CIRCUIT

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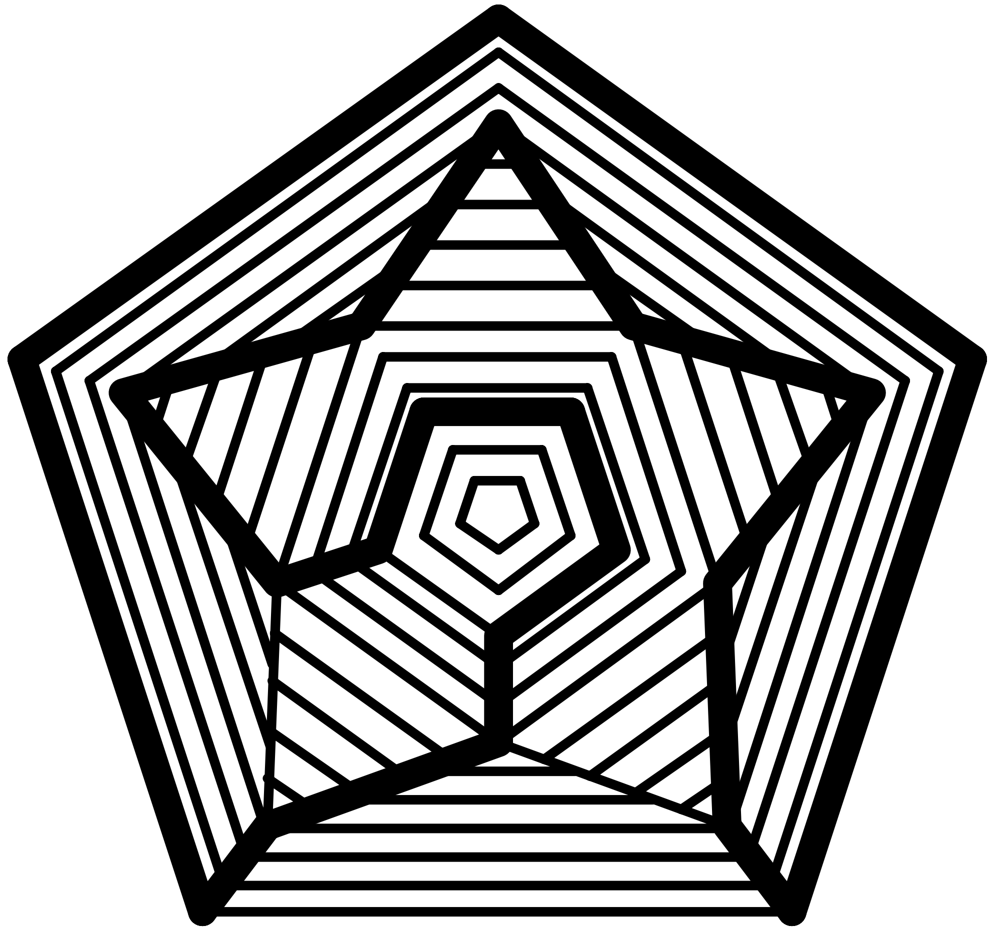

CANBERRA MATHEMATICAL ASSOCIATION

OFFICE BEARERS 1999

EDITOR *CIRCUIT* 1999

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CMA INTERNET HOME PAGE

http://education.canberra.au/projects/cma/home/html

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The objects of the Canberra Mathematical Association are to promote interest in mathematics, to encourage improvements in the teaching of mathematics and its applications, to provide means of communication among teachers and students and to advance the views of the Association on any question affecting the study or teaching of mathematics and its applications.

The Canberra Mathematical Association Logo depicts a Hamiltonian Circuit on a dodecahedron.

CONTRIBUTIONS

Circuit is always keen to receive articles, notes, problems, letters and information of interest to members of the CMA. Please contact the Editor if you wish to contribute to Circuit. If possible contributions should be submitted as Word 6 documents.

FROM THE PRESIDENT

In keeping with the CMA's goal of supporting high quality mathematics teaching and learning our association has, this year, already provided several professional development opportunities for teachers.

NOVA - Science in the News

The Academy of Science has developed several mathematics topics which can be accessed at the NOVA Science on the News site - www.science.org.au/nova. The first topics developed are suitable for senior secondary students with mathematics in sport being the opener.

AAMT 2001 - Mathematics Shaping Australia

The organising committee is now at the fine tuning stage of the program. There will be a wide range of professional development opportunities for teachers from all sectors so, as schools are pencilling in forward planning options, do look closely at choosing 15-19 January 2001 for quality PD. Not only can teachers be confident of gaining professionally from attendance at the conference but a social program, to meet the most demanding need for fun, has been planned. This is a rare opportunity for ACT teachers to meet with colleagues from Australasia, and international and national experts in mathematics and numeracy, in the comfort of their own home town.

Professional Development Activities 1999-2000

Many successful PD events have already taken place this year and we are now in the process of appraising the second semester program and planning for 2000. I am seeking help from our members to tell us what their needs are so that we can best plan to meet them. Please make contact with CMA and give us your suggestions. Every day teachers are doing great things in the classroom and CMA would like to share your expertise with other teachers. We understand how busy teachers are but ask if you could possibly send some of your ideas to us to be published in Circuit-- no contribution is too small. Material from pre-school and primary teachers is particularly sought.

Paulene Kibble
email: paulene.kibble@eom.com.au

NOTICE BOARD

INDIGENOUS MATHEMATICS

The CMA has been asked by the Indigenous Education Unit (IEU) to assist in reworking a draft document on the use of mathematics in traditional indigenous cultures. The document is designed to support teachers from all sectors in including an indigenous perspective in the ACT school mathematics curriculum.

This is an opportunity for interested CMA members to build on the existing content and be involved in producing a valuable resource. The recent publication by the

Science Teachers Association of "Ochre to Eel Traps" has demonstrated the value of working with professional associations and the IEU is confident that funds will be available to complete the project.

If you would like more details about the draft or the process please contact Rick Owens on 6205 2045 (phone), 6205 2046 (fax) or email rick_owens@dpa.act.gov.au

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AAMT VIRTUAL CONFERENCE 99

Investigating the investigative: Issues and themes in contemporary open-ended approaches to mathematics in schools
20 August - 24 September 1999

Use your computer and the Internet for powerful, up to the minute Professional Development.

No need to leave home, with all the expense and inconvenience that means!

A chance to interact with colleagues across Australia and beyond - the 1998 VC involved people from nearly 20 countries!

Select the topics that interest you from:

- Outcomes-based education
- Numeracy across the curriculum
- Applications and modelling
- Using technology to investigate in mathematics
- and more...

For teachers of mathematics K - 12.

Contact AAMT for further details or go to www.aamt.edu.au

CALL FOR CONTRIBUTIONS FROM TWO ICME9 WORKING GROUPS

Working Group for Action (WGA)6: Adult and Life-long Education in Mathematics, and WGA11: The use of Technology in Mathematics Education are both calling for contributions. ICME9 will be held in Tokyo/Makuhari, Japan from July 31 to August 6, 2000.

The email contact for WGA6 is
gfitzsimons@swin.edu.au
and for WGA11 is
kissane@central.murdoch.edu.au

FROM THE EDITOR

Marvellous, isn't it, that we can now look forward to the wonderful prospect of a GST in a year's time on most things educational, including books and membership fees for professional associations. What is it about our current crop of political leaders which enables them to ignore notions like national progress and securing the well-being of all, which I believe underpin and motivate the vast majority of educators?

So how about a poem to help sustain us against the rigours of Canberra winter and the current vicissitudes of local and national education politics and funding? It comes from *Poems on the Underground* (as in London Underground). I find it curiously fortifying and I hope it gives you something positive too.

Sometimes

Sometimes things don't go, after all,
from bad to worse. Some years, muscadel
faces down frost; green thrives; the crops don't fail,
sometimes a man aims high, and all goes well.

A people sometimes will step back from war;
elect an honest man; decide they care
enough, that they can't leave some stranger poor.
Some men become what they were born for.

Sometimes our best efforts do not go
amiss; sometimes we do as we meant to.
The sun will sometimes melt a field of sorrow
that seemed hard frozen: may it happen for you.

Sheenagh Pugh (b. 1950)

Turning to other matters, the major piece in this Circuit is Jill Middleton's contribution "What Chance Do You Have?". It bears all the hallmarks of sound teaching: a thoughtful, clear and intrinsically interesting treatment of its topic, embracing aspects of the history, politics, practical probabilities and consequent social implications of some of the common gambling activities in Australia. Truly, under most circumstances, the various forms of gambling Jill deals with appear to be "a tax on imbeciles". And after more than two years spent wearing an editorial hat, I welcome Audrey Guy's Speaking Out opinion piece, as well as George Harvey's contributions to the Problems and Activities section.

If you are just starting out on the Internet, do make a point of regularly browsing the AAMT website **www.aamt.edu.au** It has a range of links to mathematics education and mathematics related sites to get you started. It also offers you the opportunity to be part of the AAMT Internet Mailing List Community which is currently engaged in an online discussion on the topic of "ability grouping". Good browsing!

In closing, I remind you that *Circuit* only exists to serve you, its readers. So please don't be shy about forwarding responses, comments, reactions, or material for publication.

HAPPENINGS

Graphics Calculators at Stromlo High School

On Monday 24th May 1999, maths students took part in a program of activities designed to allow high school teachers to evaluate the potential of graphics calculators in the classroom. The emphasis was on mathematics and problem solving, rather than on specific instructions in the use of the technology.

These hands-on lessons were made possible through a joint venture of the Canberra Mathematics Association and Texas Instruments. Steven Thornton from the Australian Mathematics Trust conducted the lessons with the help of Brian Lannen from the Riverina Mathematics Association.

The lessons consisted of a Year 9 class doing interactive activities with a motion detector, a Year 10 class using a quadratic function to model data using a motion detector, and a Year 8 class investigating chance and data using simulations. Students really enjoyed these classes and some are keen to buy their own graphics calculators.

In the afternoon Stromlo teachers were able to participate in a workshop and discussion session on the graphics calculators.

Jan Macdonald
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Nova Website Mathematics Topic Launched

Just after 8 am at a breakfast on Friday 30 April at Becker Hall (the Australian Academy of Science Dome) in Canberra the first mathematics topics on the NOVA: Science in the News site were officially launched. Academy Fellow Professor Cheryl Praeger from the University of Western Australia performed the launch in front of an audience comprised of other Fellows of the Academy and a range of invited guests, including CMA members.

The website as a whole is managed by the Australian Academy of Science with some financial support from the Science and Technology Awareness Program of the Commonwealth Department of Industry, Science and Resources. Nevertheless, as was pointed out during the launch, without direct funding contributions from some twenty three of the thirty plus cash-strapped Australian university mathematics departments, the Nova mathematics topics would not have come to fruition.

The Nova: Science in the News website

www.science.org.au/nova

provides information on science, health, the environment and now mathematics - currently four topics with up to a dozen more to come. The format for each Nova topic is: text in non-technical language; a glossary; student activities; useful links to other websites, with brief comments; suggestions for further reading.

The hope is that Nova will be used by teachers planning lessons, students, parents, librarians, journalists, and in fact anyone seeking accurate and up-to-date information across the broad range of scientific topics, including mathematics. At this stage none of the mathematics topics, unlike the science ones, extends down to the early childhood years.

In her role as CMA President, Paulene Kibble welcomed interstate guests and Fellows of the Academy. She pointed out that it is important to develop web based mathematics learning materials which reach down as far as the early childhood level. This should be a priority, she suggested, because early positive experiences with mathematics on the web could help integrate mathematics in to the lives of children, parents and extended families, and in so doing help take mathematics out from the classroom into the wider world.

Nova mathematics is a new bright spot in the southern skies of web mathematics. Hopefully it will be able to consolidate and expand its impact in times to come, given ongoing funding.

Peter Enge

Speaking Out

Mathematics should add up to more than bums on seats.

Have you noticed that every year the ACT Education budget allocates large sums of money to the upgrading of areas such as science and technology but maths areas never seem to get a turn? You are probably not aware that when Gold Creek high school opened it had a fully equipped, custom built robotics room and dance theatre, but when it came to the maths room it was the same old uncomfortable chairs, boring tables, a whiteboard and presumably the teacher was given a pen.

When it comes to class sizes the situation is much the same. Reductions in teacher numbers in schools invariably lead to larger maths classes. Few schools seem to allocate funds solely on a per capita basis and once again maths classes subsidise other subject areas. It is hardly surprising that parents are starting to point out that the only funds the Department spends on numeracy (and literacy) go on testing to see how well teachers cope without resources!

Its time for change. We need to recognise that the basis of national intellectual capital should be adequate competencies in numeracy (and literacy). If standards are too low, much of the money being spent in other areas, such as vocational education, is less effective.

We should be arguing for adequate resources. This should include smaller classes based on educational needs, just as in those areas where class sizes are based on safety needs. Properly equipped classrooms, with comfortable furniture, a sink, an equipment cupboard with measuring instruments (cf science), and a large screen computer plus adequate access to computer labs should be standard provision.

We must insist on adequate financial resources. The old argument that some areas need more money than others has pulled the wool over people's eyes for far too long- it means that areas such as maths are left woefully under resourced. Should we sit back and watch whilst maths students subsidise students in other areas?

If we are to adequately prepare our students for the very different world and skills they will encounter in the next century we need to upgrade our philosophy and demand equal resources for them. We can't afford to do otherwise.

Audrey Guy
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`What Chance Do You Have?

The investigation of gambling chances in Australia is an interesting way to promote the learning of probability, while helping students to become aware of the probability of winning or losing in games of chance such as roulette, poker machines, lotto and lotteries.

In 1994-95 lotteries, casinos and other gambling services in Australia earned over $16 000 million revenue (ABS,1998). In 1995-96 the per capita expenditure (or amount lost for every adult in Australia) on these activities was approximately $710 with more than half of this figure coming from poker machines and casinos (Safe,1997).

Casino losers genuinely believe that they can foretell the outcome of the roulette wheel by monitoring developments carefully enough. Casinos encourage this illusion by providing roulette players with special cards to record and analyse trends and Lotto will publish sets of numbers which have won in the past!

Many people believe that if an event has not occurred as many times as expected that there is a higher than usual chance of it turning up next. This is a fallacy. In Roulette or in Lotto, each game is independent of the next. No matter what has occurred in the past, the probability of a particular event occurring remains the same for every game.

Gambling in Australia has always been a problem. In the 1790s George Barrington, ex-pickpocket principal superintendent of convicts at Parramatta was quoted as saying "The pernicious vice of gambling… was carried on to such excess among the convicts that many had been known, after losing their provisions, money, and spare clothing, to bet and lose the very clothes from their backs, exhibiting themselves as naked, and as indifferent about it as the natives of the country." (O'Hara, J., 1988, p10)

At the beginning of the 20th century there was an attempt at legislative reform led by the Protestant church, but it was unsuccessful as it was found impossible to police.

Missing Cartoon

A Wild Goose Chase, Norman Lindsay's Comment on the anti-gambling crusade. Bulletin *5 October 1905*

Legislation or prohibition will not solve our gambling problem and governments are loath to cut off such a lucrative source of revenue. "To have a source of revenue where people line up to voluntarily pay is more than they can resist" (Safe, 1997). Education is a must, so that when students are presented with the statistics they are able to evaluate the odds and estimate the expected costs. At least they could then make informed decisions about the different forms of gambling.

Let us consider the games of Roulette, Poker machines, Lotteries and Lotto, investigating probabilities, expenses and payouts.

Roulette

Missing Diagram

The gambling game of Roulette illustrates simple probability very well. In the game the wheel is set spinning and a white marble is thrown in the opposite direction and finally comes to rest in one of the slots on the wheel. The slots are numbered from 1 to 36, some being red, others black, and zero is green. The zero is included on the wheel to give the operator an advantage. All payouts are made as if there are only 36 slots. The probability of any number coming up is 1 in 37. Let us assume you bet $2 on one number, say 9, and the number 9 is spun up. You will then get a return of 36 times $2, not 37 times(This return includes your original $2).

So the probability of winning

$$= \frac{\text{No. of favourable outcomes}}{37}.$$

The return

$$= \frac{36}{\text{No. of favourable outcomes}} \times \text{Amount bet.}$$

Have the students set up a table as in **Table 1**.

From this table (see Appendix for completed table) it should become plain to the student that the casino is expected to always win eventually and that no matter which bets the punter makes the expected loss is the same. American Roulette wheels have an extra green slot which is numbered 00 which increases the casino's winnings.

Poker Machines

There are thousands of poker machines found in clubs and casinos around Australia. In NSW the minimum payout on poker machines by law is 80% but to attract customers the clubs have found that they needed to increase this to 87%. Players might get ahead for a while with a jackpot or a high payout but if they keep pulling the handle they will lose $13 out of every $100 they put through the slot. On first glance one could expect to walk up to a machine with $100, play as much as you like, and walk away with $87 in your pocket, having only lost $13. But is this right? Have the students draw up a table as in **Table 2**. This experiment looks at a player who starts with $100 and plays on a $1 machine. After 100 pulls the player is expected to have won $87 and feels lucky having had a number of payouts. The player continues to play putting all of this $87 through the machine and so on. Using the table have the students discover how many pulls the player has before she notices that she has less than $20 left. (As our results will be expected returns based on a percentage we will get parts of dollars in our answers. To get an accurate estimate we will need to assume that we can have a fraction of a pull.)

If the time taken for each pull is 10 seconds, how long will it take our gambler to lose her $80? The result is less than 2 hours. See the Appendix for the complete calculation.

Table 1 – all results are determined using bets of $1

Bet	**No. of favourable outcomes**	**Probability of winning**	**Return per $1**	**Winnings per $1**	**After 37 games the expected winnings**	**After 370 games the expected winnings**
Red	18					
Black			$\frac{36}{18}$	$\frac{36}{18} - 1 = \$1$	18 × $1 – 19 × $1 = –$1	180 × $1 – 190 × $1 = –$10
odd						
even						
a pair	2	$\frac{2}{37}$				
a column						
zero						
any number						

Table 2

Expected Amount bet	**Expected 87% return**	**No. of pulls at $1 a pull**	**Total no. of pulls**
$100	$87	100	100
$87	$75.69	87	187
$75.69	$65.85	75.69	262.69

See appendix for completed table

Investigation of the NSW "Lucky" Lotteries yields the following facts.

A $2 lottery sells 180 000 tickets and there are 10 147 prizes totalling $235 430 (including the provision for the jackpot prize). 3 384 of these are cash prizes, of which 3 382 are eligible to win the jackpot if their ticket is drawn again. First prize is $100 000 and the minimum jackpot is $500 000 with $50 000 increments each time there is no jackpot won. Students can be asked to work out the probability of winning a prize, a cash prize, first prize or the jackpot. They can determine how often the jackpot is expected to be won. Some of these probabilities have been calculated below.

The probability of winning some sort of a prize is

$$\frac{\text{Total No. of prizes}}{\text{Total No. of tickets}} = \frac{10\ 147}{180\ 000} \cong .056$$

or about 1 chance in 18.

To win first prize we have a probability of

$$\frac{1}{180\ 000} \cong 5.56 \times 10^{-6}$$

or 1 chance in 180 000 .

Perhaps this figure could be compared with the number of people in the stands of the Melbourne Cricket Ground (about 90 000 people).

After all prize winners have been selected all tickets are returned to the barrel and one ticket is drawn for the jackpot. The jackpot is won only if this drawn ticket is one of the 3 382 cash prize winners.

If P(cash) is the probability of winning a cash prize and P(jackpot) is the probability of being drawn in the jackpot draw, then **the probability of winning the jackpot is**

P(cash) × P(jackpot)

$$= \frac{3\ 382}{180\ 000} \times \frac{1}{180\ 000} \cong 1.04 \times 10^{-7}$$

or about 1 chance in $9\frac{1}{2}$ million.

The probability of the jackpot going off is the probability that any one of the 3 382 cash prize winners is drawn again, which is

$$\frac{3\ 382}{180\ 000} \cong 1.9 \times 10^{-2}$$

or about 1 chance in 53.

So we would expect the jackpot to be won about once every 53 lotteries.

Lotto

In Lotto, players select 6 numbers from numbers 1 to 45. If their numbers match those of a random draw they win a first division prize of at least $1 million. There are 2nd to 5th division prizes as well, but for this exercise I would like to concentrate on the chances of winning a first division prize.

A standard entry is $1.10 per game. System entries allow you to pick more than 6 numbers and every combination of 6 numbers becomes a separate entry, thus increasing your chances of winning. Some system tickets are described in Table 3.

Table 3

System	No of combin-ations (entries)	Cost $
System 7	7	7.45
System 8	28	29.70
System 9	84	87
System 10	210	216
System 11	462	471
System 12	924	942

System 13	1716	1749

Probability of one set of 6 numbers winning

There are ${}^{45}C_6$ possible combinations of 6 numbers from 45 numbers so the probability of any 6 numbers being drawn is

$$P(6) = \frac{1}{{}^{45}C_6} = \frac{1}{8\ 145\ 060} \cong 1.23 \times 10^{-7}$$

or 1 chance in 8 145 060 . This means that buying one game each week you are likely to win sometime in the next 156 000 years. I hope you live long enough.

What chance have we of winning in our lifetime and at what cost?

For this exercise we will assume a lifetime of lotto playing is 50 years.

Purchasing one game every week for 50 years is $50 \times 52 = 2\ 600$ games in total.

Each game has the above probability of winning so the probability of winning in our lifetime is

$$2\ 600 \times P(6) \cong 3.19 \times 10^{-4}$$

or about 1 chance in 3 133.

This would cost us a total of
$2\ 600 \times \$1.10 = \$2\ 860$

Purchasing a system 7 ticket each
week gives us 7 chances each week
or a probability each week of
$7 \times P(6) \cong 8.6 \times 10^{-7}$.

The probability of winning in our lifetime is $2\ 600 \times P(6) \cong 3.19 \times 10^{--4}$ or about 1 chance in 3 100.

Our odds are improving but at what cost?

A system 7 entry costs \$7.45 so a system 7 entry each week for the next 50 years would cost us $\$7.45 \times 2\ 600 = \$19\ 370$ (Perhaps a new car!)

What system would we need to purchase throughout our lifetime to improve our chances of winning to better than 50%?

As the chance of winning with one entry is P(6) or $\frac{1}{8\ 145\ 060}$ we would need to play half of 8 145 060 games to bring the probability of winning up to 50%. So the number of games needed is

$$\frac{8\ 145\ 060}{2} = 4\ 072\ 530$$

We play for 2 600 weeks so each week we need to play $\frac{4\ 072\ 530}{2\ 600} \cong 1\ 566.4$ games.

System 13 gives us 1 716 standard games so we would need to buy a system 13 ticket, or choose 1 567 combinations (a little tiresome), each week for 50 years to give us a better than 50% chance of winning sometime.

How much would it cost us to do this?

The cost for a system 13 ticket is \$1 749

The cost over 50 years would therefore be

$$\$1\ 749 \times 2\ 600 = \$4\ 547\ 400.$$

So to have a 50% chance of winning lotto in our lifetime we would need to spend over 4 million dollars on tickets.

Reflection

Remembering that in Australia approximately \$710 per adult is being lost in games of chance alone each year, and that this, over 50 years becomes a figure of over \$35 000. As some people do win, and some people don't bet, the amount actually lost by the rest is much higher. Can people really afford this?

Appendix

Table 1 - all results are determined using bets of $1

Bet	**No. of favourable outcomes**	**Probability of winning**	**Return per $1**	**Winnings per $1**	**After 37 games the expected winnings**	**After 370 games the expected winnings**
Red	18	$\frac{18}{37} = 0.49$	$\frac{36}{18} = 2$	$\frac{36}{18} - 1 = \$1$	18 × $1 – 19 × $1 = – $1	180 × $1 – 190 × $1 = – $10
Black	18	$\frac{18}{37} = 0.49$	$\frac{36}{18}$	$\frac{36}{18} - 1 = \$1$	18 × $1 – 19 × $1 = – $1	180 × $1 – 190 × $1 = – $10
odd	18	$\frac{18}{37} = 0.49$	$\frac{36}{18} = 2$	$\frac{36}{18} = - 1 = \$1$	18 × $1 – 19 × $1 = – $1	180 × $1 – 190 × $1 = – $10
even	18	$\frac{18}{37} = 0.49$	$\frac{36}{18} = 2$	$\frac{36}{18} = - 1 = \$1$	18 × $1 – 19 × $1 = – $1	180 × $1 – 190 × $1 = – $10
a pair	2	$\frac{2}{37} = 0.05$	$\frac{36}{2} = \$18$	$17	2 × $17 – 35 × $1 = – $1	20 × $17 – 350 × $1 = – $10
a column	12	$\frac{12}{37} = 0.32$	$\frac{36}{12} = \$3$	$2	12 × $2 – 25 × $1 = – $1	120 × $2 – 250 × $1 = – $10
zero	1	$\frac{1}{37} = 0.03$	$\frac{36}{1} = \$36$	$35	1 × $35 – 36 × $1 = – $1	10 × $35 360 × $1 = – $10
any number	1	$\frac{1}{37} = 0.03$	$\frac{36}{1} = \$36$	$35	1 × $35 – 36 × $1 = – $1	10 × $35 360 × $1 = – $10

Table 2

Expected amount bet	**Expected 87% return**	**No. of pulls at $1 a pull**	**Total no. of pulls**
$100	$87	100	100
$87	$75.69	87	187
$75.69	$65.85	75.69	262.69
$65.85	$57.29	65.85	328.54
$57.29	$49.84	57.29	385.83
$49.84	$43.36	49.84	435.67
$43.36	$37.73	43.36	479.03
$37.73	$32.82	37.73	516.76
$32.82	$28.55	32.82	549.58
$28.55	$24.84	28.55	578.13
$24.84	$21.61	24.84	602.97
$21.61	$18.80	21.61	624.58

At 10 seconds a pull the time taken would be approx. 625 × 10 = 6 250 seconds or approx. 104 minutes.

References

Australian Bureau of Statistics, Sport, Recreation and Gambling,1994-95

Jones, S.B & Couchman, K.E. (1994), 3 unit Mathematics, Book 1, Collins Dove, Victoria

McNaughton, A (1991), "Do Gambling Systems favour the rich?" in The Australian Mathematics Teacher, Vol 47, No 4

NSW Lotteries (1998), Lottery Specifications

NSW Lotteries (1998), Lotto Specifications

O'Hara, J (1988), A Mug's game, New South Wales University Press, Kensington

Reid, J (1992), A Licence to print Money, Macmillan, London

Safe, M (1997) "Counting the Cost - Casinos and Governments are cashing in on as Australian obsession : But critics fear the price is too high" in The Australian Magazine, 22-23 Nov 1997, pp 12-20

Sykes, T (1994), "Stake of the Nation - Gambling is one of Australia's fastest growing industries" in ABM: Australian Business Monthly, v 14 no. 6 Feb 1994: pp 24-33

Jill Middleton
Wanniassa High School

QUOTABLE NOTES AND NOTABLE QUOTES

Doing Mathematics: Turning Darkness into Light

“Perhaps I could best describe my experience of doing mathematics in terms of entering a dark mansion. You go into the first room and it’s dark, completely dark. You stumble around, bumping into the furniture. Gradually, you learn where each piece of furniture is. And finally, after six months or so, you find the light switch and turn it on. Suddenly it’s all illuminated and you can see exactly where you were. Then you enter the next dark room ...”

Andrew Wiles describing his seven year effort to prove Fermat’s Last Theorem

Quoted in ***Fermat’s Last Theorem*** by Amir D. Aczal

How Mathematicians Do It

Combinatorists do it as many ways as they can and they do it discretely.

([Logicians do it] or [not (logicians do it)]) and (logicians do it by symbol manipulation).

Algebraists do it in groups or algebraists do it in a ring.

Analysts do it continuously.

Real analysts do it almost everywhere.

Pure mathematicians do it rigorously.

Topologists do it openly and topologists do it on rubber sheets.

Statisticians seek significance by doing it.

Mathematicians who can do one and then one more do it forever.

Galois did it the night before.

Möbius always did it on the same side.

Markov did it in chains.

Fermat tried to do it in the margin but couldn’t fit it in.

Adapted from Web Humour at www.geocities.com

An Alternative to Lecturing?

“People have nowadays got a strange opinion that every thing should be taught by lectures. Now, I cannot see that lectures can do as much good as reading the books from which the lectures are taken.”

Samuel Johnson (1709 - 1784)

Statistics (Mathematics!) for Active and Critical Citizens

Prospective critical citizens need to be aware of the social, political and economic culture in which they live, of how and why their experiences are structured and conditioned, and of the roles played by different interest groups in society. Making students aware of the relationship between mathematics and society and the effect this has on their own situation is a key component of mathematics education. Further, genuine participation in the information society requires a critical grasp of local, regional, national and international statistical information, and must accommodate discussion, questioning, a variety of values and opinions, and a preparedness to challenge authority and to negotiate outcomes. Surely statistical literacy in this sense needs to be an integral part of every mathematics curriculum.

At the core of this notion of statistical literacy is encouraging learners to pose questions such as:

- what mathematical questions arise out of this situation?
- what mathematics is being used, or could be used, in this context?
- which groups in the community are affected by the circumstances described?
- which groups are likely to benefit from the use of mathematics in this context?
- could you look at the questions in a different way? Would this produce different answers?
- are there important factors that have been ignored?
- is there any information not given here which might help you answer your question?
- are the mathematical measures used credible?
- what is the quality of the design?
- what are the allowed inferences?
- are the conclusions warranted by the data?

Adapted from *Adults Count too Mathematics for Empowerment*
by Roseanne Benn, 1997

See also

Barnes, M., Johnston, B. and Yasukawa, K. (1995) *Critical Numeracy*, a poster presented at the Regional ICME Conference, Monash University.

Gal, I. (1996) *Statistical literacy: the promise and the challenge* a paper given to The 8th International Conference Mathematical Education, Seville, Spain)

Jill Middleton's contribution *What Chance Do You Have?* in this very issue of *Circuit* exemplifies the power of such a critical approach to statistics in a social context.

Other Ways to Knot the Tie

Two researchers, Dr Thomas Fink and Dr Yong Mao, from the University of Cambridge's Department of Physics have used the theory of random walks to explore just how many ways there are to knot a tie. (*Nature*, vol 398, p 31). Prior to this and ignoring knot eccentrics who don't publish academic papers about their doings, there have been four traditional tie knots: the four-in-hand, the windsor, the half-windsor and the Pratt. The Pratt itself is a relative newcomer, having been introduced to the world on the cover of the New York Times in 1989.

Random walk theory was originally developed to investigate the movement of gas particles and is a tool of trade for Fink and Mao in their normal domain of protein folding, polymers and colloids. They realised that the mathematical notation used for random walks, which they also applied to protein folding, could describe knots as well. “We were surprised that only one new tie knot had been revealed in the last 50 years and thought we would try our hand at it”, says Fink.

Fink and Mao developed a notation to describe the sequence of folds to the left, right or centre, involved in tying a tie and plot the associated moves onto a triangular grid. A useable tie knot cannot be completely random either as Yong Mao explains, “If the end of the tie is moved to the right for instance, its next move cannot approach can also quantify attributes such as symmetry, breadth, and how difficult the knot is to unravel.

Fink’s favourite knot is one he labels (7,3). Below we show the sequence of moves for tying the (7,3).

So, if you are a tie wearer you might like to give the (7,3) a try and investigate other alternatives to your usual knot.

Based on *A Knotty Sartorial Question* from Pass Maths on the web and

Its style, but knot as you know it by Charles Seife, in ***New Scientist***, This Week, 6 March 1999.

Exact Calculation, Language and the Brain
Without speech, we can’t add up

Our brains can make rough mathematical calculations without language, but exact calculations need language skills, a new study suggests. The discovery may point to better ways of teaching maths.

Scientists have long wondered to what extent language is required for maths. Despite their lack of language skills, even monkeys and five-month-old infants have some ability to understand numbers. They gape in surprise when they see someone put two dolls behind a screen and then raise the screen to reveal only one doll (“It all adds up”, *New Scientist*, 7 March 1998,p 42).

Also, the brain centres that process numbers seem to be different for exact and approximate calculations. Some patients with strokes or brain damage have severe difficulty with language and exact calculations, while their ability to estimate remains intact.

Elizabeth Spelke and Sanna Tsivkin of the Massachusetts Institute of Technology investigated the role of language in maths by enlisting eight adults who spoke Russian as their native language and were fluent in English. To mimic the process children go through as they learn maths, the researchers taught the subjects complex arithmetic and unfamiliar approximations. Some exercises were taught only in English, others in Russian. The teachers Spelke and Tsivkin then tested the students by giving them problems to solve in both languages. When asked to make exact calculations (does 53 plus 68 equal 121 or 127?) the students took about a second longer to come up with the answer if the question was not asked in the language they had been taught in (*Science*, vol 284, p 970). But there was no language-dependent time lag when they were asked to approxi-mate (is 53 plus 68 closer to 120 or 150?).

As part of the same study, Stanislas Dehaene and his colleagues at the French medical research organisation INSERM and Frédéric Joliot Hospital in Orsay looked at brain images of people doing calculations. Exact calculations increased the activity of speech-related areas of the brain’s left frontal lobe, while estimates increased activity in the left and right parietal lobes. These regions help control hand and finger movements and, perhaps, counting on fingers. “I was amazed that the association could be so sharp”, says Dehaene.

The findings may have implications for maths teaching. “If a child begins learning arithmetic in one language, is there a cost if the language is switched later on?” asks

Spelke. Dehaene adds that even if children have severe language problems, they could still develop their numeracy by concentrating on approximate calculations.

Brian Butterworth at University College London, who studies how the brain handles mathematics, says the results are fascinating. However, he suspects the verbal and nonverbal parts of the brain don't work alone, but interact in complex ways. People often break down exact mathematical tasks, for example, turning nine plus seven into ten plus six. "There would be big overlaps", Butterworth predicts.

Word Power by Nell Boyce
in *New Scientist*, This Week, 15 May 1999

The Real Stuff of Numeracy

... that, after all is what really matters - that pupils emerge with an interest in mathematics, a confidence to talk about it, and a willingness to engage with mathematics wherever and whenever it crosses their paths. At least Plutarch had the right idea. Two thousand years ago he saw that "the mind is not a vessel to be filled, but a fire to be kindled." Is the current round of mathematics curriculum change in Australia, being done in the name of numeracy, actually addressing what really matters?

Inspired by **All fired up** by Jonathon Osborne,
in *New Scientist*, Forum, 3 April 1999.

The Mathematically Ubiquitous Pope

It is said that the philosopher McTaggart once challenged Bertrand Russell to use the statement '1 = 2' to prove that he was the Pope.

This presented no problem for Russell: 'I and the Pope are two. But 1 = 2 so I and the Pope are one'.

Quoted by T. W. Körner in
The Pleasures of Counting

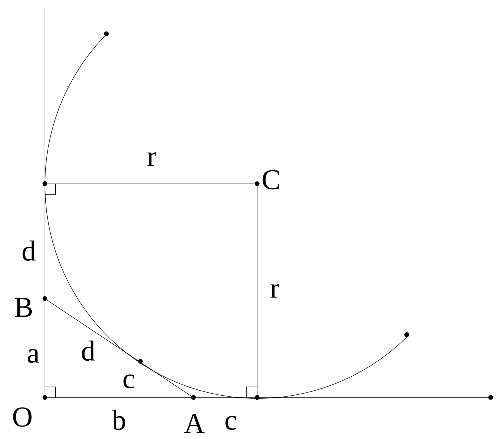
Some say the Pope is the greatest cardinal. But others insist this cannot be so, as every Pope has a successor.

From the web at geocities.com

PROBLEMS AND ACTIVITIES

Remember that we include a coding system which attempts to indicate in terms of Year levels the suitability range for each item. Thus 6 - 8 suggests an item accessible to students from Year 6 to Year 8.

CMA member George Harvey suggests a simpler solution to the **Sum Diameter** problem, published as Problem 3 in the March 1999 *Circuit*. The fact that both tangents from an external point to a circle have the same length enables us to mark lengths as in this diagram.



Hence the perimeter of ΔOAB

$$\begin{aligned} &= a + b + c + d \\ &= a + d + b + c \\ &= r + r \\ &= 2r. \end{aligned}$$

George Harvey also proposes an alternative solution to Problem 5 **Really The Largest Number**, in the same Circuit. His idea is essentially to treat equation (1) given in the solution there as the quadratic in y

$y^2 + (z - 5)y + (z^2 - 5z + 3) = 0.$

In this equation y is real if and only if
$(z - 5)^2 - 4(z^2 - 5z + 3) \geq 0.$

So y is real if and only if
$3z^2 - 10z - 13 = (3z - 13)(z + 1) \leq 0,$

and the greatest value z can take is
$z = \frac{13}{3}$

Thanks, George!

---

(1) "How Old Are You?" Times Two

6 - 10

(a) Provided that you are at least ten, if you tell me the result when you subtract the product of any one-digit number and 9 from ten times your age, I can work out your age.

How do I do it? Your age is just the sum of the last digit of your result and what remains after this digit has been removed. For example, if you are 13 and you choose the product 9 × 6 the process gives a result of 130 – 54 = 76 and I obtain your age as 7 + 6.

Explore this process and try to explain how it works.

(b) Now for a different process which, provided you co-operate, enables me again to work out your age, this time for under 10 too: Double your age, add 5, multiply by 5 and tell me the answer.

To find your age I remove the last digit (always 5) of your answer, then subtract 2 from what remains.

For example, if you are 8,

$8 \times 2 + 5 = 21$ and $21 \times 5 = 105$

so removing the 5 leaves 10

and 10 – 2 = 8.

Familiarize yourself with this process and explain why it works.

Note: Both these problems have been adapted from *The Moscow Puzzles* by Boris A. Kordemsky, translated into English by Albert Parry and edited by Martin Gardner. This classic collection of 359 mathematical recreations is well worth a look if you can lay hands on a copy.

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(2) Molecular Maths

9 - 12

A Buckminsterfullerene (buckyball for short) is a 60-atom carbon molecule only discovered in 1985 and named after pioneering US designer R. Buckminster Fuller. Imagine a model of this molecule made by connecting together the 60 carbon atoms to form a solid whose surface resembles the patchwork of a soccer ball. Each of the 60 atoms (vertices) in the model is connected to its neighbouring carbon atoms by edges representing chemical bonds. Each vertex is surrounded by two hexagons and one pentagon.
How many edges does the model have?

---

(3) Array

11 - 12

4	7	10	13	16	19	…
7	12	17	22	27	32	…
10	17	24	31	38	45	…
13	22	31	40	49	58	…
16	27	38	49	60	71	…
19	32	45	58	71	84	…
⋮	⋮	⋮	⋮	⋮	⋮	⋮

The entries in each row, and hence each column, of this symmetric array form an arithmetic sequence.

Prove:

(a) If a positive integer k is in the table, then 2k + 1 is composite.

(b) If k is not in the table, then 2k + 1 is prime.

(4) A Feynman Exercise in Algebra

11 - 12

The formula

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \qquad (1)$$

which connects the focal length f, the object distance s and the image distance s´ for an image formed by a thin lens is derived in that part of physics which deals with geometric optics. As Richard Feynman observes in *The Feynman Lectures in Physics (Volume 1)* page 27-6, the formula

$$xx' = f^2 \qquad (2),$$

with $s = x + f$ and $s' = x' + f$ is equivalent to formula (1) and much easier to work with.

Show algebraically that formulas (1) and (2) are equivalent.

(5) Calendar Capers as the Millennium Cometh

8 - 12

These problems have been adapted from some in Chapter 11 of T. W. Körner's 1996 book *The Pleasures of Counting*. They depend on the following empirical result, based on hundreds of years of astronomical observations:

1 year = 365.2422 days.

We point out that this relationship is really a statement about averages, that is 'year' should be interpreted as 'average year' and 'day' as 'average day', but we use it as a base for the calculations to follow.

(a) The Julian Calendar

This was introduced by Julius Caesar. The year is taken to have 365 days except that all years divisible by 4 have 366 days (so are leap years).

Show that the Julian calendar gives one day too many roughly every 128 years. (Thus after 128 years an event which should occur near midnight on 2 January will occur near midnight on 1 January.)

(b) The Gregorian Calendar

By 1588 the discrepancy in the Julian calendar had grown to 10 days, so Pope Gregory XIII introduced a new calendar. Ten days were omitted and Thursday 4 October 1582 was followed by Friday 15 October 1582. The new Gregorian calendar differed from the Julian calendar in that years divisible by 100 (centennial years) were to be leap years only if divisible by 400. We still use the Gregorian system, so next year is a leap year.

Show that the Gregorian calendar will have one day too many roughly every 333 years.

(c) A Soviet Calendar

In 1923 The Soviet Union adopted a new calendar which follows the Julian except that a centennial year is a leap year only if it is n × 100 where n leaves remainder 2 or 6 when divided by 9.

- **(i)** Show that the Soviet calendar will have one day too many every 4500 years.
- **(ii)** What is the first year when the Soviet and Gregorian calendars will differ?

(d) Monday Magic

Show that for the Gregorian calendar the first or last day of every odd-numbered century is a Monday.

This Monday problem, in which we have slightly modified the wording, comes originally from W. W. Rouse Ball's *Mathematical Recreations*.

THUMBNAIL (and longer) REVIEWS

Readers are welcome to contribute to this section. Reviews can cover books, periodicals, videos, software, CD ROMs, calculators, mathematical models and equipment, posters, websites etc.

NOVA Website
www.science.org.au/nova

This review looks at the four mathematics topics currently accessible on the above website. (See page 4 of this Circuit for more background on this site.) Readers should bear in mind that many of the other topics on this site, covering science, health and the environment, also have relevance for contemporary mathematics learning and teaching.

The four topics are: *When the numbers just don't add up*; *De-bugging the millennium*; *Measurement in sport - the long and the short of it*; *Calculating the threat of tsunami*. The exciting thing about the NOVA approach to content is that mathematics is a natural and integral part of each topic. A variety of situations, facts, examples, terminology, notation, key questions, issues and implications is discussed and addressed for each topic, mostly in a qualitative way. Depending on their motives and the nature of their students, teachers are then in a position to follow through suggested activities, links and whatever other activities spring to mind.

Each topic has web pages covering key text, glossary, activities, further reading and useful site links. *Measurement in sport - the long and the short of it*, exploits the coming Sydney Olympics and the Aussie preoccupation with sport to motivate the presentation of its information. The two strands cover the measurement processes used to determine the outcomes in many sporting events, and the way measurements regulate and define the very nature of particular sports. So not only do we need to know which runner was fastest, we must be sure that the running track complies with a host of specifications, and we must take account of wind (if any) assistance, etc. Measurement technology, reaction times, sports physics and the elasticity of rackets and balls (new materials) are just some of the areas traversed in this topic.

When the numbers just don't add up deals with the many ways in which numbers can be used to fudge an argument. (See also *Statistics for Active and Critical Citizens* earlier in this Circu*it*.) "A recent government report showed that the area of forest in Australia had increase from about 43 million hectares in 1992 to just under 157 million hectares in 1998." How? The definition of forest has changed. Various other standard ways to fudge arguments with numbers are exemplified. The message is that it pays to be wary when presented with numerical information.

Consider, question, be sceptical! Is more information needed? Is the situation being misrepresented? Do the numbers make sense? And the links to related sites are worth browsing.

De-bugging the millennium examines the basis of the Y2K problem, its possible consequences, and what is being done to forestall the crunch. Should we survive 1/1/2000, interest in this topic will probably collapse, but meanwhile the discussion here is succinct, with links to several other informative sites.

Calculating the threat of tsunami covers some of the history and science of these giant destructive waves, such as the ones which destroyed villages near the Sissano lagoon on the north coast of Papua New Guinea last July. “Calculating the velocity, wave height and destructive force of a tsunami for any stretch of coastline is complicated...the shape of the sea bed can produce effects that might not be predicted by a simple wave equation...harbours and headlands...cause the waves to reflect, diffract and refract, changing their direction...” Again there are interesting links, for example to material dealing with the disappearance of the Minoan civilisation from Crete about 3500 years ago, and to efforts to set up a Pacific Tsunami Warning System.

My verdict is that NOVA is a valuable new mathematics resource for teachers and students ranging across most age groups. The material is authoritative, intrinsically interesting, well presented, and shows that mathematics is embedded all around us. With more topics to come, this site should repay regular future visits. The challenge for governments is to fund public education so that all schools have access for staff and students to websites such as this.

Peter Enge

The Pleasures of Counting

by T. W. Körner
Published by Cambridge University Press, 1996

Körner, of Trinity Hall Cambridge, is the son of S. Körner who wrote on the philosophy of maths 40 years ago. In this case, like father like son, as this book is full of the best sort of philosophy and mathematics: situations, perspectives and ideas which engage interest and enthusiasm and are self sustaining. His book is aimed at all those (14 and over) who “would like to learn something of what (mathematics) looks like at a higher level.”

He intends it as ‘maths for mathematicians’ “who know very little mathematics as yet, but who, perhaps, will one day give lectures, as well as his fellow professionals” and “general readers who value maths without fearing it.” However even for those in the intended audience, Körner does not necessarily expect understanding of every detail. “Professional mathematicians consider a maths book worthwhile if they understand something new after reading it and excellent if they understand a fair amount that is new to them; anything more would lead them to suspect that the material was too easy to be worthwhile.” The book comes with exercises which range from commentaries and structured explorations of matters raised in the text to others requiring content met at the beginning of a uni course, and all the exercises are clearly signposted.

Quoting Montaigne, Körner describes his book as a “bunch of other men’s flowers”, his contribution having been to furnish the string holding them together. Of course he hopes these flowers will entice the reader to explore at least parts of the garden from which they come for herself.

The book’s five parts are titled “The uses of abstraction”, “Meditations on measurement" (Galileo, dimensional analysis, the Lorentz transformation and special relativity, the contribution of Quaker mathematician L. F. Richardson to meteorology, statistics of quarrels, defining frontiers), "The pleasures of computation" (Euclid's algorithm; net-works, sorting, Turing's theorem and several references to Donald Knuth), "Enigma variations" (the Poles, Bletchley, hard problems and Shannon's theorem), "The pleasures of thought" (growth and decay for various populations, a Greek maths lesson à la Socrates counterpoised with a Trinity Hall maths lesson, a mathematical career).

Throughout Körner does not talk down to the reader, but shows how mathematicians engage a huge range of problems, situations and ideas, from the practical to the theoretical. He has a gift for anecdote and humorous, off-beat quotations. The book includes an Appendix covering further reading with detailed comments on tens of books, a summary of notations, sources page by page, a 259 work bibliography and a useful index with topics such as the causes of World War I.

A most valuable source of mathematical inspiration and material useful for broadening out college or tertiary level courses through self contained reading activities, research, exploration of mathematical applications and satisfying exercises.

Peter Enge

Peanut Software
(It costs peanuts - nothing!)

by Rick Parris
www.exeter.edu/~rparris

Rick Parris is a teacher in the US. He has written a suite of some of the best mathematics programs available for Windows, released them free of charge, and continues to update and improve them? All of Rick's software may be accessed and downloaded directly from
http:/www.exeter.edu/~rparris

or through the AAMT home page,
http://www.AAMT.edu.au

(Free software for Macintosh is also available at this site).

The nine programs in the Peanut Software collection are recent updates of older DOS-based applications. The quality is very high and all programs are works in progress for which Parris issues upgrades and welcomes suggestions for improvements and repairs. For example, *WinPlot*, the graph plotter offers far more features than commercial products selling for around $150, while the dynamic geometry tool, *WinGeom*, will give *Cabri* and *GSP* a run for their money. The diagrams for Circui*t* Problems and Activities are created using *WinGeom*, as were the front cover and inside front cover CMA logos of a Hamiltonian circuit on a dodecahedron.

WinStat offers a useful data, statistics and probability workbench. It includes histograms, stem and leaf and box-plots, plus a variety of simulations for probability experiments. *WinLab* includes eight subprograms, ranging from conic sections and star polygons to a three body simulator. *WinArc* offers challenging and colourful problem-solving game situations including Life, Boxes, Hex, Mazes, Peg Solitaire and Rubik's cube. Each offers features immediately applicable to high school and college mathematics.

The other programs: *WinDisc* for discrete mathematics topics - networks and vertices; *WinMat* for matrix operations, *WinCalc* - a powerful calculator; and *WinFeed* for chaos and fractals may not fit directly into the traditional curriculum, but all offer features appropriate for mathematics.

At this point, it is necessary to remind the reader of two things: each of these outstanding programs is **FREE**; and the entire collection (compressed form) fits onto a single high-density disk!

The implication of these two factors is that they allow schools to send the software home for students to use, so we must now teach with the knowledge that such facilities are freely available and accessible to our students. What this will mean for homework alone, remains to be seen without even considering the scope for mathematical explorations and open-ended assessment tasks. Whatever else you do, have a look at Peanut Software.

> A much abbreviated, edited version of *Software for Peanuts* by **Stephen Arnold**, *DELTA,* Volume 38 Numbers 1 and 2, 1998, The Journal of the Mathematical Association of Tasmania.

Who is Rick Parris?

Here, in his own words, Rick explains a little about himself, how he came to write his software and why he called it Peanut.

'I teach mathematics at Phillips Exeter Academy in Exeter, New Hampshire, USA. This is a private boarding secondary school that was founded in 1781. The ages of my students range from 14 to 18 years. They come from all over the world, although most come from the USA, and most of those from the north-east part. Their goal is to attend university after graduating, so they are all highly motivated.

I have taught here for 20 years, and taught university for nine years before that.

My original career plans after I graduated from Tufts University in 1967 were to become a college mathematics professor, so I went to graduate school at Princeton. The Vietnam war messed up the plans of many of us grad students, so I began my teaching career early (in 1968, after one year in grad school), to avoid being handed a rifle.

After teaching for two years at small colleges near Boston (my birthplace in 1945), I found my way back to New Jersey and taught at Rutgers (Newark) for four years while trying to get my graduate studies going again.

That didn't work, but I was able to return to Princeton full-time in 1974 once the war had subsided, finished my dissertation (in 3-dimensional topology) in 1977, while teaching Differential Equations to Princeton engineers for three years.

By the time I had finished up at Princeton, I had decided that I would rather work with younger students. I was also tired of lecturing to 100 students at a time, and did not have much interest in spending my life trying to publish papers that no one was going to read.

Thus began my secondary school career - first at the nearby Lawrenceville School (NJ), then at Exeter.

A few of my students here got me interested in technology back in the early 1980s, and I began to write my own graphing routines for use on our DEC minicomputer. By the time I had bought my first PC in 1985, I was hooked on writing my own stand alone programs (in Pascal), there being no software of quality available back then. (I've always liked doing things myself, anyway.)

In particular, I wrote my geometry program (called Geom) then, having heard of (but not seen) the Geometric Supposer programs written by Judah Schwartz for the Apple II. (He gave me some good advice when we finally met and talked shop.)

When I made the jump from MS-DOS to Windows a few years ago, Geom become *Wingeom*, my Plot program become *Winplot*, Stats became *Winstats*, and so forth. By the way, I regard the Geometer's

Sketchpad as a very good (but flawed) imitation of my geometry program!

I used to distribute my MS-DOS programs via mailers and diskettes all over the world, but the advent of Web pages has mercifully put an end to that! Now I can just maintain my programs, upload them to my page, and let the Internet do the rest.

The programming is really just a hobby that has gotten out of control. My real job (for which I am paid) is to teach mathematics, supervise a dormitory of about 32 teenage girls, and coach various sports - chiefly long-distance running in the fall, squash in the winter, and softball (baseball) in the spring.

I have a wife (who also teaches at the same school) and two daughters, who all put up with my idiosyncrasies, and who have been enormously helpful in letting me do my job (which is very time-consuming when school is in session).

Several years ago, the software librarian at our Technology Conference felt it would improve her cataloguing system if my software could be grouped under a 'company name', so I invented one. Our school is known as 'PEA' (Phillips Exeter Academy), and the last three letters of the name, well ....

I am a bit eccentric!'

Also reprinted from *DELTA*,
Volume 38 Numbers 1 and 2, 1998.

SOLUTIONS TO PROBLEMS AND ACTIVITIES

(1) (a) Note that the product of any one-digit number d and 9 always differs from the next multiple of 10 by exactly d. If we transform the step in the example discussed in the problem statement

$$\begin{array}{r} 130 \\ -\underline{\;\;54} \\ 76 \end{array} \qquad \text{into} \qquad \begin{array}{r} 130 \\ -\;\;60 \\ +\;\underline{\;\;\;6} \\ 76 \end{array}$$

we see that the subtraction of 54 from 130 is equivalent to subtracting 6 tens and adding 6 units to 130.

The final step $\begin{array}{r} 7 \\ +\;\underline{\;\;6} \\ 13 \end{array}$

guarantees recovery of the original age because we first subtracted 6 from it (the 6 in the 60 above), leaving 7 in the tens column, then added this same 6 back to the 7, restoring it to 13.

The algebraic version of this argument follows. Let your age be the integer a ($a \geq 10$), and let d be the single digit you choose to multiply by 9.

Then
ten times your age – 9d

$$= 10a - 9d$$

$$= 10(a - d) + d$$

Since $a \geq 10$ and d is a digit, d is the units digit of $10(a - d) + d$. Hence the sum of this last digit and what remains when it is removed is $a - d + d = a$.

(b) We leave the relatively straight forward explanation of this process to you.

(2) Let E be the number of edges in the buckyball model. Since each of the 60 vertices is the meeting point of 3 edges and since each edge joins 2 vertices,

the number of edge ends $= 3 \times 60$

$= 2E$,

so E = 90.

---

(3) The n th term in rows 1,2, … ,m is easily found to be

$$\begin{aligned} a_{1n} &= 3n + 1, \\ a_{2n} &= 5n + 2, \\ a_{3n} &= 7n + 3, \\ a_{4n} &= 9n + 4, \\ &\dots \\ a_{mn} &= 2mn + m + n \\ &= k \text{ say.} \end{aligned}$$

(a) If k is in the table, then

$$2k + 1 = 2(2mn + m + n) + 1$$

and factorising gives

$$2k + 1 = (2m + 1)(2n + 1),$$

i.e. if k is in the table, then 2k + 1 is composite.

(b) If 2k + 1 is composite, then

$$2k + 1 = pq \quad (p,q \geq 2).$$

But 2k + 1 is odd, so pq is odd and both p and q are odd.

Let p = 2m + 1, q = 2n + 1,

then 2k + 1 = (2m + 1) (2n + 1).

This gives k = 2mn + m + n and hence k is in the table.

Thus we have shown that
if 2k + 1 is composite then k is in the table (*)

Hence if k is not in the table, 2k + 1 cannot be composite and so must be prime.
(The statement "if k is not in the table then 2k + 1 is prime" is the contrapositive of statement (*).

---

(4) The formula $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ (1)

can be written $\frac{s + s'}{ss'} = \frac{1}{f}$,

that is $ss' = f\,(s + s')$.

Rearranging this last formula and factorising gives

$$(s - f)(s' - f) = f^2,$$

that is $xx' = f^2$ (2)

By reversing the order of these steps, we can start with $xx' = f^2$ and deduce that

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}.$$

This establishes that formulas (1) and (2) are equivalent.

Obviously analogous pairs of formulas result from other "sum of reciprocals" relationships which occur, for example, in electrical work for resistors in parallel

$$\frac{1}{r} + \frac{1}{r'} = \frac{1}{R}.$$

and for capacitors in series

$$\frac{1}{c} + \frac{1}{c'} = \frac{1}{C}.$$

---

(5) **(a)** We can think of the Julian calendar year as having 365.25 days, since there is one leap day every 4 years. So the Julian calendar overestimates the length of the year by 365.25 – 365.2422 = 0.0078 days.

As a consequence the Julian calendar gives one day too many every $\frac{1}{0.0078} \approx 128$ years

(b) The Gregorian calendar gives 97 leap days every 400 years, so the Gregorian year has $365\frac{97}{400}$ days. Since $\frac{97}{400} = 0.2425$, this amounts to overestimating the length of the year by 0.0003 days, and gives rise to one day too many every $\frac{1}{0.0003} \approx 3333$ years.

(c) (i) The twenty natural numbers which leave remainder 2 or 6 when divided by 9, leave the same remainders on division by 90 (i.e. mod 90) as 2, 6, 11, 15, 20, 24, 29, 33, 38, 42, 47, 51, 56, 60, 65, 69, 74, 78, 83, 87. So every 9000 years the Soviet calendar gives

$2250 - 90 + 20 = 2180$ leap days,

and we can think of the Soviet calendar year as having

$365\frac{2180}{9000} = 365\frac{218}{900}$ days.

But $\frac{218}{900} - \frac{2422}{10000} = \frac{1}{45000}$, and the Soviet calendar gives one day too many every 45000 years.

(ii) The year 2800 is the first centennial leap year in the Gregorian calendar which is not a centennial leap year in the Soviet calendar.

(d) Our solution begins by pointing out the serendipitous fact that 1 January 2001 is a Monday. In other words the first day of the first week of the first month of the first year in the new millennium is a Monday - a significant or trivial observation, depending on your taste.

To keep track of which day of the week corresponds to a given date we work with remainders after division by 7 (modulo 7 arithmetic). We take Monday to be day 0 of the week, Tuesday to be day 1 of the week, … , Sunday to be day 6 of the week.

In the Gregorian calendar the century which begins on 1 January 2001 and ends on 31 December 2100, as with all centuries not containing a year which is a multiple of 400, has 24 leap years and 76 365-day years. Hence, over the course of such a century, the day count advances by

$$\begin{aligned} 24 \times 366 + 76 \times 365 &\equiv 3 \times 2 + 6 \times 1 \pmod 7 \\ &\equiv 5. \end{aligned}$$

Since
1 January 2001 is day 0, a Monday;

1 January 2101 is day $0 + 5 = 5$, a Saturday;

1 January 2201 is day $5 + 5 \equiv 3 \pmod 7$, a Thursday;

1 January 2301 is day $3 + 5 \equiv 1 \pmod 7$, a Tuesday;

and
31 December 2300 is day 0, a Monday.

Remembering that 29 February 2400 is a leap day,
1 January 2401 is day $1 + 5 + 1 \equiv 0 \pmod 7$, another Monday.

Thus the first day of the 21st century, the last day of the 23rd century and the first day of the 25th century are all Mondays. Clearly this pattern continues back before the year 2000 and into the future beyond the year 2500. This is a consequence of knowing any one assigned date in the Gregorian calendar and the fact that the Gregorian calendar cycle repeats every 400 years.