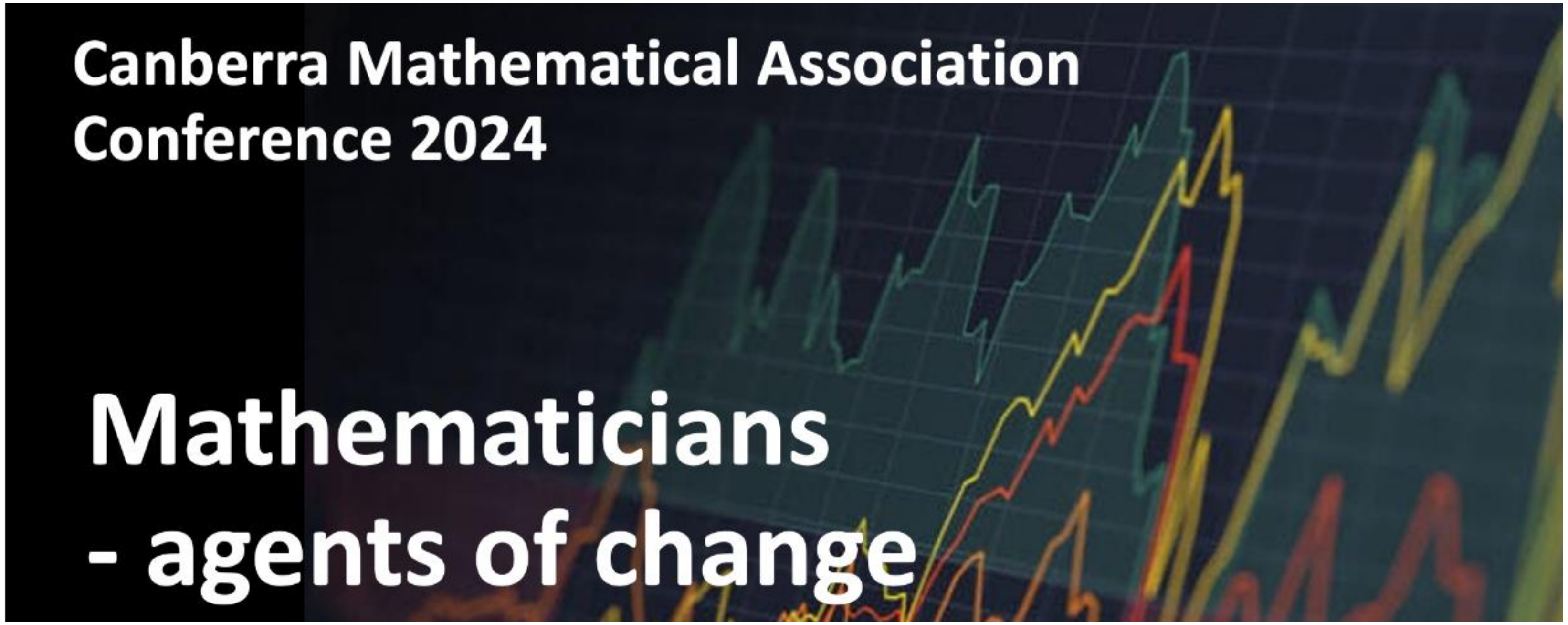

Canberra Mathematical Association
Conference 2024

Mathematicians
- agents of change

Investigating Change: Ideas for introducing Calculus

Brian Lannen murray.math@bigpond.com

Warm-up activity – turn & talk

What is it that you usually do first with your class when introducing calculus for the first time?

Limit Theory

Formula for differentiation by first principles

Overview of the notations used

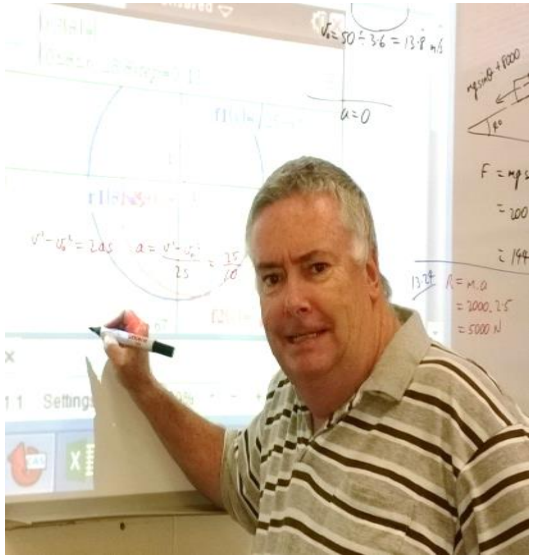
Rates of change Problems

Discussion of Infinitesimals

Story of historical context

Differentiation by Rule

Other…



Brian Lannen

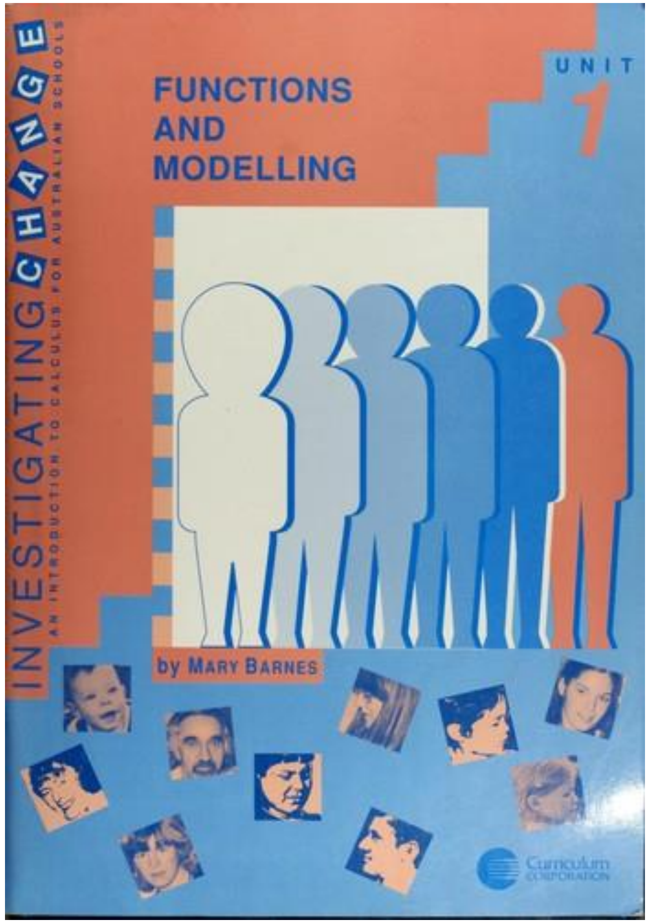
Murray Mathematics Curriculum Services

T^3 National Instructor

Brian Lannen has been teaching for over 35 years in Victoria and NSW. He has taught Physics, Maths and Science in schools, university and TAFE colleges, was a curriculum consultant in NSW and New York and has contributed to a range of text-book writing projects. He helped establish T-Cubed (Teachers Teaching with Technology) in Australia in the 1990s and is now a Senior Mentor to that association and Principal Host of the Texas Instruments Australia webinar program.

Investigating Change – Ideas for introducing Calculus Yr 11-12

Calculus not only *describes* change, but its discovery and development in the late 17th century also forever *changed* the study of mathematics. All of our senior study designs now offer at least one calculus-based course. In this session the presenter will share some of his favourite calculus activities, starting with how to introduce key concepts of differential calculus and through to a quirky Monte Carlo approach for integral calculus.



Investigating change : an introduction to calculus for Australian schools

Authors: Mary Barnes, Curriculum Corporation (Australia)

Summary: Emphasises mathematical modelling and practical applications. More emphasis is placed on meaning than on symbol manipulation. Investigative and exploratory activities encourage students to become actively involved in their learning by means of cooperative work and discussion

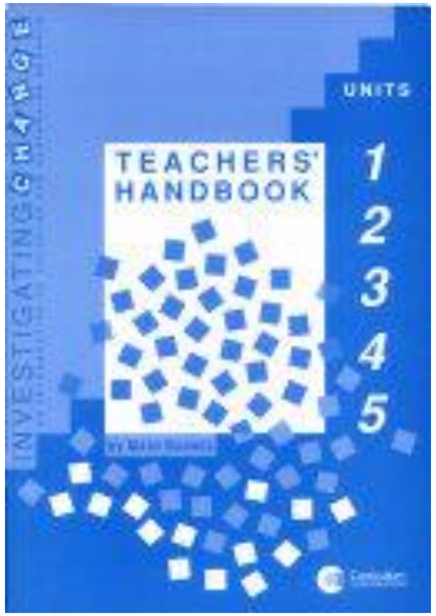
Show less ^

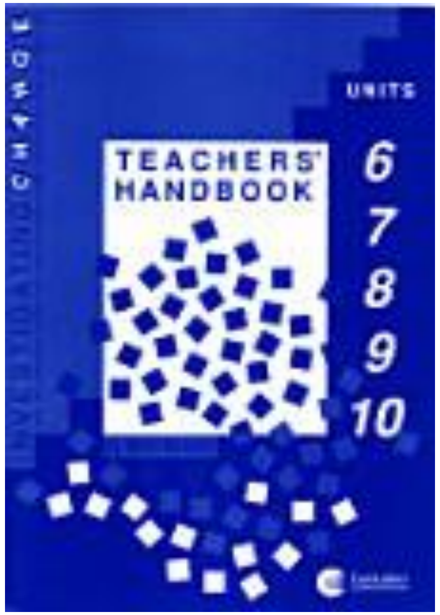
Print Book, English, ©1991-1993

Edition: View all formats and editions

Publisher: Curriculum Corp., Carlton South, Vic., ©1991-1993

Table of Contents





From Brian’s class:

Story of historical context

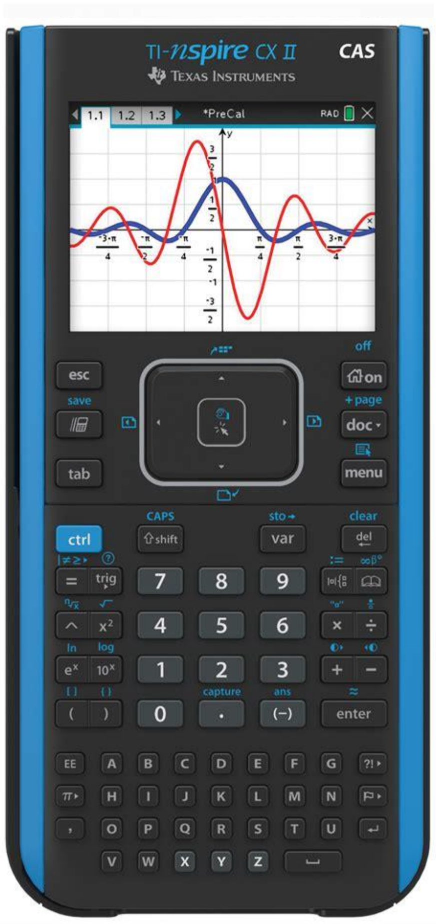
Rates of change Problems

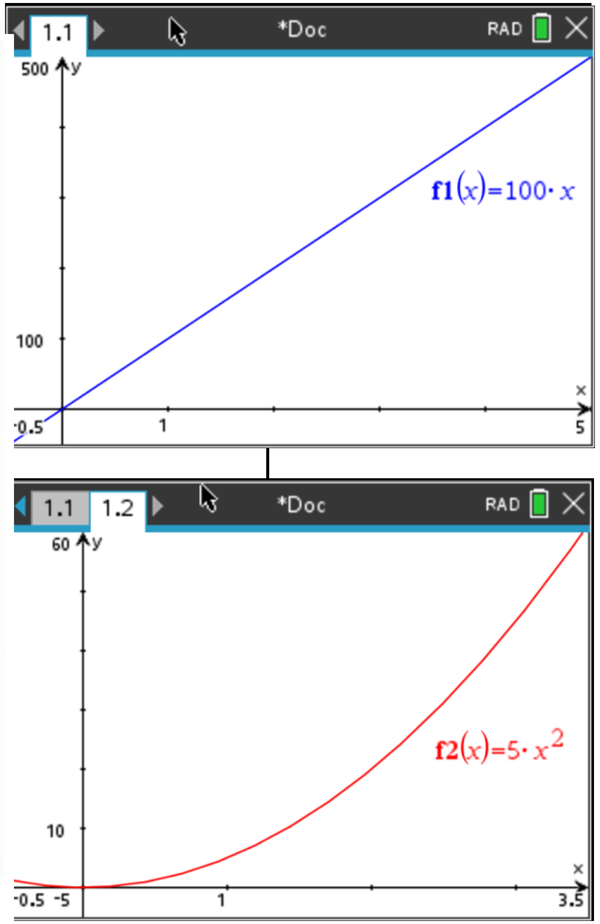
$$speed = \frac{distance}{time}$$

$$distance = speed \times time$$

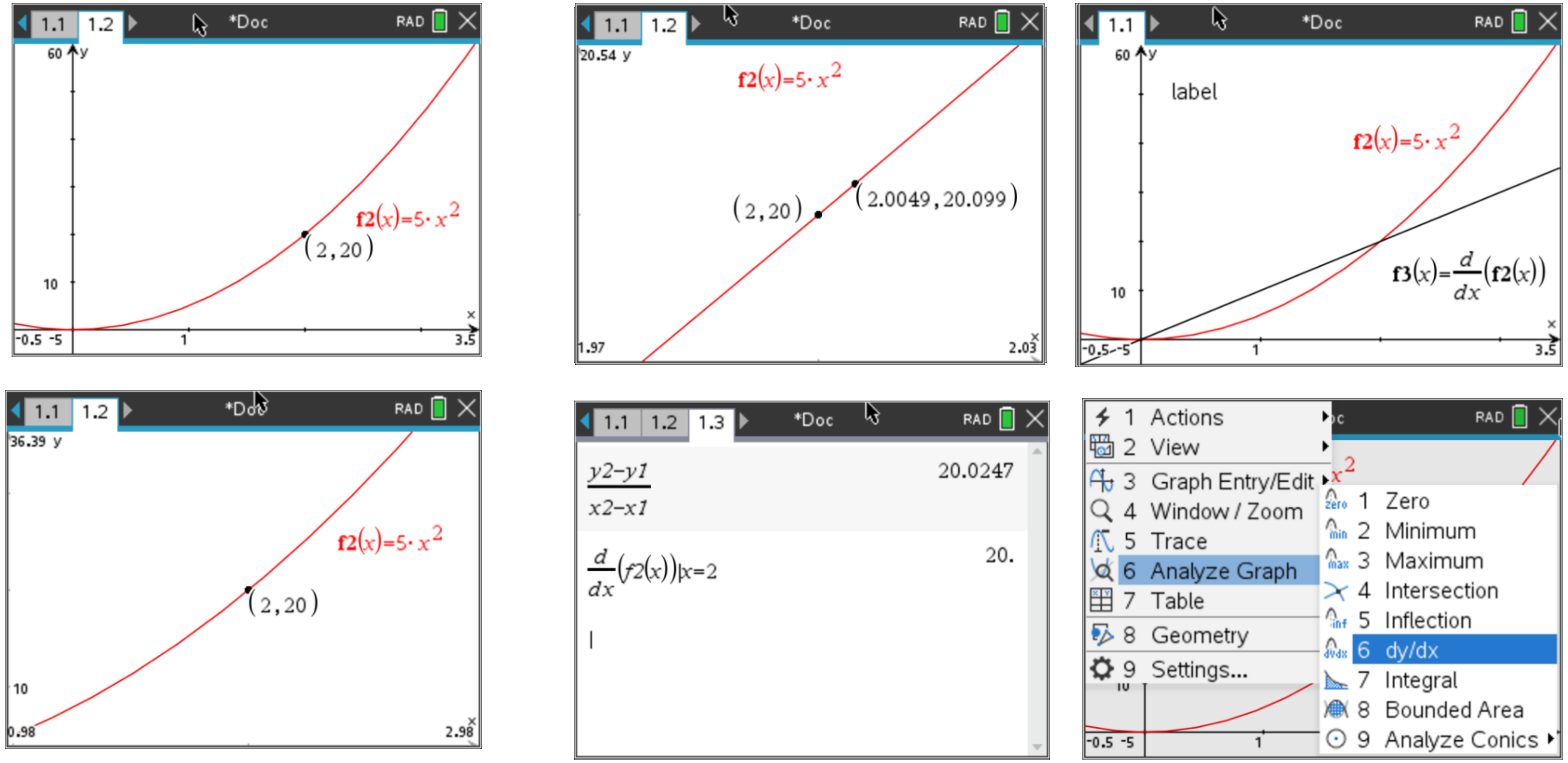
$$s = u.t + \frac{1}{2}a.t^2$$

What is the speed in each situation when *time* = 2?



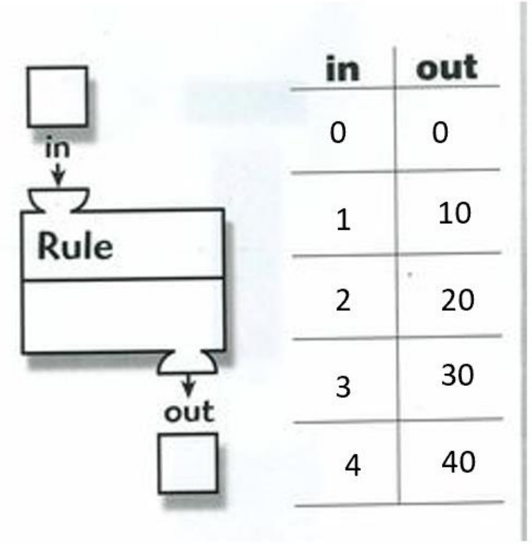


From Brian’s class:



$\frac{y2-y1}{x2-x1}$ 20.0247

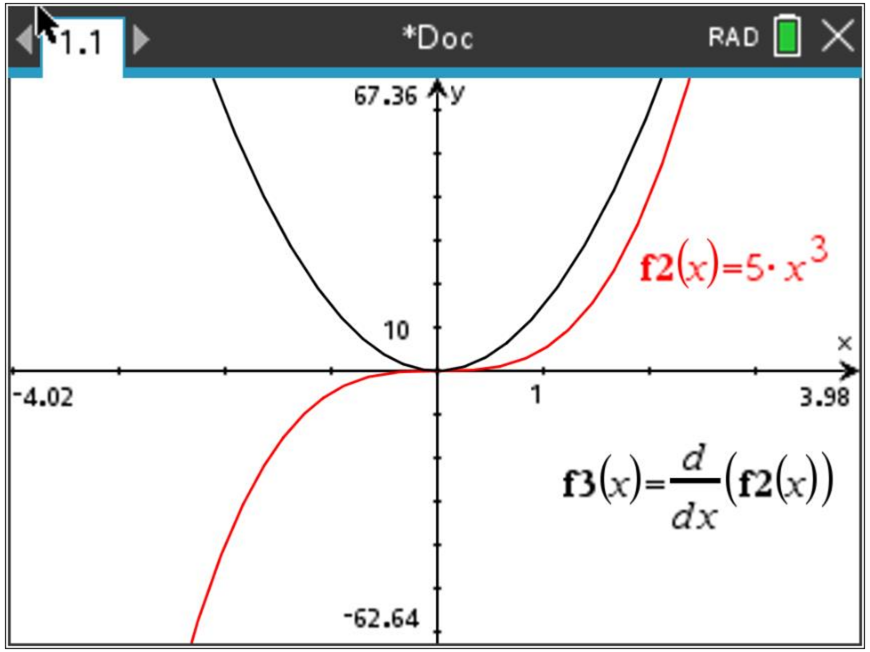
$\frac{d}{dx}(f2(x))|x=2$ 20.

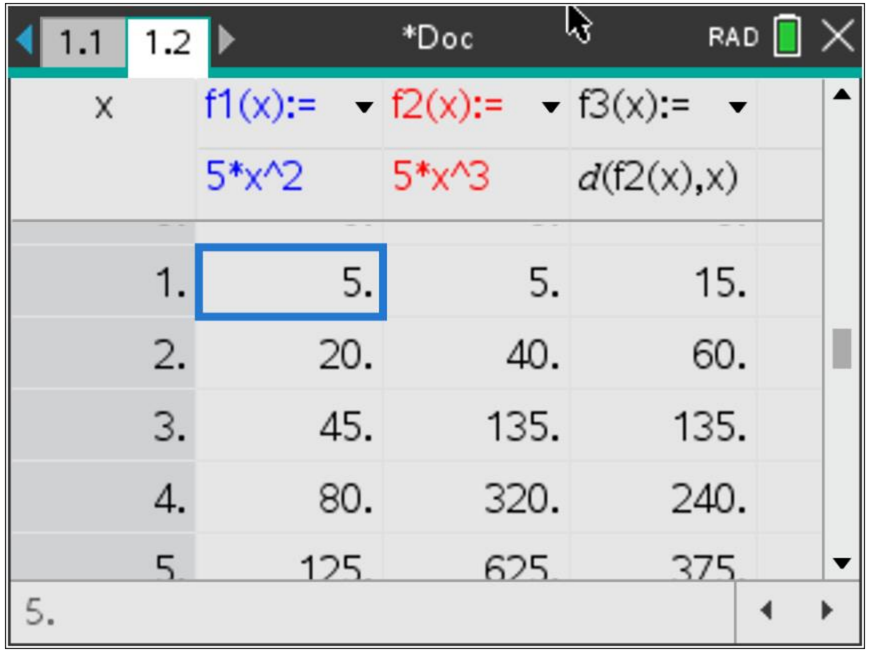


in	out
0	0
1	10
2	20
3	30
4	40

$$Y(x) = 5x^2 \Rightarrow Y'(x) = 10x$$

From Brian’s class:



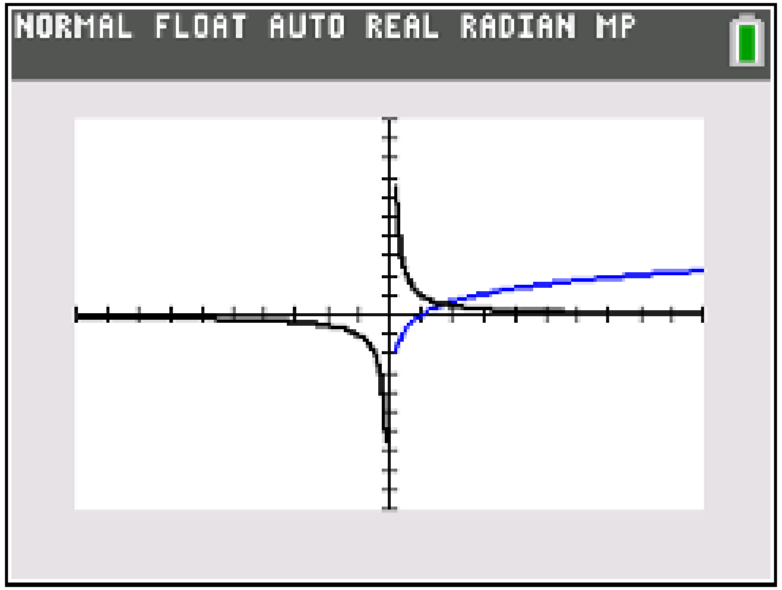


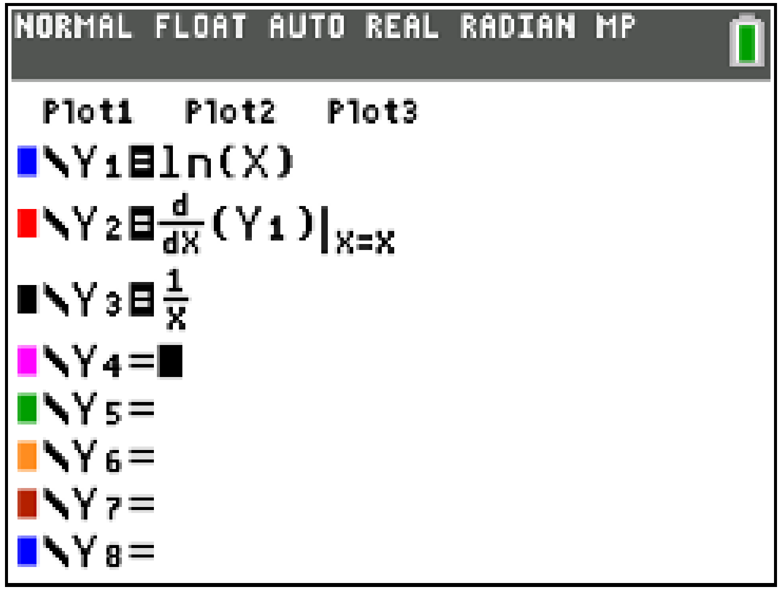
x	f1(x):=	f2(x):=	f3(x):=
	5*x^2	5*x^3	*d*(f2(x),x)
1.	5.	5.	15.
2.	20.	40.	60.
3.	45.	135.	135.
4.	80.	320.	240.
5.	125.	625.	375.

5.

$$Y(x) = 5x^3 \Rightarrow Y'(x) = 15x^2$$

Derivative of ln(x)




NORMAL FLOAT AUTO REAL RADIAN MP

Plot1 Plot2 Plot3

$Y_1 = \ln(X)$

$Y_2 = \frac{d}{dX}(Y_1)|_{X=X}$

$Y_3 = \frac{1}{X}$

$Y_4 =$

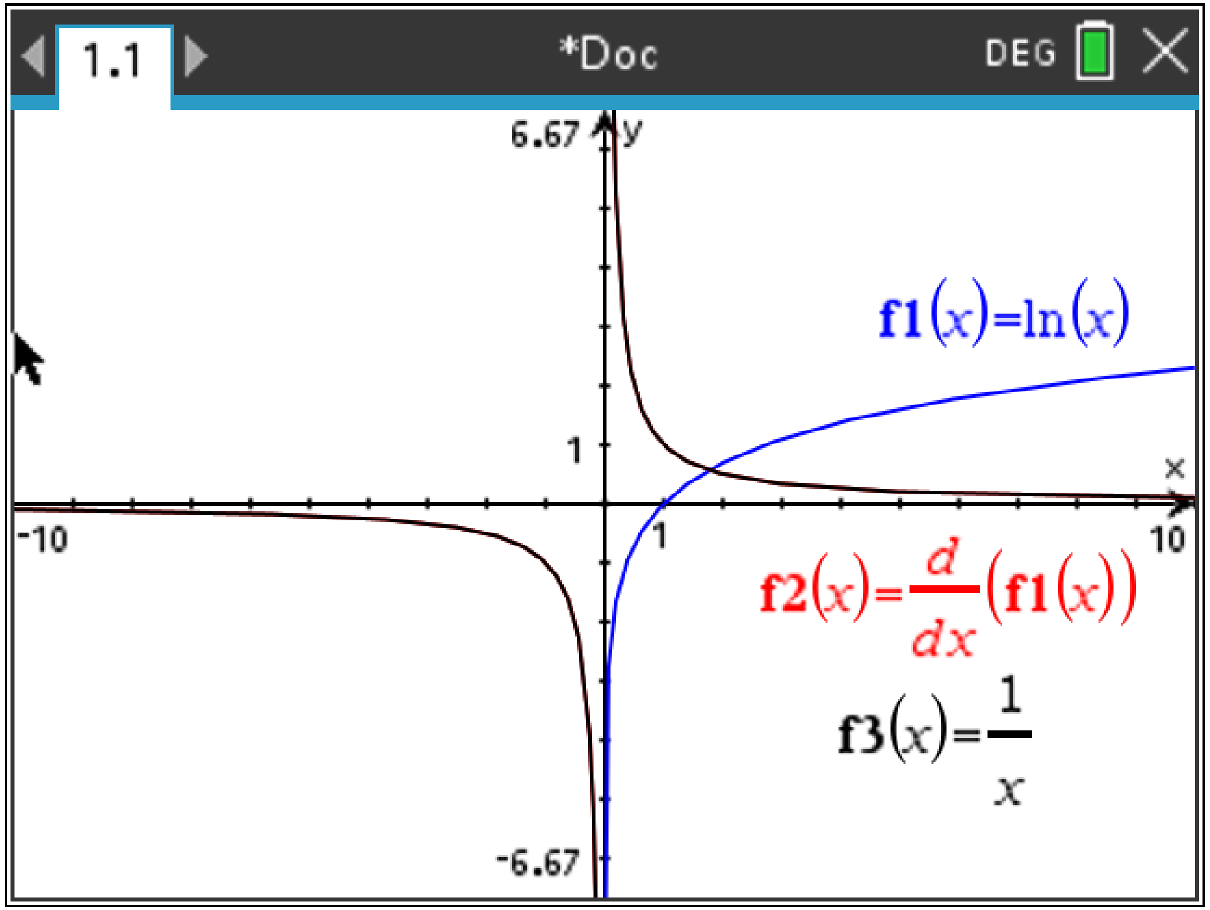
$Y_5 =$

$Y_6 =$

$Y_7 =$

$Y_8 =$

Derivative of ln(x)

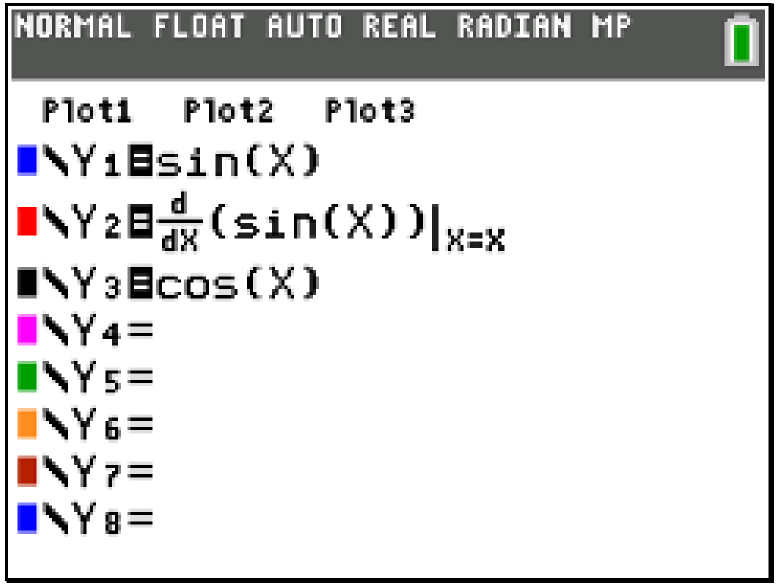


$f1(x)=\ln(x)$

$f2(x)=\frac{d}{dx}(f1(x))$

$f3(x)=\frac{1}{x}$

Derivative Trig functions



NORMAL FLOAT AUTO REAL RADIAN MP

Plot1 Plot2 Plot3

🟦 \Y1 = sin(X)

🟥 \Y2 = $\frac{d}{dX}(\sin(X))|_{X=X}$

⬛ \Y3 = cos(X)

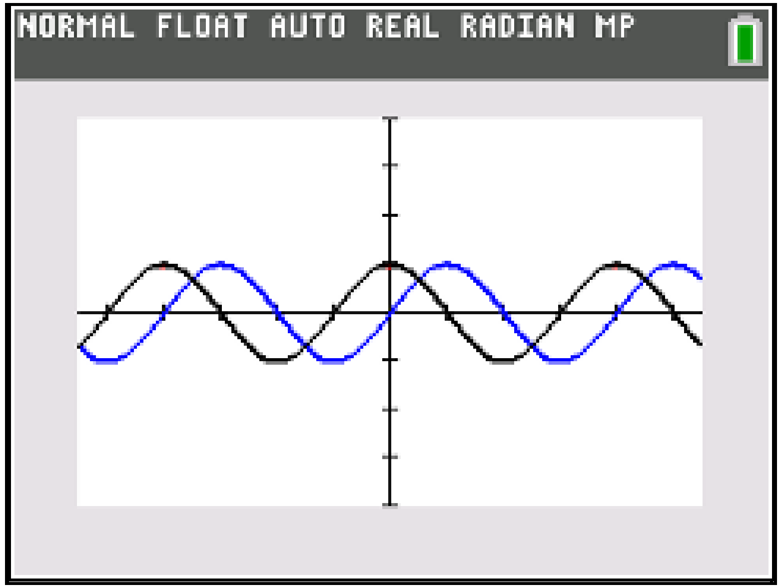
🟪 \Y4 =

🟩 \Y5 =

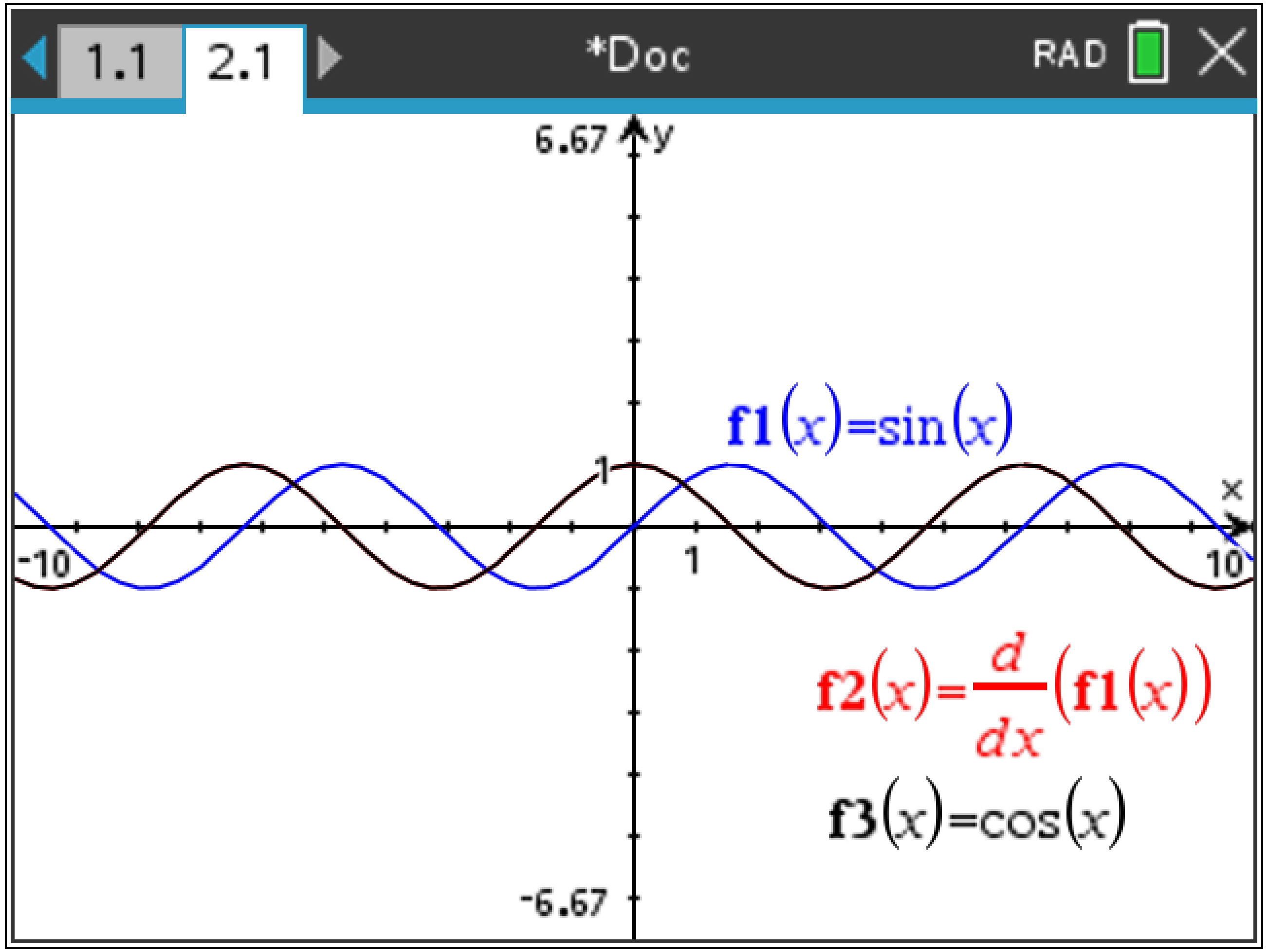
🟧 \Y6 =

🟫 \Y7 =

🟦 \Y8 =



Derivative Trig functions

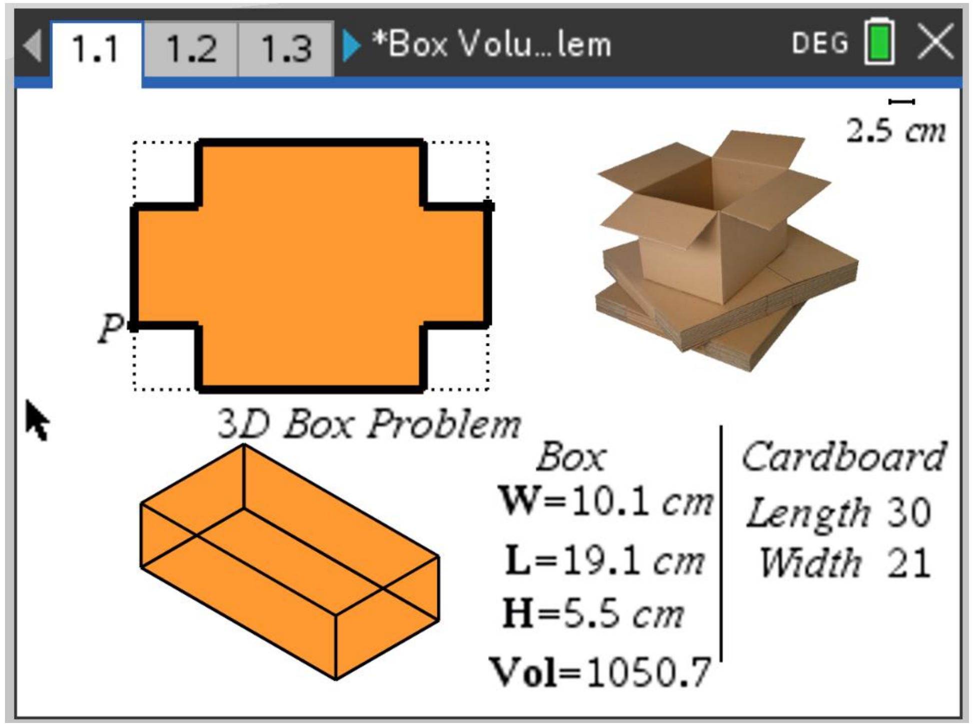


$f1(x)=\sin(x)$

$f2(x)=\frac{d}{dx}(f1(x))$

$f3(x)=\cos(x)$

Restricted domain



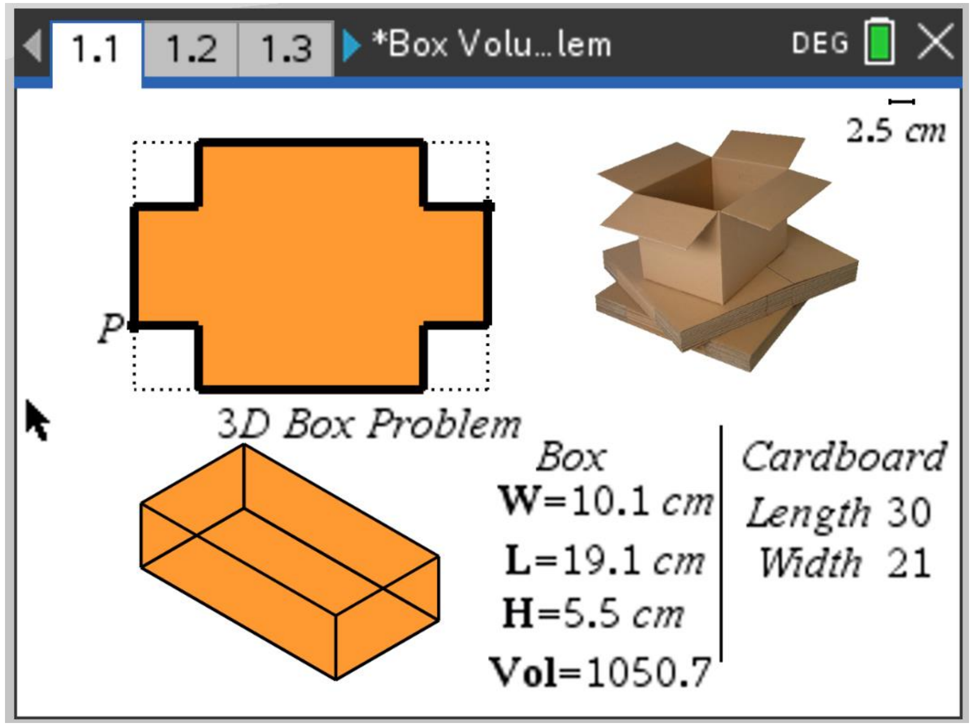
3D Box Problem

Box
W=10.1 cm
L=19.1 cm
H=5.5 cm
Vol=1050.7

Cardboard
Length 30
Width 21

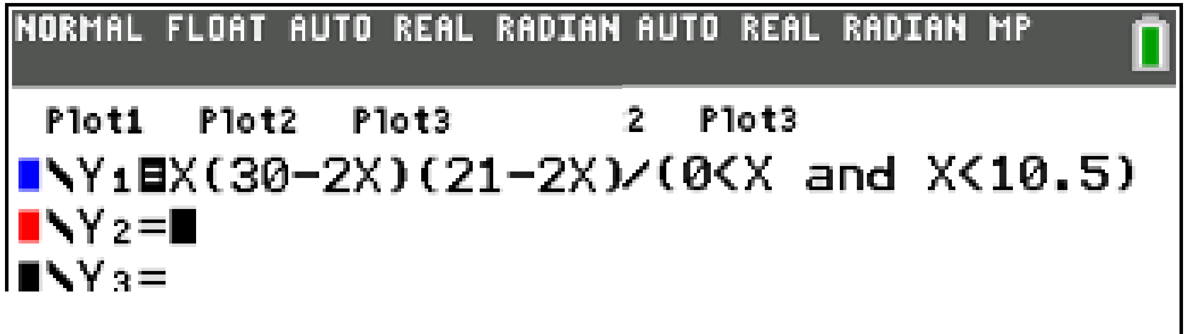
- What is the maximum possible volume of a box made from cardboard of 21 x 30 cm?
- Support your solution with appropriate graph(s)
- Any graphs should include domain restrictions relevant to the physical situation.

Restricted domain



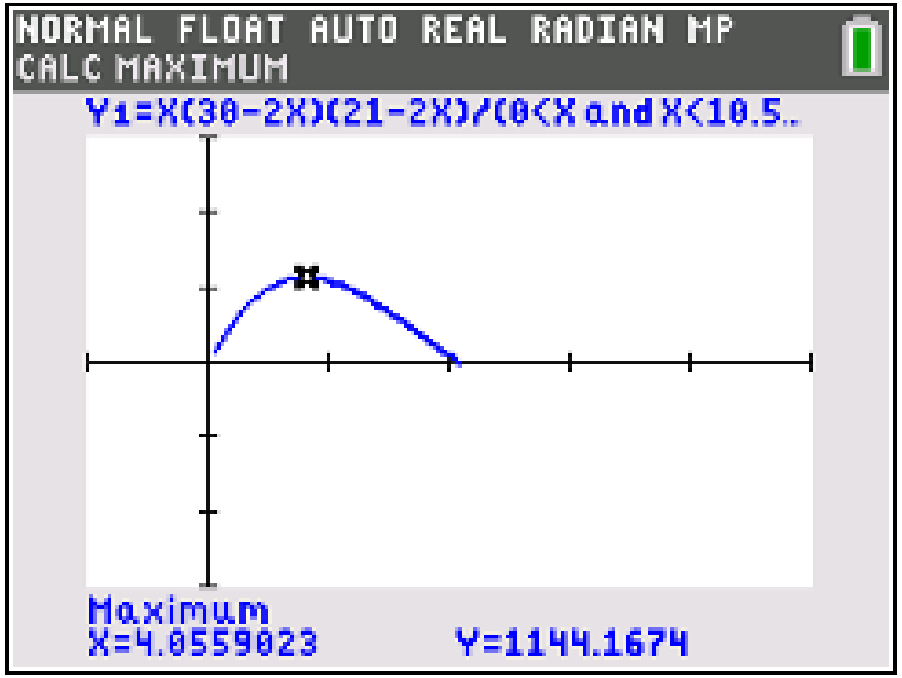
3D Box Problem

Box	Cardboard
W=10.1 *cm*	*Length* 30
L=19.1 *cm*	*Width* 21
H=5.5 *cm*	
Vol=1050.7	

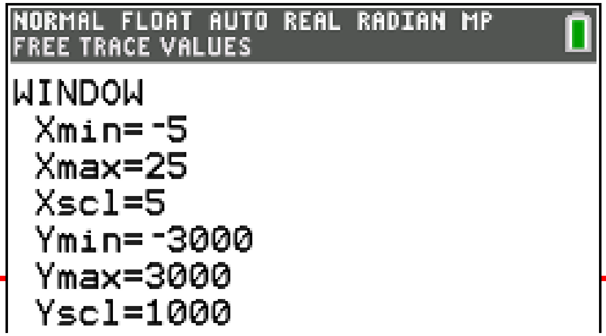


```
NORMAL FLOAT AUTO REAL RADIAN AUTO REAL RADIAN MP

 Plot1  Plot2  Plot3        2  Plot3
▌\Y₁■X(30-2X)(21-2X)/(0<X and X<10.5)
▌\Y₂=█
▌\Y₃=
```

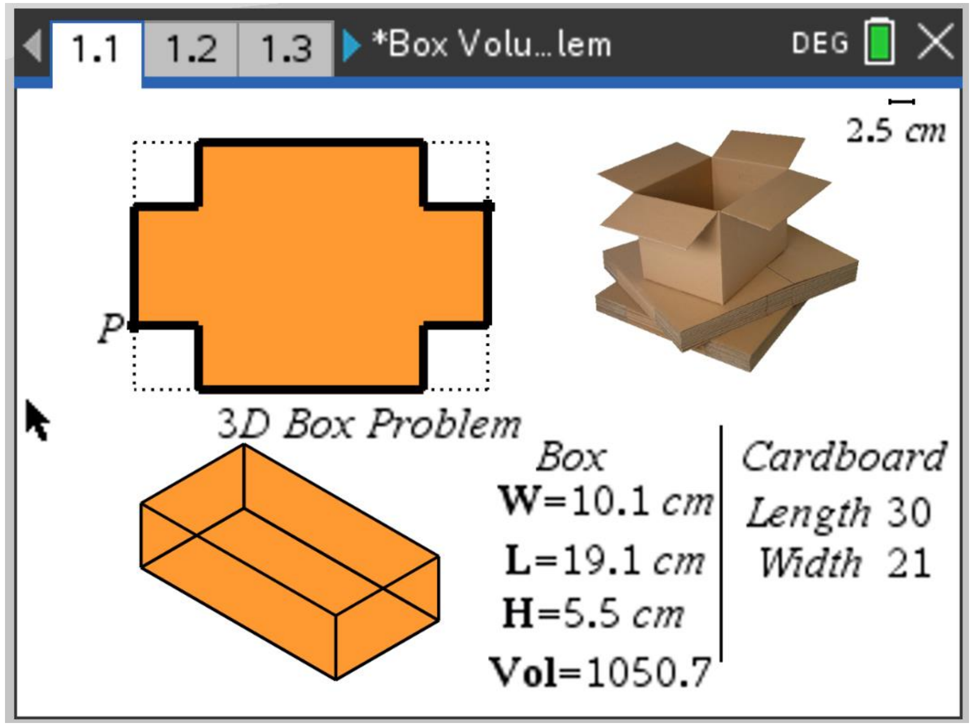


```
NORMAL FLOAT AUTO REAL RADIAN MP
CALC MAXIMUM
Y₁=X(30-2X)(21-2X)/(0<X and X<10.5…
Maximum
X=4.0559023          Y=1144.1674
```



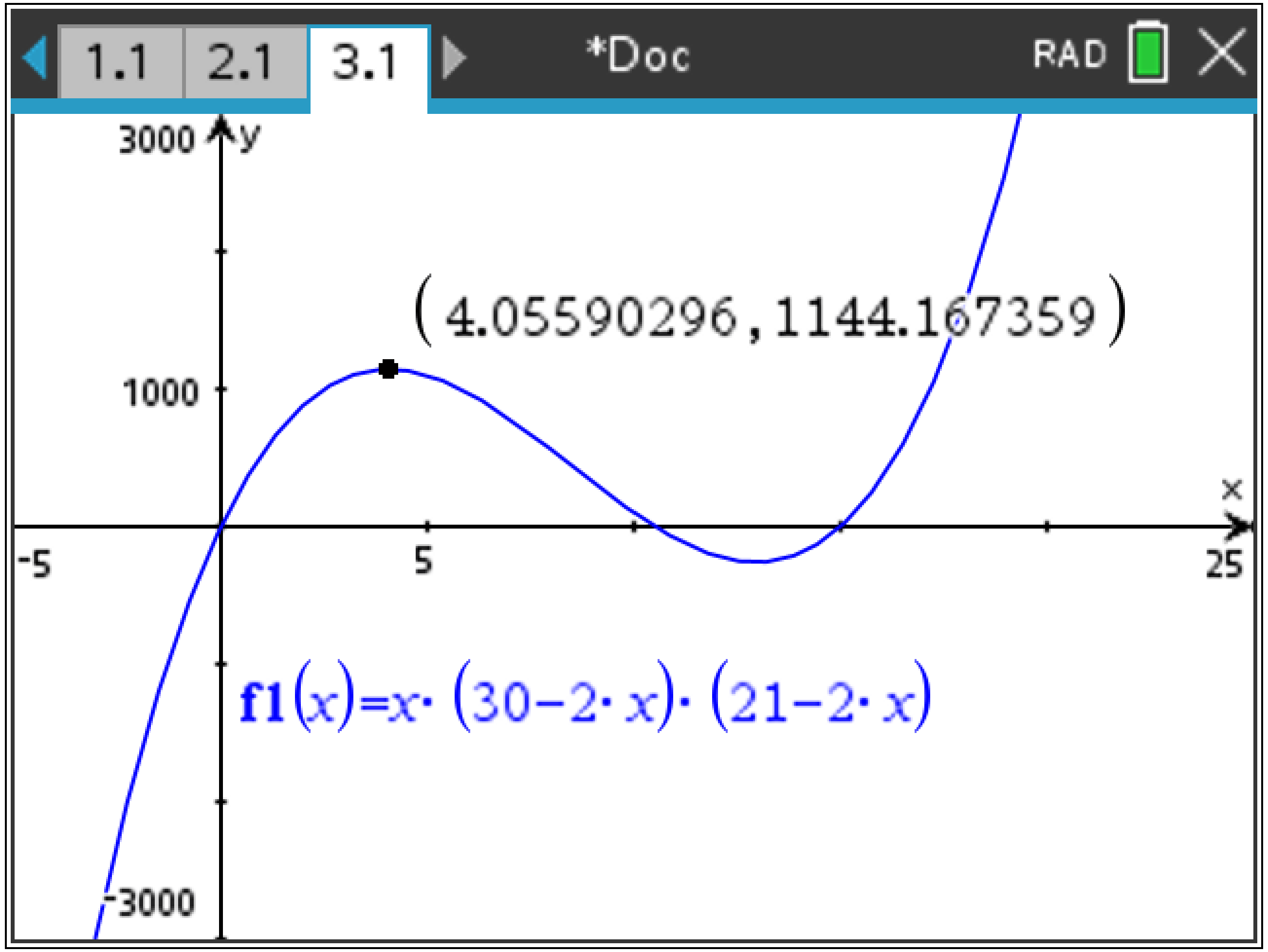
```
NORMAL FLOAT AUTO REAL RADIAN MP
FREE TRACE VALUES
WINDOW
 Xmin=-5
 Xmax=25
 Xscl=5
 Ymin=-3000
 Ymax=3000
 Yscl=1000
```

Restricted domain



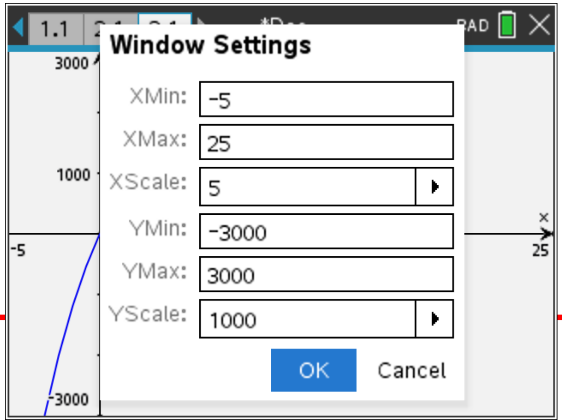
3D Box Problem

Box	Cardboard
W=10.1 cm	Length 30
L=19.1 cm	Width 21
H=5.5 cm	
Vol=1050.7	



(4.05590296, 1144.167359)

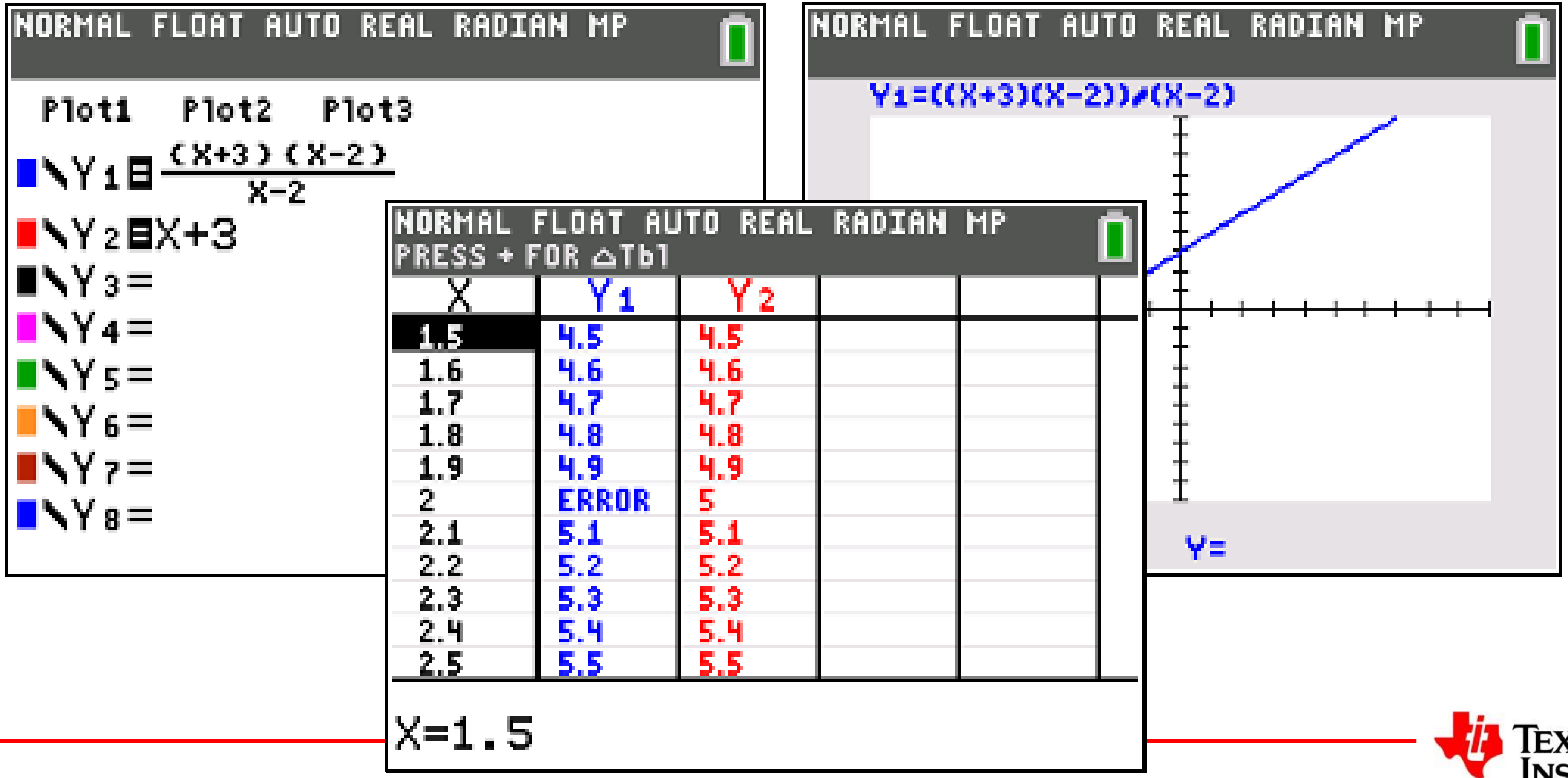
$f1(x)=x\cdot(30-2\cdot x)\cdot(21-2\cdot x)$



Window Settings

XMin: -5
XMax: 25
XScale: 5
YMin: -3000
YMax: 3000
YScale: 1000

Restricted domain – Holy graphs, Batman!



NORMAL FLOAT AUTO REAL RADIAN MP

Plot1 Plot2 Plot3

$Y_1 = \frac{(X+3)(X-2)}{X-2}$

$Y_2 = X+3$

$Y_3 =$

$Y_4 =$

$Y_5 =$

$Y_6 =$

$Y_7 =$

$Y_8 =$

NORMAL FLOAT AUTO REAL RADIAN MP

PRESS + FOR △Tbl

X	Y1	Y2			
1.5	4.5	4.5			
1.6	4.6	4.6			
1.7	4.7	4.7			
1.8	4.8	4.8			
1.9	4.9	4.9			
2	ERROR	5			
2.1	5.1	5.1			
2.2	5.2	5.2			
2.3	5.3	5.3			
2.4	5.4	5.4			
2.5	5.5	5.5			

X=1.5

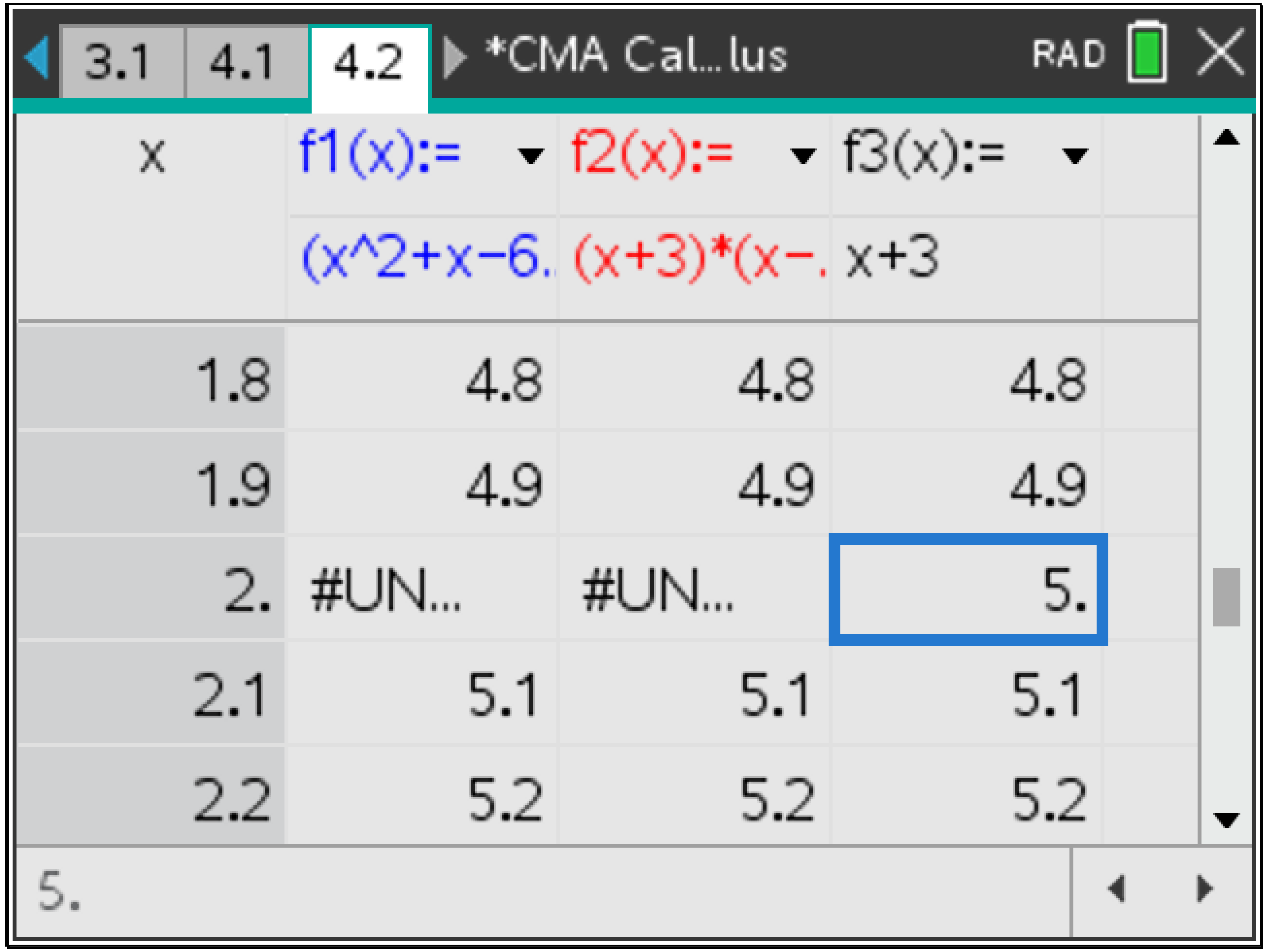
NORMAL FLOAT AUTO REAL RADIAN MP

Y1=((X+3)(X-2))/(X-2)

Y=

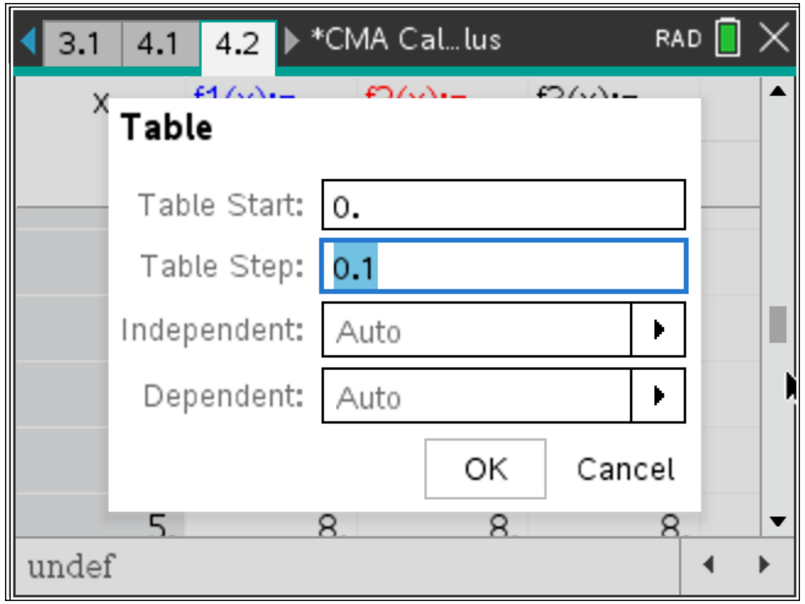
TEXAS INSTRUMENTS

Restricted domain – Holy graphs, Batman!



x	f1(x):=	f2(x):=	f3(x):=
	(x^2+x−6.	(x+3)*(x−.	x+3
1.8	4.8	4.8	4.8
1.9	4.9	4.9	4.9
2.	#UN...	#UN...	5.
2.1	5.1	5.1	5.1
2.2	5.2	5.2	5.2

5.



Table

Table Start: 0.

Table Step: 0.1

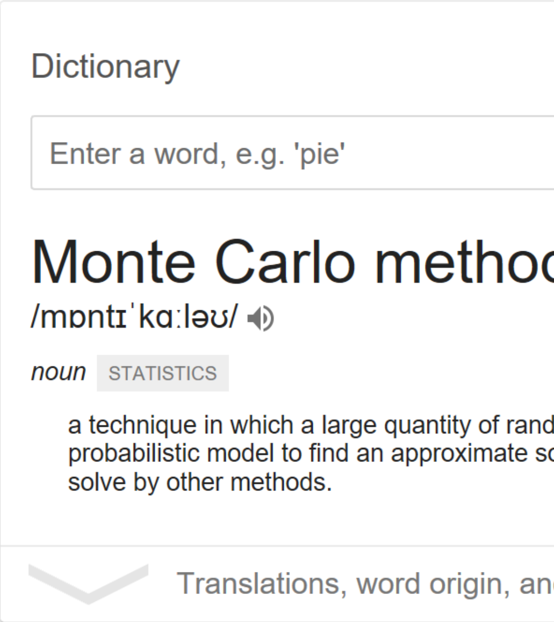
Independent: Auto

Dependent: Auto

undef

Monte Carlo Simulation:

This method us … lts that
may otherwise b



Dictionary

Enter a word, e.g. 'pie'

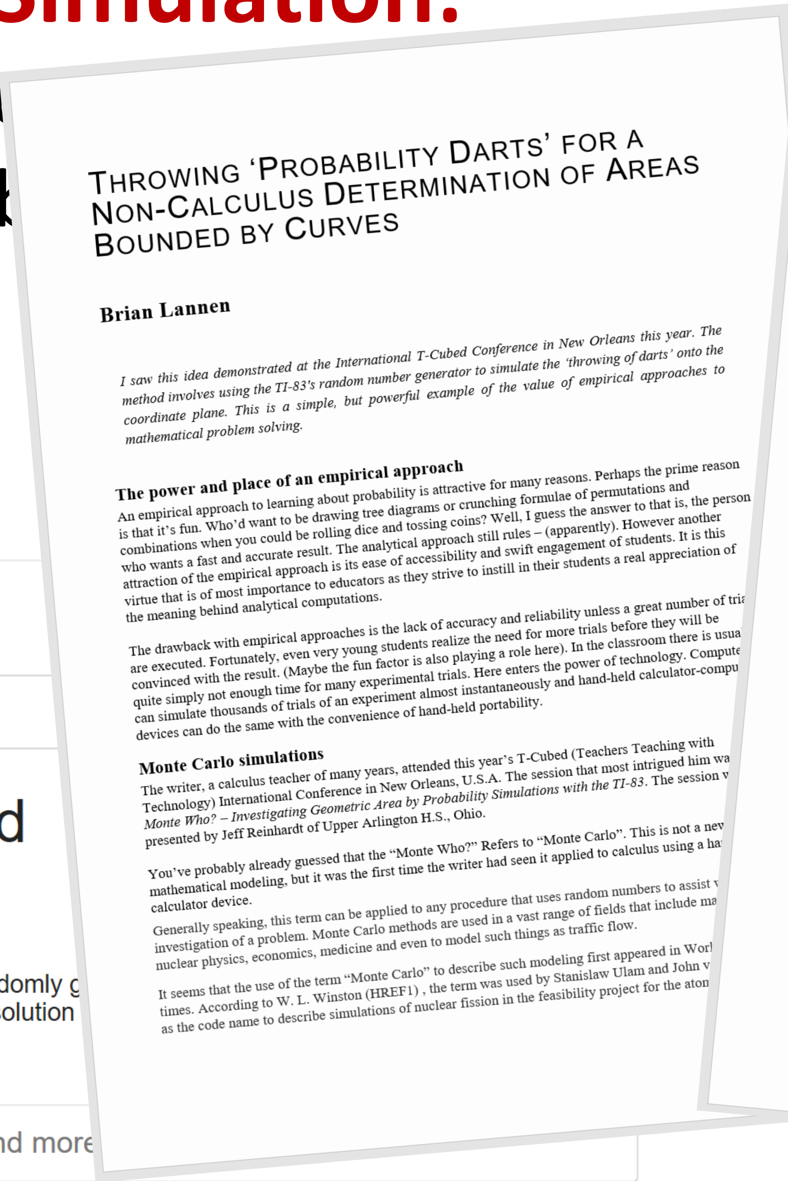
Monte Carlo method

/mɒntɪˈkɑːləʊ/

noun STATISTICS

a technique in which a large quantity of randomly g
probabilistic model to find an approximate solution
solve by other methods.

Translations, word origin, and more



THROWING 'PROBABILITY DARTS' FOR A NON-CALCULUS DETERMINATION OF AREAS BOUNDED BY CURVES

Brian Lannen

I saw this idea demonstrated at the International T-Cubed Conference in New Orleans this year. The method involves using the TI-83's random number generator to simulate the 'throwing of darts' onto the coordinate plane. This is a simple, but powerful example of the value of empirical approaches to mathematical problem solving.

The power and place of an empirical approach

An empirical approach to learning about probability is attractive for many reasons. Perhaps the prime reason is that it's fun. Who'd want to be drawing tree diagrams or crunching formulae of permutations and combinations when you could be rolling dice and tossing coins? Well, I guess the answer to that is, the person who wants a fast and accurate result. The analytical approach still rules – (apparently). However another attraction of the empirical approach is its ease of accessibility and swift engagement of students. It is this virtue that is of most importance to educators as they strive to instill in their students a real appreciation of the meaning behind analytical computations.

The drawback with empirical approaches is the lack of accuracy and reliability unless a great number of tria
are executed. Fortunately, even very young students realize the need for more trials before they will be
convinced with the result. (Maybe the fun factor is also playing a role here). In the classroom there is usua
quite simply not enough time for many experimental trials. Here enters the power of technology. Compute
can simulate thousands of trials of an experiment almost instantaneously and hand-held calculator-compu
devices can do the same with the convenience of hand-held portability.

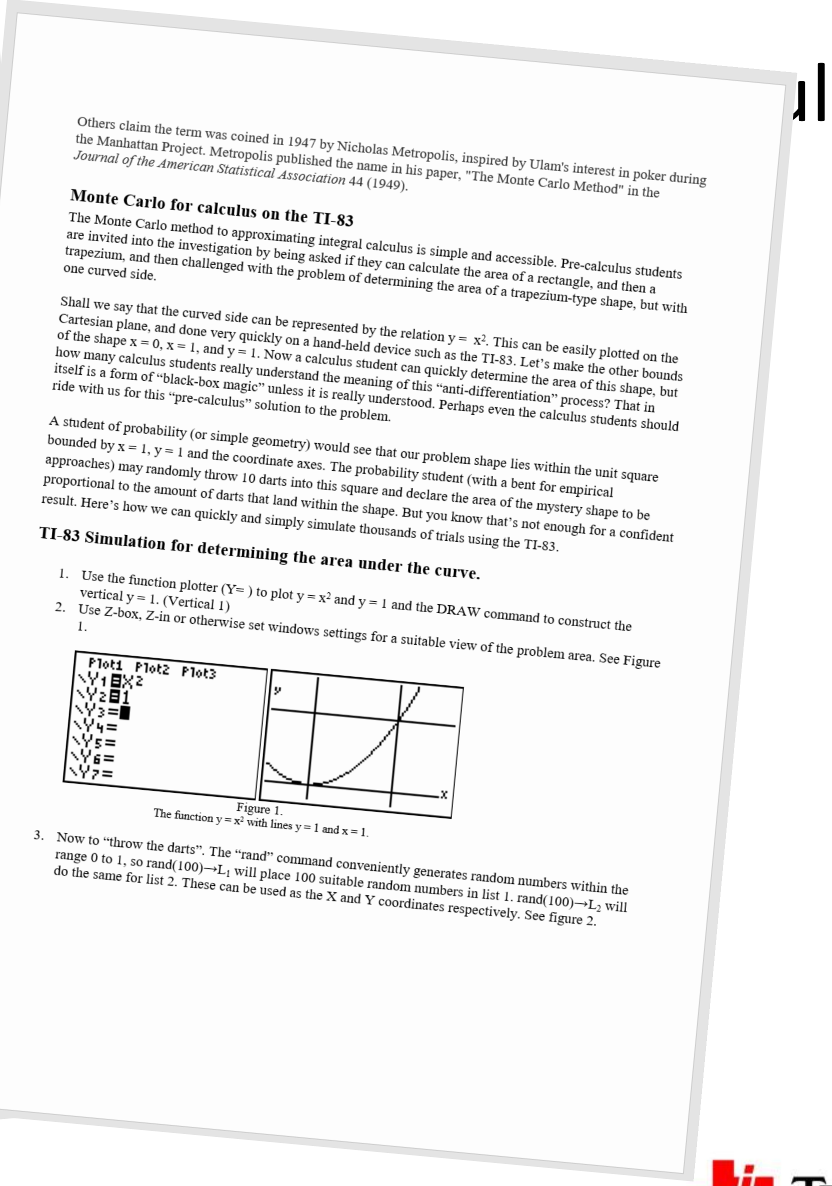
Monte Carlo simulations

The writer, a calculus teacher of many years, attended this year's T-Cubed (Teachers Teaching with
Technology) International Conference in New Orleans, U.S.A. The session that most intrigued him wa
Monte Who? – Investigating Geometric Area by Probability Simulations with the TI-83. The session v
presented by Jeff Reinhardt of Upper Arlington H.S., Ohio.

You've probably already guessed that the "Monte Who?" Refers to "Monte Carlo". This is not a nev
mathematical modeling, but it was the first time the writer had seen it applied to calculus using a ha
calculator device.

Generally speaking, this term can be applied to any procedure that uses random numbers to assist v
investigation of a problem. Monte Carlo methods are used in a vast range of fields that include ma
nuclear physics, economics, medicine and even to model such things as traffic flow.

It seems that the use of the term "Monte Carlo" to describe such modeling first appeared in Worl
times. According to W. L. Winston (HREF1) , the term was used by Stanislaw Ulam and John v
as the code name to describe simulations of nuclear fission in the feasibility project for the aton



Others claim the term was coined in 1947 by Nicholas Metropolis, inspired by Ulam's interest in poker during the Manhattan Project. Metropolis published the name in his paper, "The Monte Carlo Method" in the *Journal of the American Statistical Association* 44 (1949).

Monte Carlo for calculus on the TI-83

The Monte Carlo method to approximating integral calculus is simple and accessible. Pre-calculus students are invited into the investigation by being asked if they can calculate the area of a rectangle, and then a trapezium, and then challenged with the problem of determining the area of a trapezium-type shape, but with one curved side.

Shall we say that the curved side can be represented by the relation $y = x^2$. This can be easily plotted on the Cartesian plane, and done very quickly on a hand-held device such as the TI-83. Let's make the other bounds of the shape $x = 0$, $x = 1$, and $y = 1$. Now a calculus student can quickly determine the area of this shape, but how many calculus students really understand the meaning of this "anti-differentiation" process? That in itself is a form of "black-box magic" unless it is really understood. Perhaps even the calculus students should ride with us for this "pre-calculus" solution to the problem.

A student of probability (or simple geometry) would see that our problem shape lies within the unit square bounded by $x = 1$, $y = 1$ and the coordinate axes. The probability student (with a bent for empirical approaches) may randomly throw 10 darts into this square and declare the area of the mystery shape to be proportional to the amount of darts that land within the shape. But you know that's not enough for a confident result. Here's how we can quickly and simply simulate thousands of trials using the TI-83.

TI-83 Simulation for determining the area under the curve.

1. Use the function plotter (Y=) to plot $y = x^2$ and $y = 1$ and the DRAW command to construct the vertical $y = 1$. (Vertical 1)
2. Use Z-box, Z-in or otherwise set windows settings for a suitable view of the problem area. See Figure 1.

Figure 1.
The function $y = x^2$ with lines $y = 1$ and $x = 1$.

3. Now to "throw the darts". The "rand" command conveniently generates random numbers within the range 0 to 1, so rand(100)→L_1 will place 100 suitable random numbers in list 1. rand(100)→L_2 will do the same for list 2. These can be used as the X and Y coordinates respectively. See figure 2.

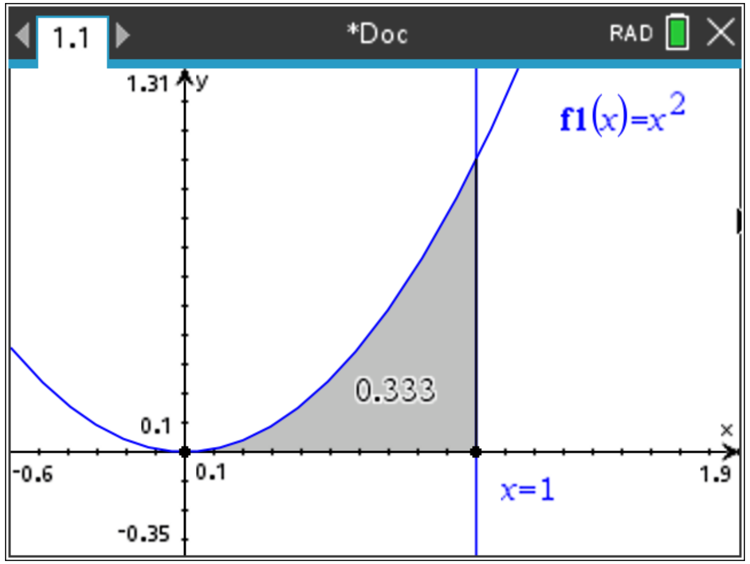
Monte Carlo method
From Wikipedia, the free encyclopedia

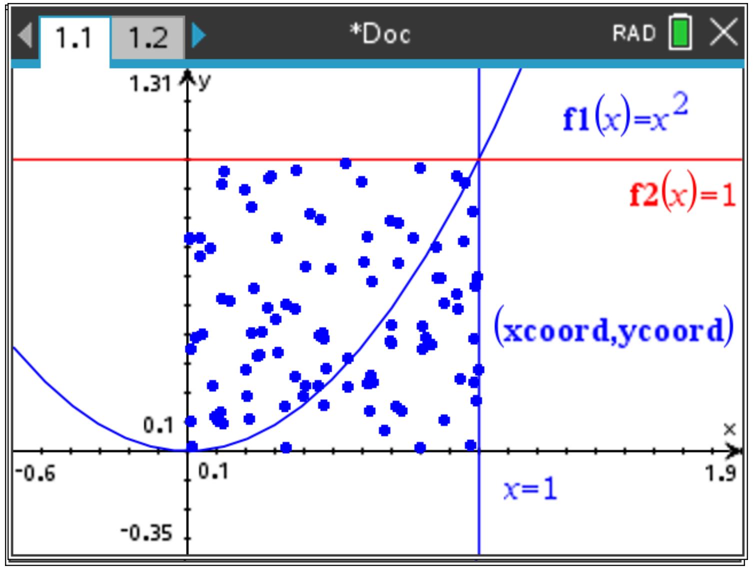
Monte Carlo methods (or **Monte Carlo experiments**) are a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results. Their essential idea is using randomness to solve problems that might be deterministic in principle. They are often used in physical and mathematical problems and are most useful when it is difficult or impossible to use other approaches. Monte Carlo methods are mainly used in three distinct problem classes:[1] optimization, numerical integration, and generating draws from a probability distribution.

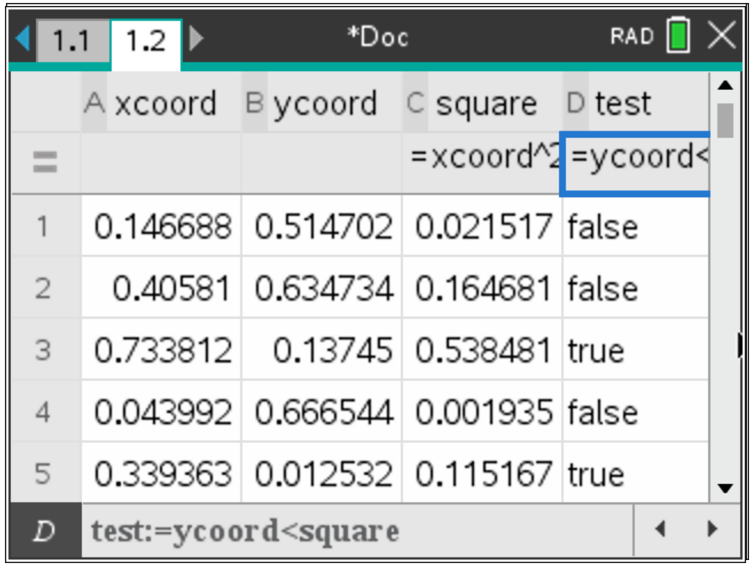
Being secret, the work of von Neumann and Ulam required a code name. A colleague of von Neumann and Ulam, Nicholas Metropolis, suggested using the name *Monte Carlo*, which refers to the Monte Carlo Casino in Monaco where Ulam's uncle would borrow money from relatives to gamble.[12]

Monte Carlo methods were central to the simulations required for the Manhattan Project, though severely limited by the computational tools at the time. In the 1950s they were used at Los Alamos for early work relating to the development of the hydrogen bomb, and became popularized in the fields of physics, physical chemistry, and operations research. The Rand Corporation and the U.S. Air Force were two of the major organizations responsible for funding and disseminating information on Monte Carlo methods during this time, and they began to find a wide application in many different fields.

Monte Carlo Simulation:

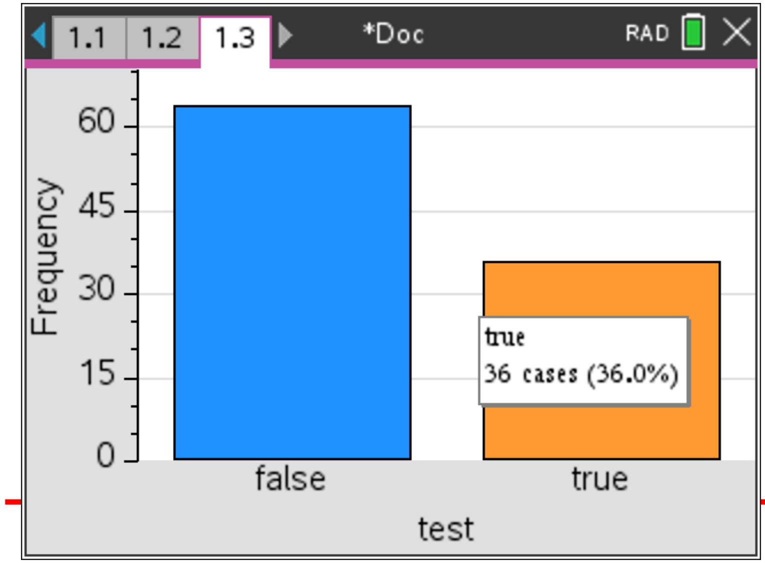






	A xcoord	B ycoord	C square	D test
=			=xcoord^2	=ycoord<
1	0.146688	0.514702	0.021517	false
2	0.40581	0.634734	0.164681	false
3	0.733812	0.13745	0.538481	true
4	0.043992	0.666544	0.001935	false
5	0.339363	0.012532	0.115167	true

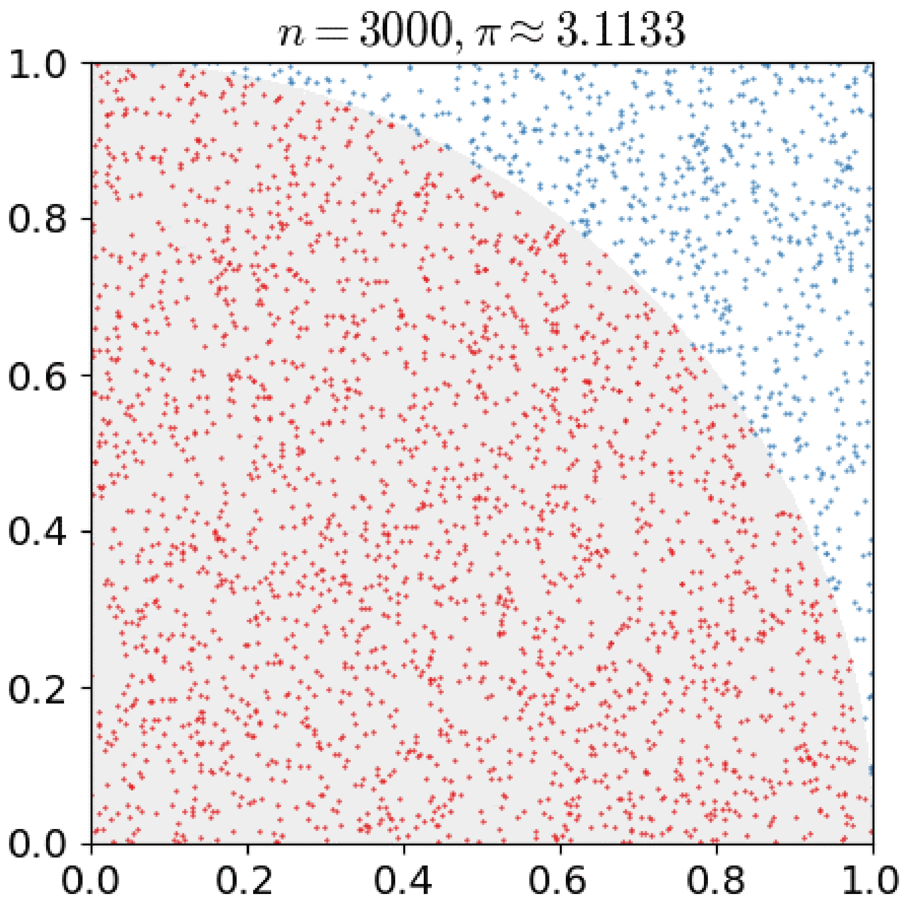
D test:=ycoord<square



Challenge:

Could you use this method to find an approximation for pi?

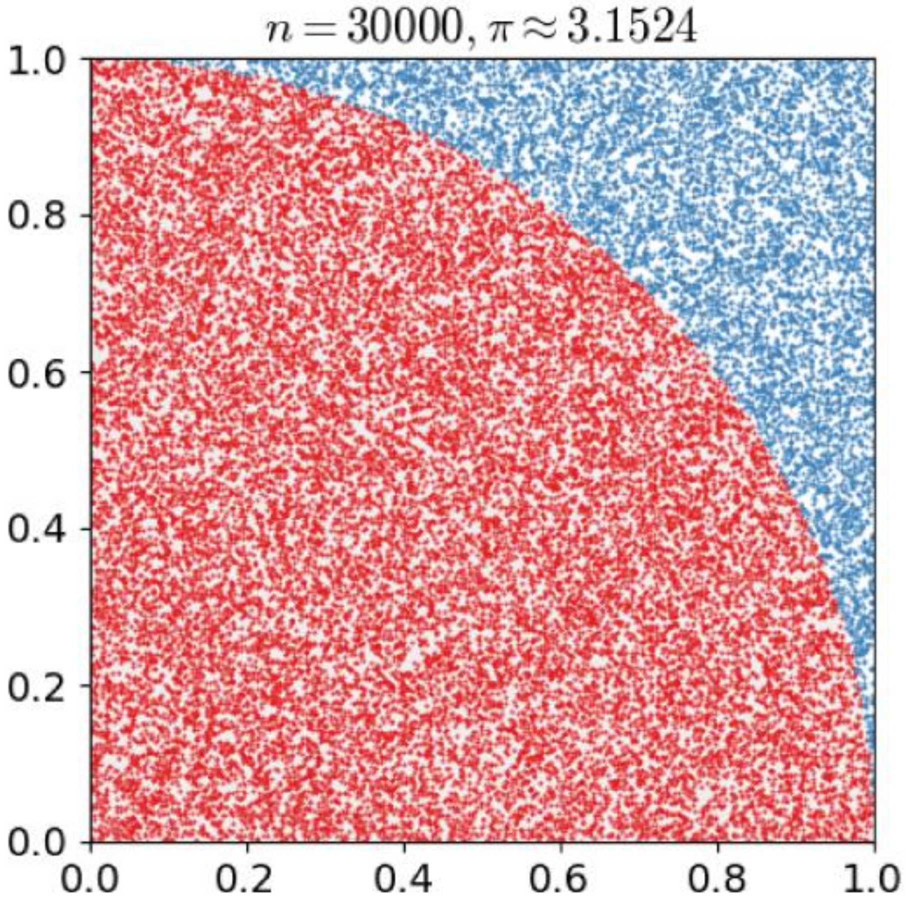
TEXAS INSTRUMENTS



Monte Carlo method applied to approximating the value of π. After placing 30,000 random points, the estimate for π is within 0.07% of the actual value.

As points are randomly scattered inside the unit square, some fall within the unit circle. The fraction of points inside the circle over all points approaches pi/4 as the number of points goes toward infinity. This animation represents this method of computing pi out to 30,000 iterations.

By nicoguaro - Own work, CC BY 3.0,
https://commons.wikimedia.org/w/index.php?curid=14609430



Monte Carlo method applied to approximating the value of π. After placing 30,000 random points, the estimate for π is within 0.07% of the actual value.

As points are randomly scattered inside the unit square, some fall within the unit circle. The fraction of points inside the circle over all points approaches pi/4 as the number of points goes toward infinity. This animation represents this method of computing pi out to 30,000 iterations.

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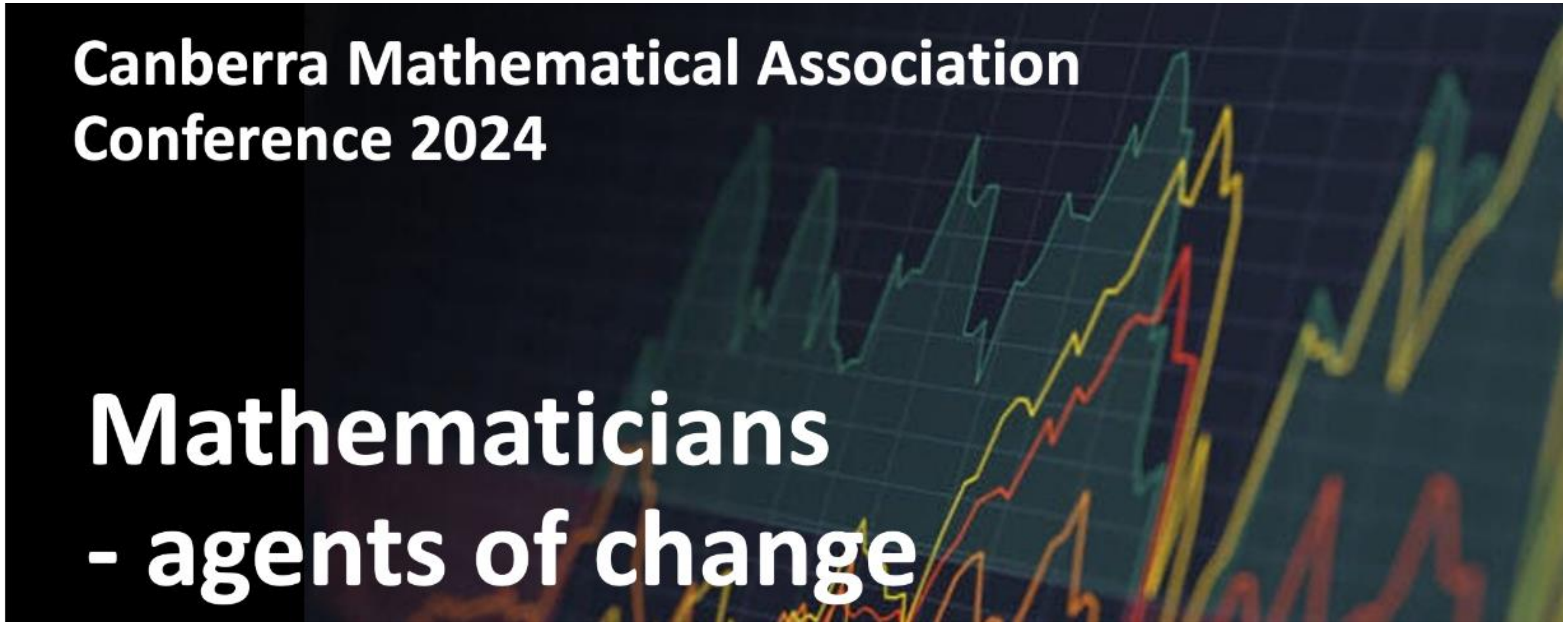
https://commons.wikimedia.org/w/index.php?curid=14609430



Investigating Change: Ideas for introducing Calculus

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