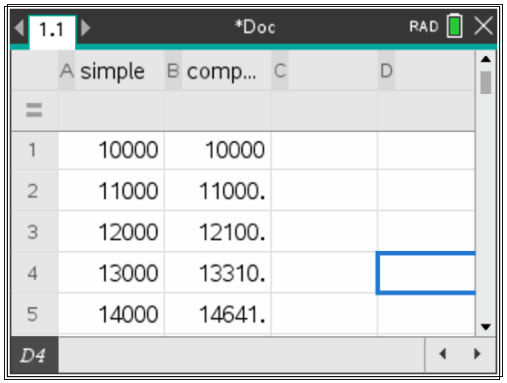




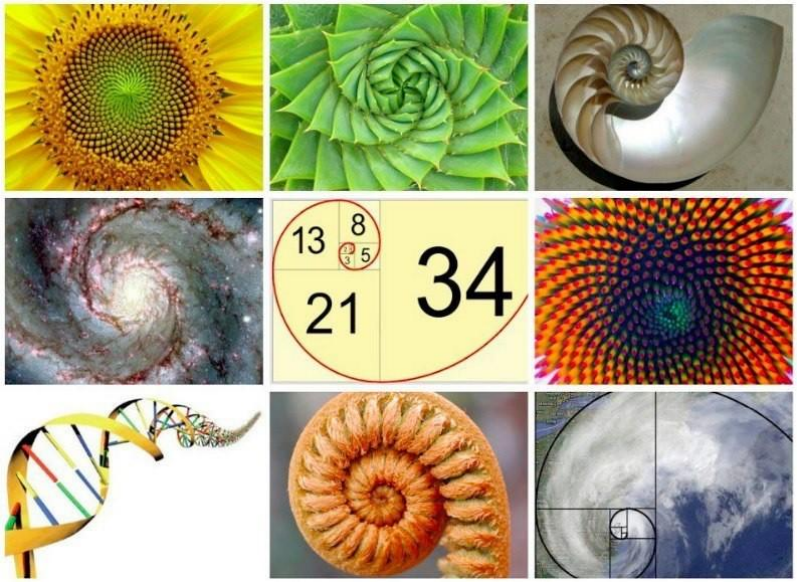
What is a “difference equation” and why is the Fibonacci sequence considered a “second order difference equation”?

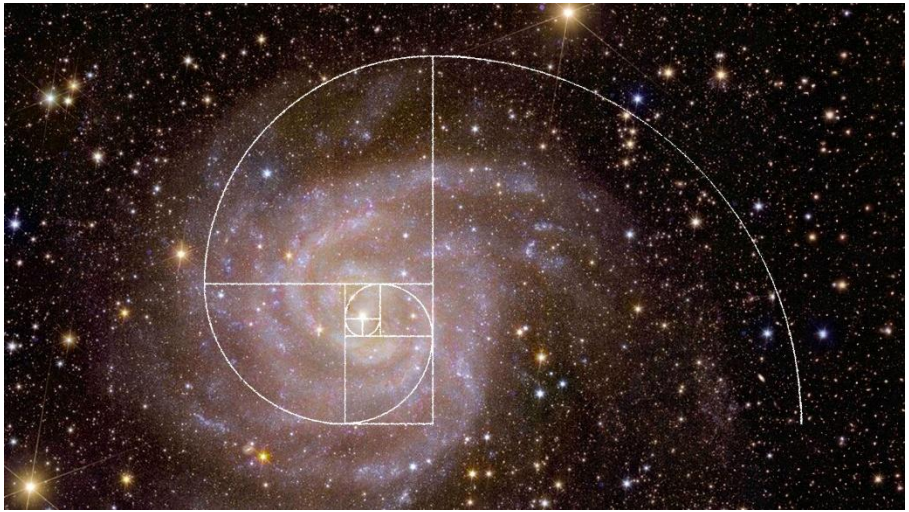




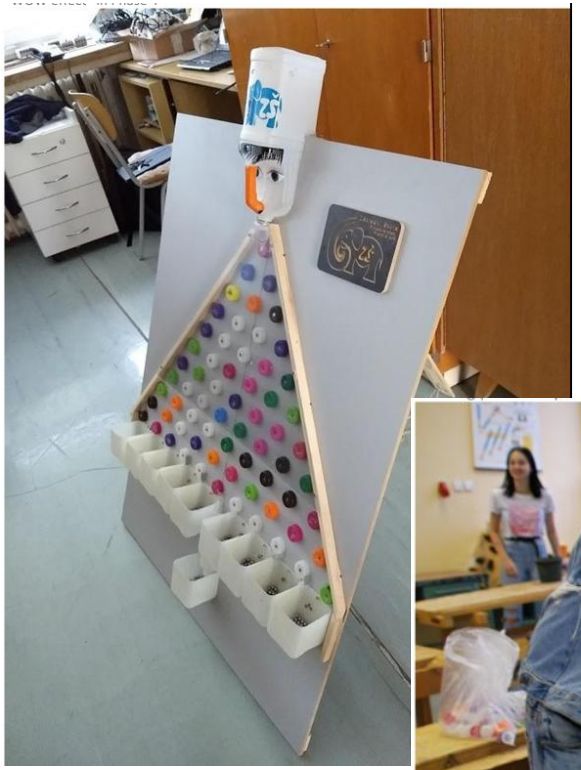


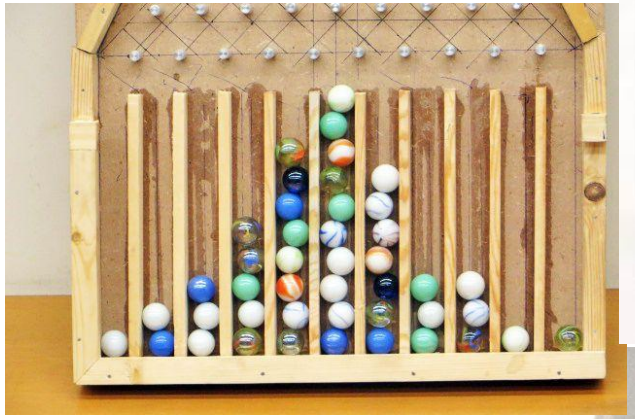
STEM connections

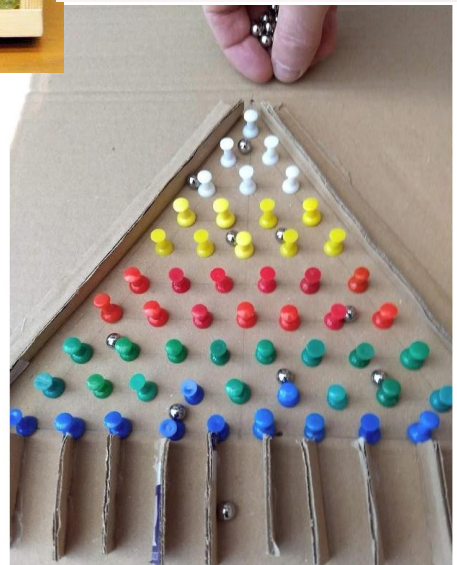




STEM connections







Just a few more interesting links:

https://www.mathsisfun.com/data/quincunx.html

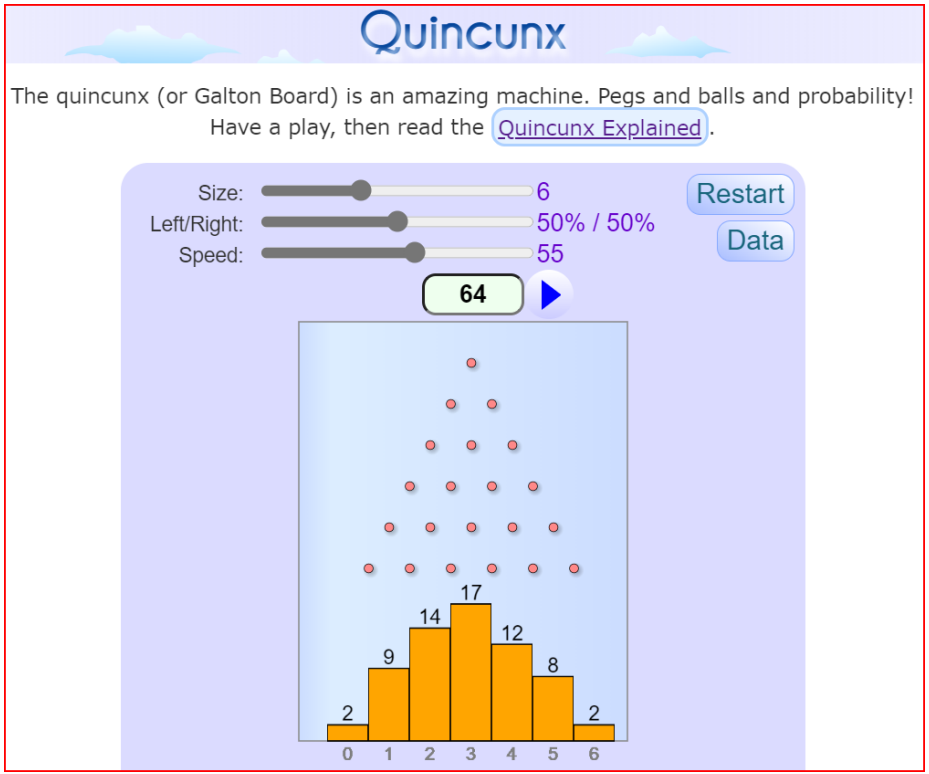
https://www.mathsisfun.com/data/quincunx-explained.html

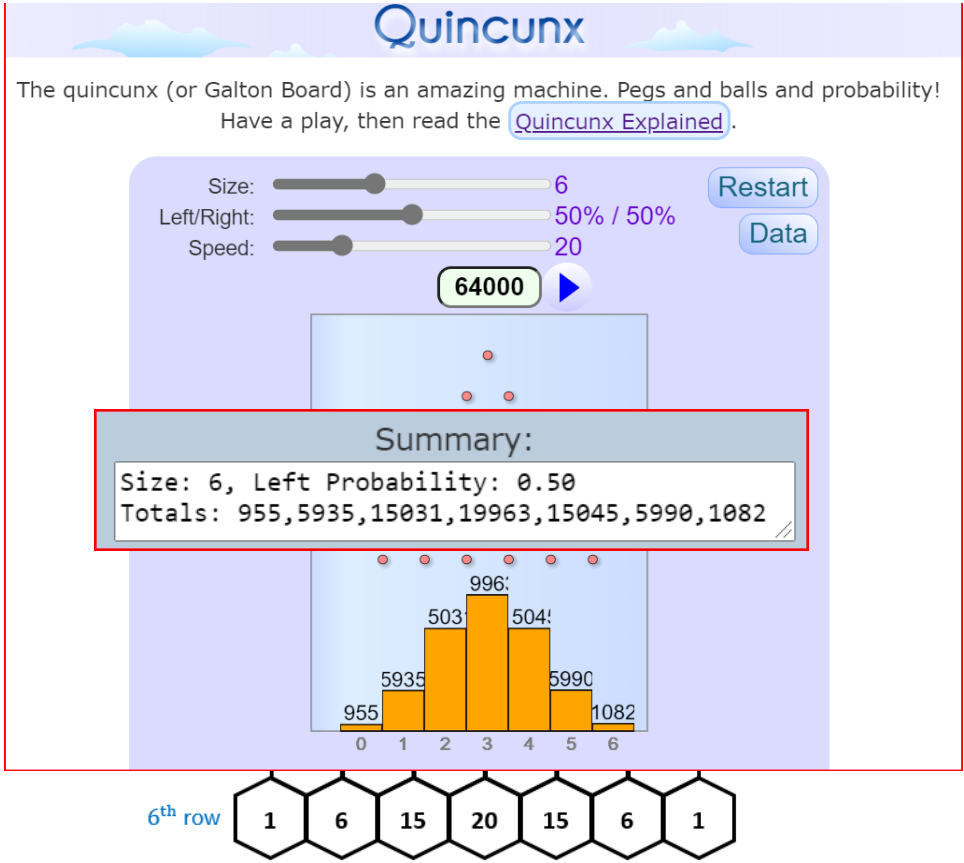
https://galtonboard.com/

Classroom Activities: Pascals Triangle Hidden Gems - Texas Instruments - content

Results from a simulator:

[Quincunx (Galton Board) (mathsisfun.com)]





Resources for more activities

Triangular Numbers

Teachers Teaching with Technology

Teacher Notes & Answers

7 8 9 10 11 **12**

TI-30XPlus MathPrint™ | Activity | Student | 30 min

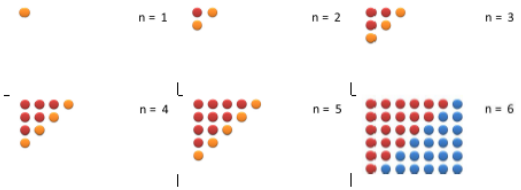
Introduction to Induction

What is the sum of the first *n* whole numbers?

There are several ways this problem can be solved. Given any value for *n* you could add the numbers up, one by one, but if *n* is large this could take a lot of time. A formula would be a much quicker way to determine such a sum. In this activity you will work with a range of visual and numerical methods to arrive at a formula. However, the formula is based on observation, intuition and a relatively small sample of numbers. There are many cases where formulas seemed to work, but are later found to be flawed. In the final stage of this activity you will prove that your formula works for all whole number values for *n*.

Visual Observation

The series of diagrams below shows one way to visual sums of the first *n* whole numbers. In each case the new row (orange) shows the quantity being added. The diagrams show why the pattern is referred to as 'triangular' numbers. The last representation includes a duplication of the pattern.



Question: 1.

The following questions refer to the last pattern (n = 6).

a) How many dots in the last pattern?

b) Explain how you determined this quantity.

c) What is the sum of the first 6 whole numbers?

Question: 2.

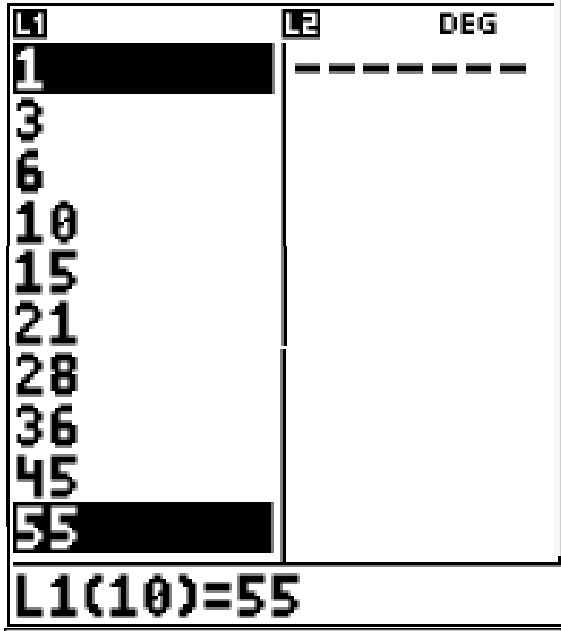
Determine the sum of the first 7 whole numbers without using 'addition'.

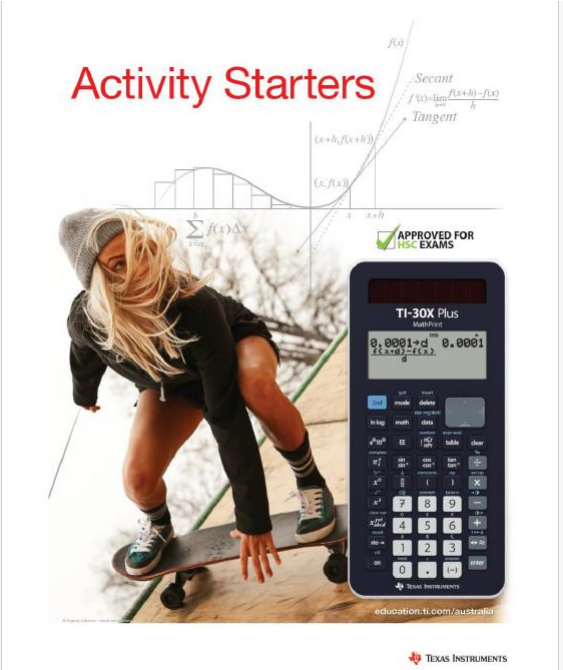
Question: 3.

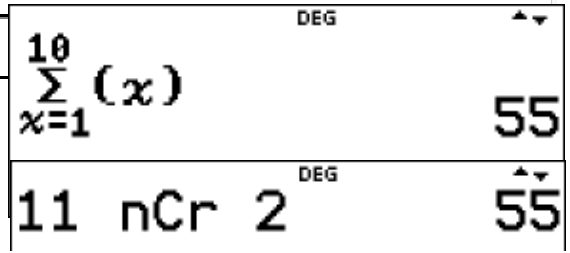
Generalise your answer to Question 2 for the sum of the first *n* whole numbers.

Author: P. Fox

Texas Instruments







Texas Instruments

Tetrahedral Numbers

Teachers Teaching with Technology™
Professional Development from Texas Instruments

Student Worksheet

7 8 9 10 11 **12**

TI-30XPlus MathPrint™ | Activity | Student | 50 min

Finding Patterns

What are the Tetrahedral numbers? The prefix 'tetra' refers to the quantity four, so it is not surprising that a tetrahedron consists of four faces, each face is a triangle. This triangular formation can sometimes be found in stacks of objects. The series of diagrams below shows the progression from one layer to the next for a stack of spheres.

Row Number	1	2	3	4	5
Items Added					
Complete Stack					

Question: 1.

Create a table of values for the row number and the corresponding quantity of items that are added to the stack.

Answer:

Row	1	2	3	4	5
Items:	1	3	6	10	15

Question: 2.

Create a table of values for the row number and the corresponding quantity of items in a complete stack.

Answer:

Row	1	2	3	4	5
Items:	1	4	10	20	35