Short Circuit

Newsletter of the Canberra Mathematical Association INC

VOLUME 12 NUMBER 7 JULY 2021

NEWS AND COMMENT

In preparing this edition of Short Circuit your editor, on a whim, looked back at an edition of Circuit from 1999. Some things have not changed. We are still urging readers to contribute their puzzles, articles, comments, reviews, letters to the editor, poems and so on. At that time too there was a curriculum review under way. We have reprinted, on page 2, a quote on that topic from July 1999 that appears again to be relevant.

The proposed collaborative response with QAMT to the draft curriculum review did not happen. However CMA councillors are about to meet to prepare an independent response. We expect to bring news of this in the next Short Circuit.

We report proudly that CMA councillor Sue Wilson has received her PhD.

Sue Writes:

'To all those CMA members who have attended my conference presentations and workshops updating the progress of my PhD research on maths anxiety, I was heartened by your support and encouragement, and very grateful for your feedback. My thesis was submitted at the end of 2019 and I graduated last year and received my testamur Covid-style through Australia Post. In April this year I was able to attend a Graduation ceremony.'

Sue has provided a summary of her thesis. It can be found on page 6.

CANBERRA MATHEMATICAL ASSOCIATION

Inside:

Coming Events:

CMA conference: 7 August at ADFA

Theme: '19, 20, 21 – What's next?'

AAMT virtual conference 29-30 September. Theme: 'Future Proofing'

AGM: 10 November.

Wednesday Workshop:

MEMBERSHIP

Memberships run from 1 Jan to 31 Dec. each year. Membership forms can be accessed from the CMA website: http://www.canberramaths.org.au

Membership of CMA includes affiliation with the Australian Association of Mathematics Teachers and a subscription to one of two AAMT journals.

As a member, you are entitled to attractive rates for the CMA annual conference and CMA professional development events.

CMA members may attend conferences of the AAMT affiliates in other states, MAV, MANSW, etc. at member rates.

PUZZLES

1. Transposed digits

Austin had to multiply a positive integer by 35. Instead, he multiplied by 53 and this caused the result to be too high by 540. What was Austin's result?

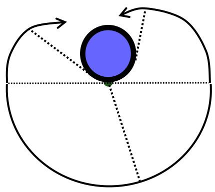
2. Fair game

In a gambling game a coin of diameter y is thrown onto a grid of squares each of side x. If when the coin comes to rest it is wholly within any square, the player wins. To make it a fair game what should the ratio y/x be?

3. Goat

There are various puzzles to do with the area of grass available to a grazing goat tethered near an obstacle. This one has an exact solution but a plausible estimate would be commendable.

The goat is attached by a tether 5m long to a stake immediately next to a water tank that has a diameter of 3180mm. The area that the goat can graze has a semi-circular portion and two curved parts where the tether winds around the tank. What area can the goat reach?



PLUS ÇA CHANGE...

The Real Stuff of Numeracy

... that, after all is what really matters - that pupils emerge with an interest in mathematics, a confidence to talk about it, and a willingness to engage with mathematics wherever and whenever it crosses their paths. At least Plutarch had the right idea. Two thousand years ago he saw that "the mind is not a vessel to be filled, but a fire to be kindled." Is the current round of mathematics curriculum change in Australia, being done in the name of numeracy, actually addressing what really matters?

From *Circuit*, July 1999

PRECISION

What we mean by words like *exact, precise, correct*, and *accurate* is something that arises in the mathematics classroom. How often have you observed a student futilely writing down all the decimal places in a calculator display in their answer to a question involving measurement?

CMA councillor, Sue Wilson, communicated an observation that could be a starting point for a discussion about these matters. Sue writes:

> 'I heard someone (possibly a politician) talking about the vaccine. They said (about blood clots?) that the research finding is that they happen to 1 in 666,666 people, "so it's very exact". I imagine that the actual figures quoted were 1.5 per million. I thought it could be the basis for an interesting question about significant figures.'

Of course, 1.5 per million is very close to 1 in 666,666 but the two proportions convey very different impressions.

Was the original 1.5 per million figure intended to be understood as an exact proportion? Probably not. More likely, we would be justified in believing that the observed incidence of clotting is somewhere between one and two cases per million. We would certainly not expect that after the next two million doses of vaccine *exactly* three people will experience a blood clot.

As the speaker said, 1 in 666,666 is very exact. We might dismiss the quote of six significant digits as a clumsy attempt to give a veneer of statistical validity to the argument, annoying in ordinary circumstances but problematic and potentially mischievous in the midst of a pandemic with people reluctant to have the vaccine for fear of blood clots.

This was an item in a TV news or current affairs program. For critically numerate viewers it may be that a figure quoted to such a silly level of precision would undermine the credibility of the speaker. For others, the impact is harder to predict. How would you react?

CANBERRA MATHS TALENT QUEST

The National Mathematics Talent Quest has provided a venue to showcase the creative thinking skills of students in Australia for many years. To be eligible to enter the national quest a project has to be successful in a similar quest at the state level. Students throughout the ACT have put considerable time and effort into mathematics assignments and projects yet they have not had a mechanism where they could get local or national recognition and encouragement for their work.

The executive of CMA has decided that we should launch our own mathematics quest this year. The Canberra Mathematics Quest has the following aims:

- To promote mathematics in schools and in the community
- To develop students' appreciation of the scope of mathematics
- To encourage creativity and develop thinking skills

Entry to the Quest

Students may participate in the quest in one of three categories:

- Submit an **individual** entry
- Be part of a **small group** (up to 6 students)
- Be part of a **whole class** entry (7 or more students)

The work must be submitted by **Friday 13th August 2021**.

Entry is free.

All students from Kindergarten to Year 12 in the ACT may submit an entry.

Any topic that involves a mathematical investigation is acceptable. The topic may be something that the student has found interesting, or it could be something the class has been given as a task.

Information about presentation formats, how to submit an entry, the rubric that will be used in judging the entries, and other things, is now available in the form of a booklet downloadable from the CMA website. (See the 'upcoming events' section.)

CMA ANNUAL CONFERENCE 2021

Saturday 7th August 2021, at ADFA

"19, 20, 21, …What's next?"

Teachers and educators in all sectors are warmly invited to attend this year's conference.

The 2021 Conference committee is also pleased to call for **expressions of interest in being a presenter** at the conference. All presentations to do with the teaching, learning or use of Mathematics are welcome. If you can incorporate the conference theme or respond to it, so much the better.

For further details, please contact Valerie Barker: vnwb@internode.on.net .

To register for the conference, click here.

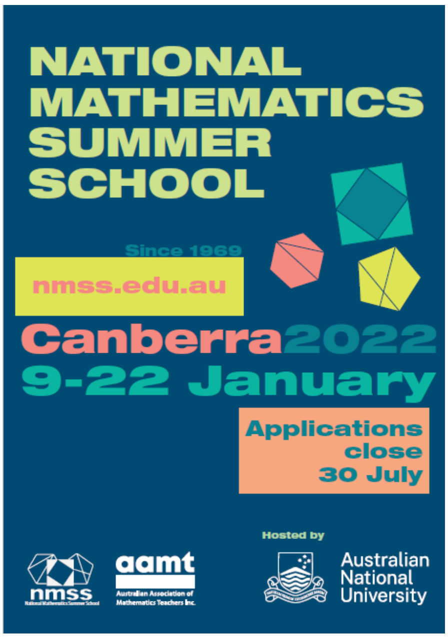
AAMT

Call for presenters and registration to attend the AAMT e-conference, 29-30 September, is now available.

Click this link to the revamped AAMT website to register as a presenter or an attendee.

NMSS

Go to the CMA website or directly to the National Mathematics Summer School website for details.



NEWSLETTER OF THE CANBERRA MATHEMATICAL ASSOCIATION INC

PO Box 3572
Weston ACT 2611
Australia

E-mail: canberramaths@gmail.com

We're on the Web!
http://www.canberramaths.org.au/

ABOUT THE CMA

The Canberra Mathematical Association (Inc.) is the representative body of professional educators of mathematics in Canberra, Australia.

It was established by, among others, the late Professor Bernhard Neumann in 1963. It continues to run - as it began - purely on a volunteer basis.

Its aims include

* the promotion of mathematical education to government through lobbying,
* the development, application and dissemination of mathematical knowledge within Canberra through in-service opportunities, and
* facilitating effective cooperation and collaboration between mathematics teachers and their colleagues in Canberra.

THE 2021 CMA COMMITTEE

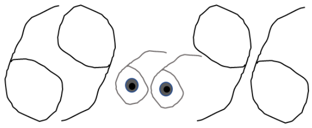
President	Bruce Ferrington	Radford College
Vice Presidents	Jo McKenzie	ACT Education Directorate
	Aruna Williams	Erindale College
Secretary	Valerie Barker	
Treasurer	Paul Kruger	Saint Mary MacKillop College
Membership Sec.	Paul Turner	
Councillors	Peter McIntyre	University of NSW Canberra
	Theresa Shellshear	Australian Catholic University
	Heather Wardrop	
	Andrew Wardrop	
	Anna Williams	Mother Teresa Primary School
	Sue Wilson	
	Yuka Saponaro	
	Jane Crawford	Brindabella Christian College

Short Circuit is edited by Paul Turner. **ISSN 2207-5755**

Find us on Facebook

http://www.facebook.com/pages/Canberra-Mathematical-Association/110629419011275

TRANSPOSITION ERRORS



What is the difference between 42 and 24? In a literary sense, one of them, according to the book *The Hitchhiker's Guide to the Galaxy,* is the answer to the ultimate question of life, the universe and everything; while the other, according to the nursery rhyme, is the number of blackbirds baked in a pie.

In an accounting sense, each is the other with its digits transposed.

In a mathematical sense, the difference is 18.

Reconciliation of sums of numbers from two sources is a common accounting task. A frequent and exasperating event that causes sets of numbers to have different sums when they should be the same is the transposition error. These errors typically occur when there is manual data entry somewhere in the data assembly process. It is easy for one's fingers to type 42 by mistake while reading 24 from a document, for example, and not only do the fingers sometimes fail to follow what the brain is thinking, but in the act of comparing lists the eye can easily fail to distinguish between pairs of numbers that differ only by the exchange of two digits.

The mathematical sense of difference can point to the existence of a transposition error. Observe that the difference 42 – 24=18 is a number divisible by 9. It turns out that whenever a pair of adjacent digits is exchanged the difference between the resulting number and the correct number is always a number divisible by 9.

To explain this, consider our base 10 place-value system of numeration. (This was invented in India and transmitted to Europeans a mere thousand years ago by Arabic speakers, along with the idea of using alphabetic symbols to stand for numbers that we cannot or do not wish to specify.)

We can represent any positive whole number by

$$M + 10^{k+1}a + 10^{k}b + N$$

where a and b are adjacent digits in a numeral and M and N stand for the leading and trailing digit strings, if present. The same number with the digits a and b transposed is represented

$$M + 10^{k+1}b + 10^{k}a + N$$

When the second of these is subtracted from the first, we obtain $10^{k+1}(a-b) + 10^{k}(b-a)$ and this can be rewritten as $(10^{k+1} - 10^{k})(b-a)$ or more simply, $10^{k} \times 9 \times (a-b)$. Thus, whatever the values of a, b and k, we always obtain a multiple of 9.

While it is perhaps less likely to occur, it could happen that a single correct digit is interposed between a transposed pair. For example, 294 ↔ 492. In such a case, after representing the numbers as before and subtracting one from the other, we obtain

$$(10^{k+2} - 10^{k})(b-a) \quad \text{or } 10^{k} \times 99 \times (a-b).$$

Thus, a transposition of this kind creates a difference that is divisible by both 9 and 11. If there are two digits between a transposed pair, the difference is divisible by 999, that is, it is a multiple of both 9 and 37.

There is a logical feature of this observation that should be noted. The reasoning says that the existence of a transposition implies a difference that is divisible by 9. Thus, we can make the contrapositive statement that a difference not divisible by 9 is not due to a transposition error. However, the inverse implication does not hold: A difference divisible by 9 may or may not be due to a transposition error. Divisibility by 9 suggests transposition as a possible cause but does not guarantee it.

PT

SUMMARY OF THESIS—DR SUE WILSON

International studies report maths anxiety in pre-service primary teachers can impact on their emotional and academic well-being. This study focussed on addressing participants' understanding of their emotional responses and anxieties towards mathematics using the readings and reflection processes of bibliotherapy. Participants emphasised the impacts of former teachers and of negative testing regimes.

This thesis argues that insights from their maths anxiety experiences can result in participants developing a more positive mathematical identity, to more empathy in their future classrooms and potentially increased effectiveness in their future teaching.

It also discusses implications and recommendations for future research, education practice and policy.

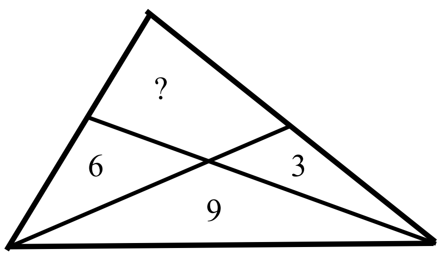
I am looking forward to sharing the implications of my study with CMA members at this year's CMA conference in August. My thesis argues against framing experiences of maths anxiety as a negative construct, and argues that teachers who reflect on and understand their past experience can potentially be more empathetic and effective teachers and enable a wider range of students to develop more positive relationships with mathematics. Is this a challenging idea? Let's discuss it at the conference.

Dr Sue Wilson

A HARD QUESTION

A contributor to a LinkedIn forum posed the following (difficult) problem.

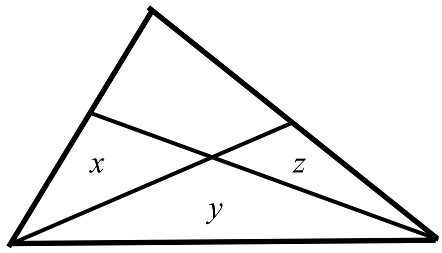
Compute the missing area.



A reader found the solution by appealing to something called the *ladder theorem*, which says:

If x, y and z refer to the areas of the respective regions, and Δ is the area of the whole triangle, then

$$1/\Delta + 1/y = 1/(x + y) + 1/(y + z)$$

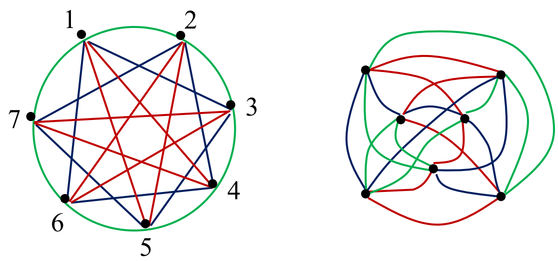


But, how might one prove the ladder theorem?

PUZZLE SOLUTIONS from Vol 12 No 6

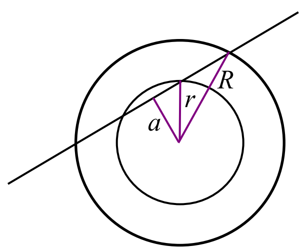
1 Different or not

The graphs are not the same. In the graph on the left, there is a blue edge connecting every second vertex in the green path, but in the graph on the right there are two instances where what should be a blue edge is red. The graphs could be made the same by swapping the colourings of these two pairs of edges.



2. Annulus

The division into three equal parts cannot be done if the radius of the inner circle is less than 1/3 of the radius of the outer circle. In the bounding case, the line contains the diameters of both circles.



With a construction like the one shown, we can deduce that each of the parts of the line is $2\sqrt{(r^2 - a^2)}$ and also $2/3\ (\sqrt{(R^2 - a^2)})$. Given that each of the parts measures 2, we have $r^2 - a^2 = 1$ and $R^2 - a^2 = 9$. Hence, $R^2 - r^2 = 8$. We do not need to know the two radii because the area of the annulus is $A = \pi(R^2 - r^2)$ and therefore, $A = 8\pi$.