SHORT CIRCUIT

Business Name

VOLUME 17 NUMBER 1 JANUARY 2026

NEWS AND COMMENT

The crew at Short Circuit and the 2026 CMA committee wish our readers a happy new year.

Many of you will be away from your inbox as this edition goes out. It will be waiting for your return from what we hope has been a refreshing break. Others, technologically connected, will have immediate access to Bruce Ferrington's piece on page 4 on the potentials in education to

> … *[encourage] our students to find their own mathematical connections in art, in music, in sport and science, in language and in history, in skateboarding and in scuba diving.*

Some will choose to dive first into the article on page 2 from Ed Staples about an apparently new discovery in triangle geometry. Something like it happened in 1899 with Morley's theorem, and again years later with an extension to that theorem involving the external angles of a triangle.

But a discovery from mathematics teacher Nadia Laklalech is distinct from either of these, being about a triangle's altitudes rather than its angle trisections.

We include on page 7 an extract from a journal for students published by the Mathematical Association of South Australia: *South Australia's Future Mathematicians*, July 2001. It explains, without proofs, what Morley's theorem and its extension are about.

The CMA annual conference is set for Saturday 28th March. Contact Valerie Barker through the CMA email address if you would like to be a presenter.

CANBERRA MATHEMATICAL ASSOCIATION

NEWSLETTER

The CMA newsletter, Short Circuit, is distributed monthly to everyone on our mailing list, free of charge and regardless of membership status.

That you are receiving Short Circuit does not imply that you are a current CMA member but we do encourage you to join.

Short Circuit welcomes all readers.

CMA MEMBERSHIP

Memberships run from 1 Jan to 31 Dec. each year. Membership forms may be downloaded from the CMA website:
http://www.canberramaths.org.au

The benefits of Membership of CMA may be found on the website.

Inside:

Puzzles—p.6

CMA council 2026—p.5

Contributed articles—pp.2,4

A DISCOVERY

From Ed Staples

It seems that just about everything there is to know about a triangle has been documented, at some time or another, by someone somewhere. But quite recently a maths teacher, Nadia Laklalech, shared a story with three people (including myself) collaborating on LinkedIn.

Nadia, who lives and teaches in France, is an enthusiastic and talented mathematics teacher who has, over the time I've been involved with the group, had a special interest in geometric constructions and proofs.

Recently while cleaning out and reviewing old university material, Nadia found a line she'd written in her notes. She shared her story with us:

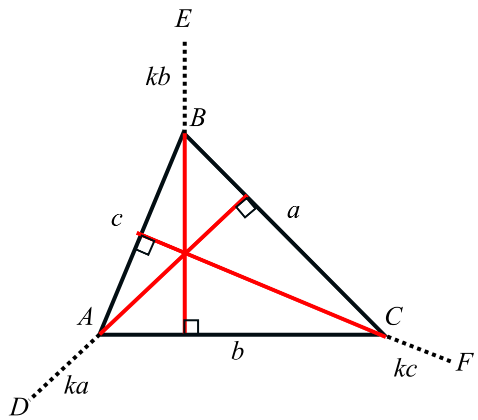
> *I started from a sentence that I had written down by hand a long time ago in my old notebook. It read 'There is another variant of Morley's theorem using extensions of heights and proportionality with 'faces'…*

She didn't quite know what to think about the statement, but remembered Morley's theorem, the one that shows that the trisections of the three angles of any triangle result in an equilateral triangle being formed around the triangle's centre. It's quite a mysterious result in that it applies to any triangle at all. As to the idea of 'extensions of heights and proportionality of faces' Nadia could not quite remember what that meant, but it was enough to motivate her to play around.

Andrew Wiles, the mathematician who proved Fermat's Last theorem, once stated in part that 'doing maths is like entering a dark mansion – you go into the first room …you stumble around, bumping into furniture, gradually you learn where each piece of furniture is…and finally after 6 months or so, you find the light switch and turn it on…'.

In Nadia's play, she stumbled around for a while, drawing triangles and 'height lines' (altitudes as they are more commonly known) but couldn't quite make sense of the connection between height extensions, 'faces' and Morley's theorem.

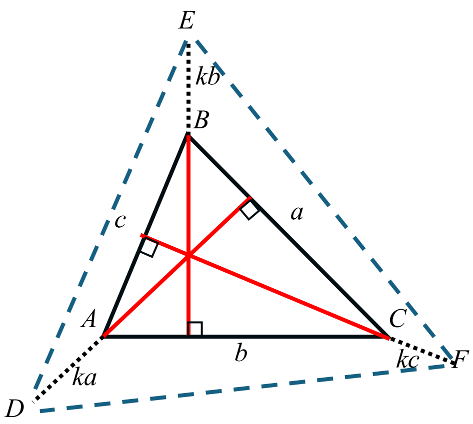
At some point in her play, she struck upon the idea of drawing any scalene triangle, say triangle ABC with sides a, b, and c, extending the altitudes h_1, h_2 and h_3 by some constant factor, say k, of the side length it was perpendicular to. Specifically, the altitudes were extended by the amounts ka, kb and kc to new points that she labelled D, E and F (see figure 1).



(figure 1)

The value of k chosen in figure 1 is clearly less than 1(but it need not be) and the length of the extensions are proportional to their respective triangle sides. It was a diagram like this that alerted Nadia to the possibility of constructing another triangle with D, E and F as its vertices as shown in figure 2.

As Nadia stared at the figure, she began to wonder whether the new triangle was equilateral (note that something was mentioned in the university notes about an equilateral triangle). Being proficient with the dynamic software *GeoGebra*, Nadia hastily constructed figure 2, and began to play with k as a variable.



(figure 2)

To her surprise, there was one and only one value of k that produced an equilateral triangle. According to *GeoGebra*, the value was approximately 0.5773502692. She recognised that the true value was almost certainly $\tan 30°$ or $1/\sqrt{3}$.

GeoGebra allows just about anything to vary, and so, at $k = 0.577352692$, Nadia's next step was to shift the vertices of the original triangle around. Astonishingly, the new triangle (subsequently christened the Laklalech triangle) appeared to remain equilateral!

But why? Nadia began searching the net. Surely such a mathematical diamond would have been discovered by someone, at some stage, somewhere? Alas, there was nothing there, nothing at all.

Nadia's next step was to formally prove that, for any triangle ABC with sides a, b, and c, not all equal, extensions to the altitudes of length of ka, kb and kc through the vertices will create a Laklalech triangle DEF if and only if $k = 1/\sqrt{3}$. (Note that it makes no difference if the original triangle is obtuse-angled). Of course, when the original triangle is equilateral, any non-zero k will work.

After a few days it was done, and in two ways - the first using Euclidean geometry and the second using complex numbers. For anyone interested in seeing these proofs, please contact the editor.

As a final note, it is relatively easy to prove, using coordinate geometry techniques, that the value $k = 1/\sqrt{3}$ creates the Laklalech triangle (of course that's only half of what is required). Any triangle can be represented with vertices in the form of A (0,0), B (p,q) and C $(r,0)$. If we apply altitude extensions of lengths

$$|AD| = \frac{1}{\sqrt{3}}|BC|,\ |BE| = \frac{1}{\sqrt{3}}|AC| \text{ and } |CF| = \frac{1}{\sqrt{3}}|AB|$$

then it turns out that the coordinates of the Laklalech triangle are given by

$$D\left(-\frac{q}{\sqrt{3}}, \frac{p-r}{\sqrt{3}}\right), E\left(p, q+\frac{r}{\sqrt{3}}\right), F\left(\frac{r+q}{\sqrt{3}}, -\frac{p}{\sqrt{3}}\right)$$

Expressions for distances can be found that will verify that triangle DEF is indeed equilateral.

CONFERENCE 2026

Save the date: the 2026 CMA Annual Conference will be held at ADFA on Saturday 28th March. The theme is *Mathematics: Are You Game?*

If you have contributions of ideas, experiences or research to share at the conference, contact Valerie Barker via canberramaths@gmail.com.

THREE QUESTIONS

From Bruce Ferrington.

I have three questions that I like to use in the classroom to challenge students to think creatively in Mathematics.

Question 1 – *Is there another way?*

Once you have a solution, can you find other ways to do it? Can you look at your process from a different perspective or use a different method? Asking students to do this means that I need to have posed the original question or problem in a way that is open to multiple solutions or can be solved using different approaches. It means asking the right questions in the first place. It means expecting different answers as well as different procedures.

Question 2 – *What connections can you see?*

I want to encourage my students to look for these connections, to find links to their previous learning, to find links to the real world and to find applications for what they have discovered. A big part of this step is to draw on the use of imagery and metaphor in the language of maths. I want to hear my students say things like, 'The equals sign is like a man on a tight rope – whatever is on one side has to balance what is on the other side'. Find a picture. Make a picture. Describe a picture. Represent this abstract idea with something real. Use language to tell me the story.

Question 3 – *Is that always true?*

Once we have a solution, or many solutions, I want to find a rule, a proof, a generalization that we can refer back to in the future. And once we have these rules and generalisations, can we find any exceptions? Does the rule hold true in all contexts? What about with negative numbers? What about with fractions? What about with algebra?

These three simple questions help me to get my students to start thinking differently about the mathematical process. And by sowing these seeds of thought, we can anticipate a harvest of action.

Creativity – the putting together of two things to make a new thing.

In Mathematics we call this *addition*. Or if one of the things has a negative value, we call it *subtraction*. Or if we keep putting those things together multiple times, we could call it *multiplication*. Or if we…

This process of creativity – putting things together to make a new thing – is embedded in the nature of Mathematics. It is no surprise then that the story of mathematics is rich in discoveries resulting from this creative process.

Archimedes noticed the water overflow when he lowered his body into the bath tub. 'Eureka!' he proclaimed, understanding that the displacement of the water must be equal to the volume of the object being submerged [and the buoyant force exerted on the object was equal to the weight of the displaced water].

Fibonacci looked at a pair of breeding rabbits and saw an astonishing sequence of numbers which now bears his name.

Sir Isaac Newton was awoken from his daydreaming by an apple falling on his head, leading to a sudden epiphany on the natural force called gravity.

We should be encouraging our students to find their own connections, creatively bringing disparate elements together to make new and exciting discoveries and understandings. And we can start this process by having our students stand up and walk out of the classroom door and into the world where they live. For it is in the real world that they will find the real-world questions and learn that there is no 'one size fits all' answer.

We should be encouraging our students to find their own mathematical connections in art, in music, in sport and science, in language and in history, in skateboarding and in scuba diving. And in encouraging this search for connection, education can truly start to become the process of liberation.

ABOUT THE CMA

The Canberra Mathematical Association (Inc.) is the representative body of professional educators of mathematics in Canberra, Australia.

It was established by, among others, the late Professor Bernhard Neumann in 1963. It continues to run - as it began - purely on a volunteer basis.

Its aims include

- the promotion of mathematical education to government through lobbying,
- the development, application and dissemination of mathematical knowledge within Canberra through in-service opportunities, and
- facilitating effective cooperation and collaboration between mathematics teachers and their colleagues in Canberra.

NEWSLETTER OF THE BUSINESS NAME INC

PO Box 3572
Weston ACT 2611
Australia

E-mail: canberramaths@gmail.com

We're on the Web!
http://www.canberramaths.org.au/

THE 2026 CMA COMMITTEE

President	Aruna Williams	Erindale College
Vice President(s)	Bruce Ferrington,	Radford College
	Peter McIntyre	University of NSW Canberra
Secretary	Valerie Barker	
Treasurer	Jane Crawford	Covenant Christian School
Membership Sec.	Paul Turner	
Councillors	Theresa Shellshear	Australian Catholic University
	Heather Wardrop	
	Andrew Wardrop	
	Yuka Saponaro	Melba Copland Secondary School
	Bernadette Matthew	Mother Teresa School
	Linda Goth	
	Katrina Simms	

Theresa Shellshear is CMA's COACTEA representative.

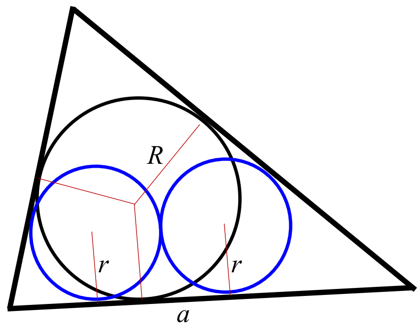
Bruce Ferrington is CMA's AAMT representative.

Short Circuit is edited by Paul Turner. **ISSN 2207-5755**

Find us on Facebook

http://www.facebook.com/pages/Canberra-Mathematical-Association/110629419011275

PUZZLE



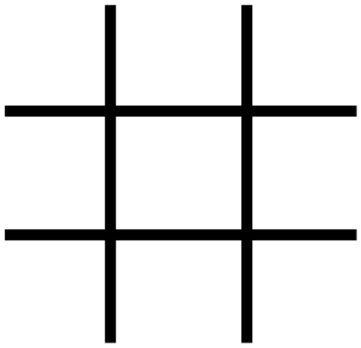
A triangle has incircle with radius R. Two smaller circles, both with radius r, are drawn inside the triangle so that they just touch one another, and the sides are tangent to them. The side that is tangent to both small circles has length a.

Discover an equation that relates the quantities r, R and a.

GAME

Math-Mat-Tic

From “Maths Games” by Chris Betts (Landmark Press: 1993)



This game is played on a normal noughts and crosses grid but numbers are used instead of noughts and crosses symbols.

Only the digits from 1 to 9 can be used and each can only be used once.

The winner is the first person to make three in a row add up to 16.

Supplementary Rule – You cannot put 9 in the centre position. (Why is this an important development in the game?)

THE MIRACLE OF MORLEY"S THEOREM

Dr. Branislav Cabric and Despot Stojanovic
Faculty of Sciences, Kragujevac, Yugoslavia

Morley's Theorem

In 1899, when almost all facts about significant points, lines and circles associated with triangles seemed to have been revealed, Morley discovered a new theorem concerning the trisectors of the angles of a triangle.

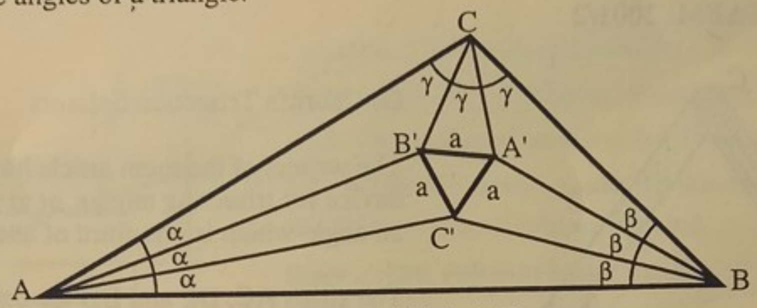


Fig 1. Illustration of Morley's Theorem

Morley states that: If, in any triangle ABC, each interior angle is trisected, these trisectors always meet at the vertices of an equilateral triangle, the *Morley triangle*, A'B'C' in the diagram above. [4,5]

We found that if we made an inflexible hard-wire model of ΔA'B'C' and flexible thread models of Δs ABC', AB'C and A'BC, we obtained a mechanical model of Morley's theorem, which at the same time could be used as an instrument for trisecting angles. We have called this a 'trisector net'.

Moreover, we extended our study to the exterior angles of ΔABC, (see Fig. 2 on the next page), using the computer program *CorelDRAW*. Again, the three intersection points always formed an equilateral triangle. This result we have called *the expanded Morley theorem*.

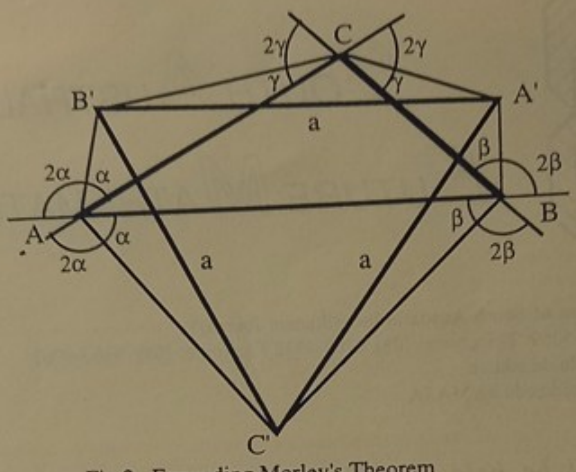


Fig 2. Expanding Morley's Theorem

The Expanded Morley Theorem states that: In any triangle ABC, the trisectors of adjacent exterior angles always meet at the vertices of an equilateral triangle, the *Exterior Morley triangle*, A'B'C' in the diagram above

Fittingly, this was discovered exactly one hundred years after Morley's original theorem.

References

(4) Newman, D.J., *The Morley Miracle* The Mathematical Intelligencer, **18**, 31 (1996).

(5) Dudley, U., *A Budget of Trisections* Springer-Verlag, New York, 1987

(6) Dudley, U., *The Trisectors* The Mathematical Association of America, Washington, 1994.