SHORT CIRCUIT

Newsletter of the Canberra Mathematical Association INC

VOLUME 12 NUMBER 1 JANUARY 2021

NEWS AND COMMENT

The first meeting of the new 2021 Council was held on December 7, 2020. Among the many other things discussed was the issue of 'maths anxiety'. This is shaping up to be a theme for 2021 and a possible focus for professional development initiatives.

Scenarios can be identified that contribute to maths anxiety for students, but simple remedies seem elusive. Teachers too can be affected.

In the primary sector maths is just one of many topics in the curriculum and it may not be every teacher's favourite. It can be for some a source of discomfort and trepidation.

Mathematics is often taught in secondary schools by teachers whose specialisations do not include maths. Many do this well but there may be times when it will seem embarrassing for the teacher to have to say in class, 'I don't know how to explain this' or 'I will look for a solution to this problem for next time' or 'Time is short and we can skip the rest of this topic'.

The more confident maths teachers are probably not so disturbed to find themselves in such a situation. They know that maths actually is hard and that easy answers do not always come to mind on cue. They know also that maths is usually a collaborative enterprise in which colleagues bounce ideas between them in search of an elegant solution or any solution at all.

Whether in primary or secondary teaching, it helps to be aware of resources that have been collected by others. One need not be shy about borrowing and sharing ideas. The renowned Isaac Newton, for example, claimed that he had seen further than others because he had stood on the shoulders of giants.

Now, with these thoughts as preamble, you may wish to turn to page 3 to find some resources assembled by Anna Williams at Mother Teresa Primary School in Harrison.

This edition of Short Circuit is early for the **festive season** and January 2021.

For one thing, we wanted to send it out before the cut-off date for the job opportunity with the Maths Trust, announced on page 3. The Australian Maths Trust is another trove of resources not to be overlooked, with more being developed.

CMA would like to wish its members and all readers of Short Circuit a pleasant and relaxing summer break.

The conjunction between Jupiter and Saturn on 21 December will see the least angular gap between the planets since 1623. In this sense, 2020 has been a year worth waiting for.

Coming Events:

Wednesday Workshop:

MEMBERSHIP

Memberships run from 1 Jan to 31 Dec. each year. Membership forms can be accessed from the CMA website: http://www.canberramaths.org.au

Membership of CMA includes affiliation with the Australian Association of Mathematics Teachers and a subscription to one of two AAMT journals.

As a member, you are entitled to attractive rates for the CMA annual conference and CMA professional development events.

CMA members may attend conferences of the AAMT affiliates in other states, MAV, MANSW, etc. at member rates.

CANBERRA MATHEMATICAL ASSOCIATION

PUZZLES

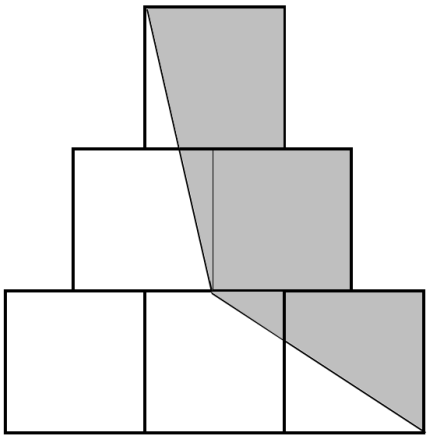
1 Numbers like Ramanujan's

The ordinary looking number 1729 is known as Ramanujan's number due to his casual remark to G. H. Hardy that it is the smallest number that can be expressed as the sum of two cubes in two different ways. Ed Staples reports that the number 1729 is even less ordinary than we had supposed since, when its digits are reversed and 9271 is added to 1729, the sum is 11,000. The question arises under what conditions this can happen, and how many other numbers have this property? (Ramanujan's birthday falls, irrelevantly, on December 22.)

2 Hard circles

In the solutions to puzzle number 4 from the December edition (see page 6), a procedure is described in which a chain of six circles is drawn inside a triangle. It is left as a difficult challenge to prove that the chain is indeed closed.

3 Shady squares



What fraction of the total area of the six squares is shaded?

4 From A to B

I want to go from A to B, a distance of 100 km. A previously unknown law of nature dictates that my speed in km/h must be equal to my current distance from B. Will I ever reach my destination?

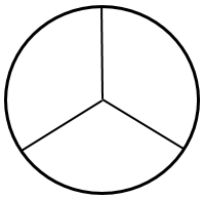
5 Cryptic digits

A nine digit counting number has the digit pattern ABCABCBBB where A, B and C are distinct. It is divisible by every integer from 1 to 17. Can you determine the number?

6 Fibonacci

The sequence of Fibonacci numbers begins with 1, 1, 2, 3, ... and continues according to the rule that each new number is the sum of the previous two. It is claimed that the greatest common divisor of two Fibonacci numbers is always another Fibonacci number. Is there a non-zero chance of success if you decide to search for a counterexample to this claim?

7 Mercedes sector



This famous logo consists of three congruent sectors. If the circumference of the circle is 3π, find the perimeter of one of the sectors.

8 2020 conjunction

The planet Jupiter orbits the sun in nearly 12 earth years while Saturn takes nearly 30 years. The planes of the orbits are nearly the same. Like the two hands of a clock, Jupiter overtakes Saturn periodically.

(a) What is the least number of years after which both planets have completed a whole number of orbits?

(b) Using the clockface analogy, when after time zero Jupiter first reaches 12 o'clock, Saturn has advanced to what time in minutes in its orbit?

(c) While Saturn completes two orbits, how many times does Jupiter lap Saturn? Thus, from the point of view of an observer stationed far above the orbital plane, how many years elapse between these conjunctions?

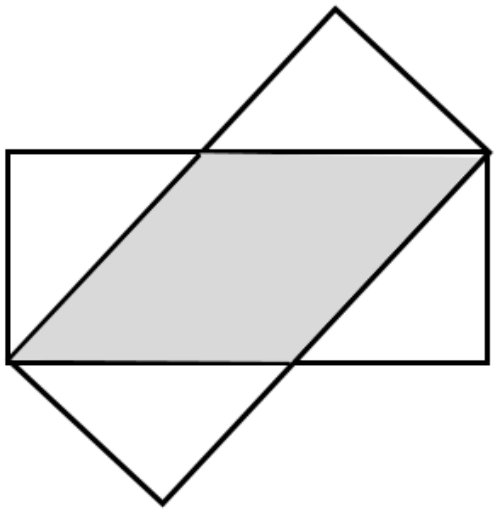
9 Factors

A certain integer has exactly 8 divisors and two of these are 15 and 21. Find the sum of all 8.

MORE PUZZLES

10 Rectangles

These two overlapping rectangles form a rhombus that is 5/8 of the area of each rectangle. What is the ratio of the length of the longer side of the rectangle to the length of the shorter side?



ADVANCED HUMOUR

$$\begin{aligned}\frac{d}{dx}\frac{1}{x} &= \frac{\cancel{d}}{\cancel{dx}}\frac{1}{x} \\ &= \frac{1}{x}\frac{}{x} \\ &= -\frac{1}{x^2}\end{aligned}$$

AUSTRALIAN MATHS TRUST POSITION

Nathan Ford, Chief Executive Officer of the Australian Maths Trust, writes:

> I am excited to let you know that we are expanding our team of in-house mathematicians at the Trust and will be adding two new roles to help us deliver on our objectives in 2021. … Both roles:
>
> a. Are for 1-year (there may be the possibility for extension),
>
> b. Are part-time (0.5FTE),
>
> c. Have a salary of $44,460.
>
> Applicants can be based anywhere in Australia (they do not need to be Canberra based) and should contact us at hr@amt.edu.au with an EOI including a short cover letter and a CV. EOI should be submitted to us by COB 23 December 2020.
>
> Regards,
> Nathan

Job descriptions can be accessed at the AMT website. Select the 'About' tab, then 'News and Events'.

MATHS AT MOTHER TERESA SCHOOL

Anna Williams is a Year One teacher and Numeracy Coordinator at Mother Teresa School in Harrison, ACT. Anna is now also a CMA councillor with a passion for improving CMA's service to teachers working in the primary sector.

As an initial contribution, Anna has made available two documents listing resources that have been assembled by and for teachers at her school and for parents and teachers grappling with remote learning due to the pandemic. The listings are laid out attractively with explanations and annotations. Each entry has a link to a relevant site.

Rather than reprinting the full lists, we have put the google.docs links in the following titles. Just mouse-click to connect:

1 Mother Teresa resources

2 Number games and activities

NEWSLETTER OF THE CANBERRA MATHEMATICAL ASSOCIATION INC

PO Box 3572
Weston ACT 2611
Australia

E-mail: canberramaths@gmail.com

We're on the Web!
http://www.canberramaths.org.au/

ABOUT THE CMA

The Canberra Mathematical Association (Inc.) is the representative body of professional educators of mathematics in Canberra, Australia.
It was established by, among others, the late Professor Bernhard Neumann in 1963. It continues to run - as it began - purely on a volunteer basis.

Its aims include

* the promotion of mathematical education to government through lobbying,
* the development, application and dissemination of mathematical knowledge within Canberra through in-service opportunities, and
* facilitating effective cooperation and collaboration between mathematics teachers and their colleagues in Canberra.

THE 2021 CMA COMMITTEE

President	Bruce Ferrington	Radford College
Vice Presidents	Jo McKenzie	ACT Education Directorate
	Aruna Williams	Erindale College
Secretary	Valerie Barker	
Treasurer	Paul Kruger	Saint Mary MacKillop College
Membership Sec.	Paul Turner	
Councillors	Peter McIntyre	University of NSW Canberra
	Theresa Shellshear	Australian Catholic University
	Heather Wardrop	
	Andrew Wardrop	
	Anna Williams	Mother Teresa Primary School
	Sue Wilson	
	Yuka Saponaro	
	Jane Crawford	

Short Circuit is edited by Paul Turner. **ISSN 2207-5755**

Find us on Facebook

http://www.facebook.com/pages/Canberra-Mathematical-Association/110629419011275

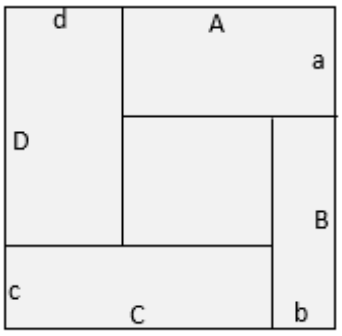
PUZZLE SOLUTIONS from last edition

1 Slices of 3D

(a) Cube: A great circle must include the vertices at opposite ends of a body diagonal as these are furthest apart, but the plane of this circle can also include two more vertices. So, four vertices can belong to a great circle.

(b) Tetrahedron: The four vertices are not in the same plane but any three are. So, two great circles are needed.

2 If but not only if

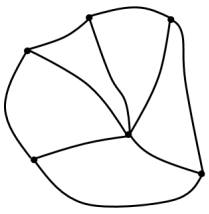


Five rectangles are arranged inside a square, as shown. We prove that if the outer rectangles have equal area, then the central rectangle must be a square. The following explanation was suggested by Ross Pure, a former ANU Senior Secondary College student, now a mathematician in the private sector.

If the outer rectangles all have the same dimension $X \times x$, then the central rectangle is a square of side $X - 2x$. If the shapes are not the same, there must be a rectangle with smaller width than its neighbour. Suppose $b < a$. Then, $Aa = Bb$ implies $A < B$. Now, $A + d = B + a$ implies $a < d$. Then, $aA = dD$ implies $D < A$ and $A + d = D + c$ implies $d < c$. Continuing in this way, we find $b < a < d < c < b$ which is impossible. Hence, the outer rectangles are congruent and the central rectangle is a square.

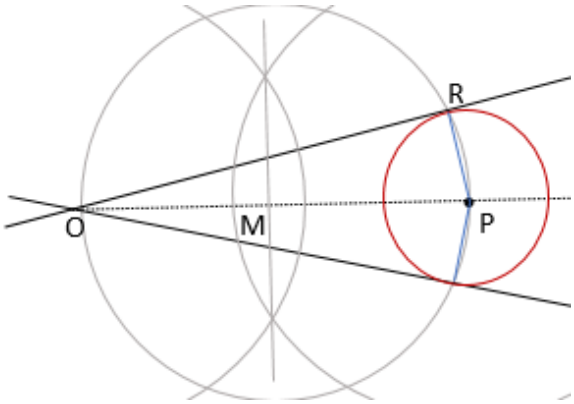
On the other hand, if the outer rectangles are different and there is a central square, then the areas must be different.

2 Pentagon colouring



Keep the colour of one region fixed and vary the others. Discard reversals. Thus, there are 4×3=12 combinations. John H. Conway observed that 12 is also the number of faces on a dodecahedron and asked in how many ways is it possible to partition each face of a dodecahedron in this way and to use the 12 colourings, one for each face, so that no two adjacent regions have the same colour.

4 Constructions

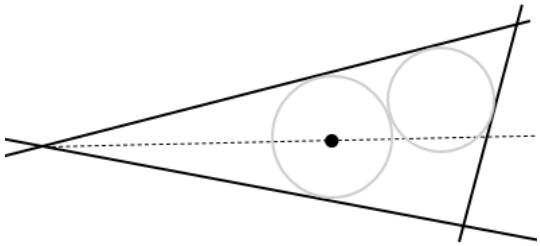


(a) Given two intersecting lines and a point P on the angle bisector from O, the problem is to construct a circle with P as centre such that the two lines are tangent to it. Locate the midpoint M of OP by constructing arcs with equal radius about O and P and connecting their intersections with a line. A circle with centre M and radius MP is drawn to locate point R on one of the lines. Since OP is a diameter of this circle, angle ORP is a right angle. Then the circle with centre P and radius PR is the one required since the intersecting lines are tangent to it.

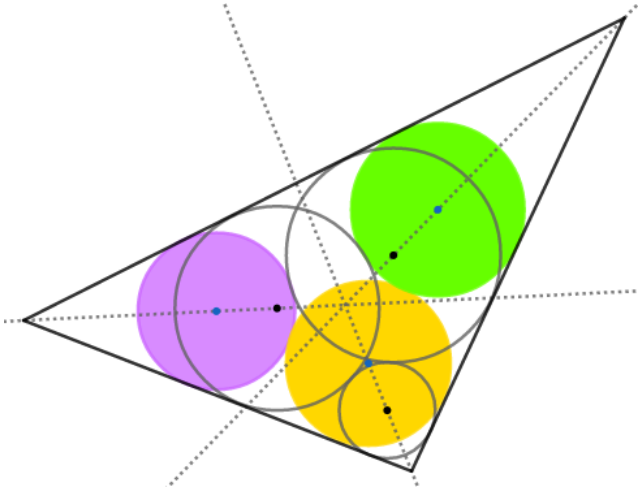
(b) No straight-edge and compass construction for part (b) has come to light, as far as we know.

PUZZLE SOLUTIONS continued

However, the idea leads to a remarkable result. In effect, we asked for a second circle to be drawn in the next vertex of a triangle so that it would be tangent to the first circle, as in the diagram below.



The process can be continued to make a chain of circles in the triangle. At each step, a new circle is placed in the next vertex so that it is tangent to the previous circle. It turns out that the sixth circle,



which is tangent to the fifth, is also tangent to the first. Thus, the chain repeats after six steps.

This puzzle editor is working hard, and needs help, to find a proof of this astonishing fact (one that is more convincing than a picture). It is left as a challenge to the interested reader, until next time.