SHORT CIRCUIT

Newsletter of the Canberra Mathematical Association INC

VOLUME 12 NUMBER 2 FEBRUARY 2021

NEWS AND COMMENT

The Welcome Event for 2021 has become a Welcome Workshop Day, on Saturday morning 20th February, hosted by Anna Williams and the Mother Teresa Primary School. Details are on page 5.

CMA will also have a presence, and has helped to organise speakers, at the annual All Colleges Day on 28th January, which is a professional development day for government sector colleges and some high schools.

On page 3 you will find information about the 2021 International Mathematical Modelling Challenge. This activity is suitable for middle to upper high school students. Teams from the ACT have done well in past years, some reaching the International level.

Continuing the mathematical modelling theme, we feature an article contributed by Peter McIntyre about Population Modelling. The article, beginning on page 3, will spread over several issues. It is clear from this month's introductory section that the ideas are very relevant in this time of plague, not to forget the wider environmental issues.

As usual, Short Circuit invites contributions from its readers. Your ideas and experiences are valuable and are worth sharing!

miniMaths Book 3 – Maths In Nature Inquiries

Thanks to another ACT Nature Play grant (2020), we have been able to produce the third book in the miniMaths: Maths In Nature Inquiries series for early years teachers and learners. This book presents another ten tasks and activities that can be used outside the classroom to encourage young minds to appreciate many deeper maths concepts. From making circles to hugging trees, each activity has a deliberate mathematical purpose and is linked to the Early Years Learning Framework. Workshops to promote this book will be conducted in the second half of Term 2.

Memberships, Individual and Institutional are due for renewal. Download and fill in the form from the website and email it to the membership secretary. (The 2021 forms should be up on the website soon, along with other website updates but, if not, the 2020 form will do.)

CANBERRA MATHEMATICAL ASSOCIATION

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Coming Events:

Saturday 20th February: Welcome Workshop Day—Creating a Mathsmosphere… Michelle Tregoning at Mother Teresa Primary, Harrison, 8:30 a.m.—12. (3 hrs TQI)

Wednesday Workshop:

MEMBERSHIP

Memberships run from 1 Jan to 31 Dec. each year. Membership forms can be accessed from the CMA website: http://www.canberramaths.org.au

Membership of CMA includes affiliation with the Australian Association of Mathematics Teachers and a subscription to one of two AAMT journals.

As a member, you are entitled to attractive rates for the CMA annual conference and CMA professional development events.

CMA members may attend conferences of the AAMT affiliates in other states, MAV, MANSW, etc. at member rates.

PUZZLES

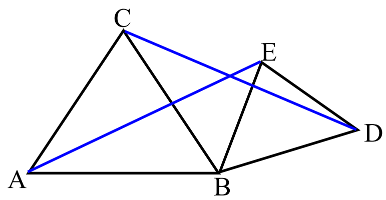
1 A coin game for two

Fifty coins are arranged in a line. The denominations of the coins are selected randomly from the local currency. Players take turns to take a coin from either end of the line. The winner is the player who has, at the end, taken the most money. Is there a strategy for either the first or the second player never to lose the game?

2 Eyes: blue, brown and green

On a bleak island there are 100 people with blue eyes, 100 people with brown eyes, and one green-eyed seer. All are perfectly logical. Each person is unaware of their own eye colour (it could be any colour whatsoever) but they can see the 200 other people's eye colours. They may not communicate with one another in any way. Every evening, a boat leaves the island. People wishing to leave may present themselves to the captain. Those who correctly state their eye colour will be permitted to board the boat but those who give their eye colour incorrectly will be executed. Each morning, the green-eyed seer looks at the assembled people and says: I see someone with blue-eyes. Who can leave the island and when?

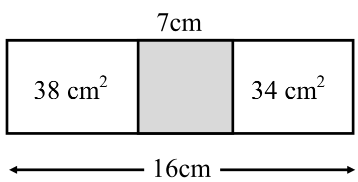
3 Amazing hinged triangles



Equilateral triangles ABC and BDE share a common point B. There is something noteworthy about the angle formed by the intersection of the lines AE and CD. Explore!

4 Missing middle

What is the area of the shaded rectangle?

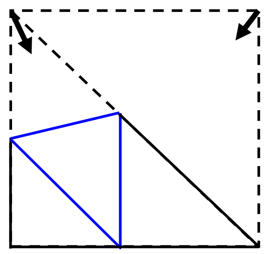


5 Probability but no counting

You pick three random points on a circle. The points are joined by lines to make a triangle. What is the probability that the centre of the circle is inside the triangle? What strategies could you use for this? Does it make a difference whether the centre is strictly inside the triangle or is allowed to be on a line?

6 Paper fold

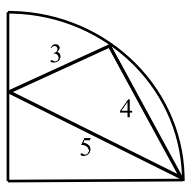
A square of paper is folded along a diagonal to make an isosceles triangle. Then, one of the acute angles is brought down to touch its opposite side in such a way that part of the diagonal edge is now perpendicular to that side, making a new triangular region.



If the square has side 2 units, what is the area of the triangular region coloured blue in the diagram?

7 Half the picture is never enough

What is the area of the quarter circle in this diagram?



8 Convenient shopping

I bought four items from the 7-11, and the total came to \$7.11. Remarkably, the product of the four prices was also \$7.11! What were the four prices?

9 Real sums

If the sum of three real numbers is 5 and the sum of their squares is 9, we could have 1, 2, 2 as the three numbers. But are there any other possibilities?

IMMC

Team registrations for the 2021 **International Mathematical Modelling Challenge** (IMMC) open during the first week of Term 1.

Will your team be the one to make Australia the international winner for 2021?

Last year, 100 teams, involving about 350 students from Years 7 to 12, participated in solving a real-world problem based on Flash Sales. Teams came from most of Australia's states and territories, public and private school alike. In 2020, following Australia's national phase of the IMMC, two teams were selected as Australia's entry to the International phase of the Challenge: a team from North Sydney Boys High School in NSW, and a team from Caulfield Grammar School in VIC. Both received an honourable mention at the international level - well done to both teams!

The international winners for 2020 were Anglo Chinese School in Singapore and the Canford School in the United Kingdom, for their innovative work and reports. Links to last year's problem can be found at: https://immchallenge.org/Contests/2020/2020_IMMC%20_Problem.pdf.

Team registration for 2021 will be open until 4 March (teams must choose a period of up to 5 consecutive days between 8 March and 30 March). Registrations are processed through the IMMC website (https://www.immchallenge.org.au) where you can find further information and resource materials designed to help teachers develop modelling skills in their students. A consolidated resource guide is available for **free** download from https://www.immchallenge.org.au/files/IM2C-Teacher-and-student-guide-to-mathematical-modelling.pdf.

Changes to IMMC for 2021

The 2021 IMMC will be supported in Australia by monetary and in-kind contributions from not-for-profit organisations the Australian Council for Educational Research and the Consortium for Mathematics and its Applications. The Challenge will also be supported by in-kind contributions from universities, research organisations and mathematical associations throughout Australia. The IMMC uses a large amount of volunteer labour, as well as funds from not-for-profits. However, due to the financial constraints on many not-for-profits in 2020/2021 arising from the impacts of COVID-19, these generous contributions will not cover the running costs for IMMC in 2021.

We need to ask for a small contribution from participating schools to continue to run the IMMC in 2021. To this end, a small registration fee will be asked per team which will range from $12.50 to $25 per student depending on the number of team entries from the school. As always, your support of IMMC is very much appreciated.

Kristy Osborne

POPULATION MODELLING

By Peter McIntyre

(This is the first instalment of a three-part piece written some years ago, addressed to students.)

1 Introduction to Population Modelling

When mathematicians talk about playing with a model, chances are they don't mean a model plane or boat. They are probably talking about a mathematical model—a set of equations that describe in mathematics how a particular system works. There are mathematical models for many things, such as the planets revolving about the sun, heating iron ore in a blast furnace, pollution in a lake, how prices vary on the stock exchange, the spread of diseases and how populations (people, animals, bacteria, viruses, etc) change with time.

Population modelling started a long time ago, and one of the earliest modellers was Fibonacci (1170 - 1250). In his book Liber Abaci, he modelled a rabbit (continued on page 6)

NEWSLETTER OF THE CANBERRA MATHEMATICAL ASSOCIATION INC

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We're on the Web!
http://www.canberramaths.org.au/

ABOUT THE CMA

The Canberra Mathematical Association (Inc.) is the representative body of professional educators of mathematics in Canberra, Australia.
It was established by, among others, the late Professor Bernhard Neumann in 1963. It continues to run - as it began - purely on a volunteer basis.

Its aims include

* the promotion of mathematical education to government through lobbying,
* the development, application and dissemination of mathematical knowledge within Canberra through in-service opportunities, and
* facilitating effective cooperation and collaboration between mathematics teachers and their colleagues in Canberra.

THE 2021 CMA COMMITTEE

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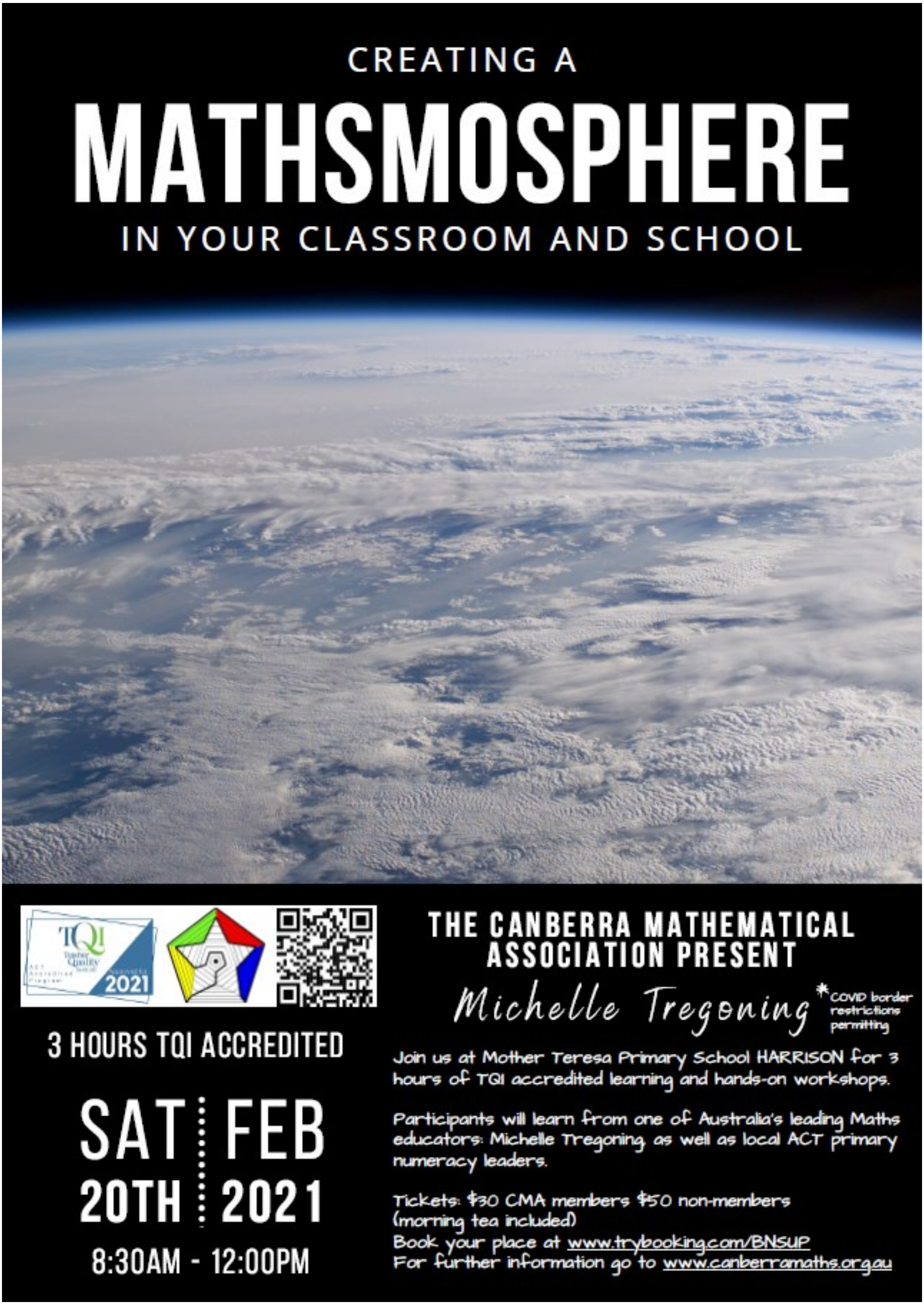

CREATING A
MATHSMOSPHERE
IN YOUR CLASSROOM AND SCHOOL
2021
3 HOURS TQI ACCREDITED
SAT 20TH
FEB 2021
8:30AM - 12:00PM
THE CANBERRA MATHEMATICAL ASSOCIATION PRESENT
Michelle Tregoning
*COVID border restrictions permitting
Join us at Mother Teresa Primary School HARRISON for 3 hours of TQI accredited learning and hands-on workshops.
Participants will learn from one of Australia's leading Maths educators: Michelle Tregoning, as well as local ACT primary numeracy leaders.
Tickets: $30 CMA members $50 non-members
(morning tea included)
Book your place at www.trybooking.com/BNSUP
For further information go to www.canberramaths.org.au

POPULATION MODELLING part I continued

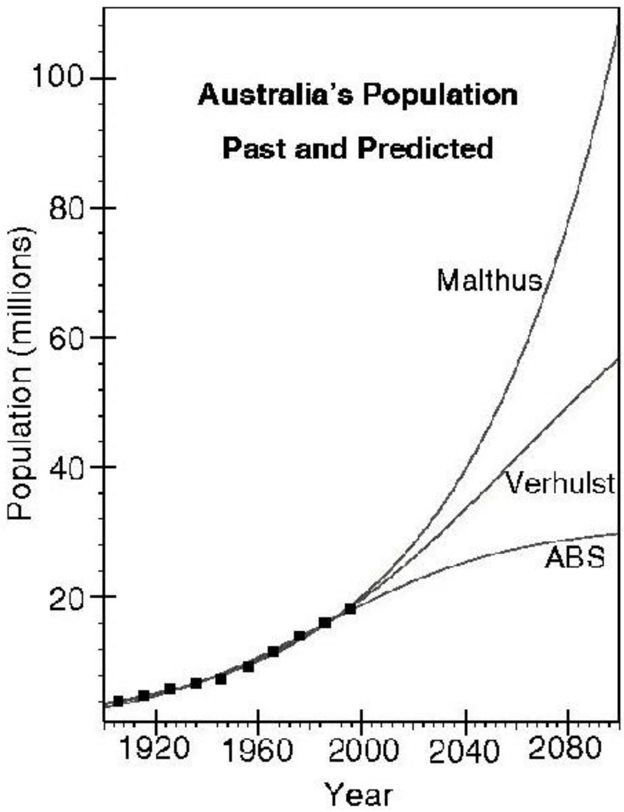
population, starting with one pair of baby rabbits.

If each adult pair of rabbits produces only one pair of baby rabbits each month, and if baby rabbits take one month to become adults, the numbers of pairs of rabbits in successive months are given by the famous Fibonacci numbers: 1, 1, 2, 3, 5, 8, 13, 21, and so on. The next number is found by adding the previous two numbers. Fibonacci numbers are also found elsewhere in nature. If you look at a pine cone, you will find the 'petals' spiral in two directions. The number of petals it takes to get once around is almost always a Fibonacci number. The same thing occurs in pineapples, sunflowers and many other flowers.

Much later than the time of Fibonacci, Thomas Malthus (1766 - 1834) in England startled the world by predicting that food would run out sometime in the future, because of the rapid increase in the human population. Based on the data he had at the time, Malthus predicted that the world population would increase exponentially, doubling every 40 years, thereby increasing at a faster and faster rate. (Forty years is the current doubling time of the world's population.) If you start with the number 1 and keep doubling it, you will see an example of exponential growth.

The models of Fibonacci, Malthus and some other scientists all predict that the population will grow faster and faster. This is an alarming prospect, but does not seem to happen in experiments performed when there are limited resources, such as food and space to live in. Experiments with small animals and fungi in the laboratory, and with larger animals in fenced areas in the field show that as the resources start to run out, the reproduction rate reduces and the rate of growth slows down. The Belgian scientist Pierre Verhulst (1804 - 1849) while at the Belgian military school, the Ecole Royale Militaire, developed a model, called the logistic model, which took into account these observations. He introduced the idea of a 'carrying capacity' or maximum sustainable population that the environment will support.

We can illustrate the Malthus exponential model and the Verhulst logistic model by looking at the population of Australia since 1900. The small dark squares on the graph below show the Australian Bureau of Statistics figures for the number of people in Australia (in millions) up until 1996.



If we model these data with an exponential curve (the Malthus model), we get the top curve in the figure. The middle curve is the Verhulst model. Both these curves fit the population numbers up to the present time well (year 2000 ed.), but predict quite different future populations.

According to the exponential (Malthus) model, the population will continue to grow at a faster and faster rate, with a predicted population of about 109 million people in the year 2100, and about 587 million people in 2200. The Verhulst model predicts that the population will keep on growing, but at a slower and slower rate; the predicted population in 2100 is about 57 million people, and the population

POPULATION MODELLING part I continued

would level off eventually at about 83 million people.

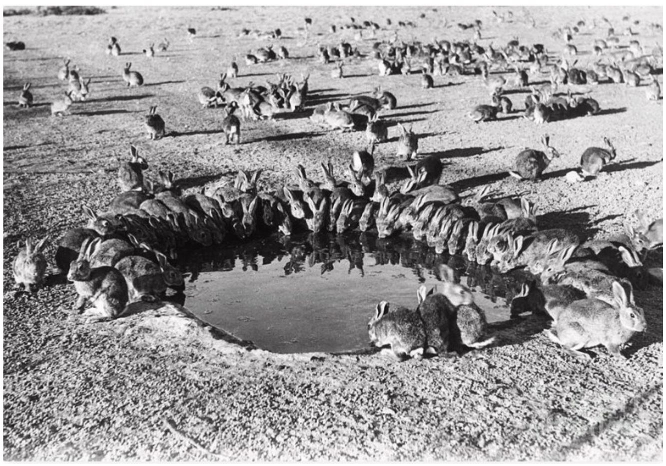
The Australian Bureau of Statistics uses a mathematical model to predict the population of Australia well into the future to assist in planning for the number of people who will be living here. The predictions of their model are shown as the bottom curve in the figure. It has the shape of a Verhulst curve, but levels out much faster than the middle curve, predicting a population in 2100 of about 30 million people, and a maximum population of about 31 million.

Prediction is one powerful aspect of a mathematical model. By putting in the numbers we know, such as for the Australian population, we can predict what a population will be in the future, according to our model. Of course, the accuracy of our predictions depends on how good our model is, that is, how well it describes the phenomena that affect population growth.

Another important use of a population model is to predict what will happen to the population if something changes, for example if the birth rate drops, if the number of immigrants is decreased, or if, say in a war, many people die. Predicting changes in a population is particularly relevant to populations of animals, insects and plants which have become serious pests after being brought into Australia from overseas. These include rabbits, foxes, mice, cane toads and European carp among the animals, and prickly pear, Paterson's curse, salvinia, mimosa and scotch thistles, to name but a few of the plants. The populations of some of these have reached very high levels at times, causing serious problems for farmers and the environment.

How do we control such pests? Often there are a number of possible ways, but which one is best? Population models can be modified to include the effect of the release of a predator, the spread of a disease in the pest population, the effect of poisoning or some other control measure. It is then possible to use the models to predict what would happen to the population if the different control strategies were tried. The models can also be used to find the best way of carrying out a particular control measure. Sometimes the modelling is done together with small-scale experiments, but often only the mathematical model can be used because the experiments are too risky or too expensive.

In using a population model, we put the starting conditions and parameters (number of animals, how quickly they breed, etc) into our equations and predict the population at some later time. What if we change the starting conditions only slightly? We will end up with nearly the same final answer, right? Not necessarily. In some models, for example a variation on the Verhulst logistic model, with particular parameters, we find that the population does not change steadily towards some ultimate population, as we saw in modelling the Australian population, but changes rapidly and unpredictably with time. We say the model exhibits chaos: it loses its ability to predict, because a small change in the starting conditions produces a large change in how the population varies with time.



Rabbits drinking at a waterhole before the introduction of myxomatosis.

(To be continued…)

PUZZLE SOLUTIONS

Solutions to puzzles from the January edition.

1 Numbers like Ramanujan's

Four digit numbers can be represented 1000a+100b+10c+d. With reversals adding to 11000, we have 1000(a+d)+100(b+c)+10(b+c)+(a+d)=11000. Hence, 91(a+d)+10(b+c)=1000. We deduce that a+d=10 and b+c=9. If repeated digits are allowed, there are 90 possibilities. If not, there are 48.

2 Hard circles

An explanation and proof of the six-circles theorem (too long for this column) has been devised by your puzzles editor. It can be accessed at the Mathematical Whetstones website.

3 Shady squares

Comparing the first and second rows of squares, there is a pair of congruent triangles, one shaded and one not, so that two full squares are shaded. In the third row, half of half of three squares is shaded. Thus, 2 3/4 squares are shaded out of 6. The fraction is 11/24.

4 From A to B

The situation is similar to that for radioactive decay. In the puzzle, speed is the rate of change of distance and it depends on the current distance just as radioactive decay is the rate of change of mass, which depends on the current mass. Thus, the diminishing distance to B has a 'half-life' and there is always some distance remaining.

There is a differential equation explanation in which the unattainability of the goal is signalled by the failure to exist of log 0.

5 Cryptic digits

The 9-digit number ABCABCBBB is divisible by every positive integer up to 17. Hence, it must be divisible by the primes 2, 3, 5, 7, 11, 13, 17. Since it is divisible by 3^2 and 2^4 it has two further factors of 3 and three more of 2. Therefore the number is more than the product 12252240. To make the 1st and 4th digits of this number the same, we must multiply by further factors of 5. Thus, 61261200 is a candidate. Multiplying this again by 5 gives 306306000, which is the required number.

6 Fibonacci

The question asks whether the greatest common divisor of two Fibonacci numbers is also a Fibonacci number. The answer is 'yes' but the proof is not immediate. In fact, if k is the greatest common divisor of m and n, the greatest common divisor of the mth and nth Fibonacci numbers is the kth Fibonacci number.

7 Mercedes sector

The curved part of one sector has length π. Since the circumference is 3π, the radius is 3/2. Hence, the perimeter of a sector is $3+\pi$.

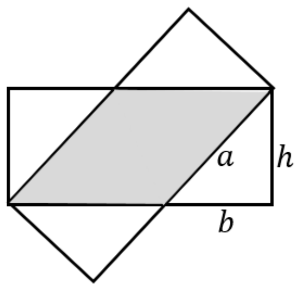
8 2020 conjunction

(a) 60 years. (b) 24 minutes. (c) Jupiter overtakes Saturn 3 times in 60 years. Therefore, conjunctions occur every 20 years. (The 2020 conjunction was special because it was particularly close.)

9 Factors

The full list of divisors must be 1, 3, 5, 7, 15, 21, 35, 105 and the sum of these is 192.

10 Rectangles



The triangles are congruent. The rhombus area is ah and this is $5/8\ h(a+b)$. Thus, $a = 5/3\ b$. By Pythagoras, $h = \sqrt{(a^2 - b^2)}$. Then, the required ratio is

$$h/(a+b) = \sqrt{(a^2 - b^2)}\ /\ (a+b)$$
$$= \sqrt{(a-b)}\ /\ \sqrt{(a+b)}.$$
$$= \sqrt{(2/3\ b)}\ /\ \sqrt{(8/3\ b)}$$
$$= 1/2$$

Someone may wish to look for a general solution.