## SHORT CIRCUIT

Newsletter of the Canberra Mathematical Association INC


## NEWS AND COMMENT

The restrictions due to Covid-19 have disrupted life as we knew it in many ways. Several CMA events have been cancelled and other dates have been changed or made tentative. Teaching and learning have had to find alternative procedures.
Disruptions can lead to good outcomes, though. Let us aim for improvements that outweigh the annoyance and the tragedy of this epidemic.

In this edition we publish an article from a reader. (See page 3.) Contributions from readers engaged with education are very welcome in the pages of Short Circuit. This one springs from an anecdotal observation that touched off some pithy remarks relating to pedagogy, particularly in the online context.

## Conference

2020: With Hindsight, Creating a Vision
The organisers are proceeding optimistically towards presenting the annual conference on 15 August at ADFA.
If you or anyone you might wish to nominate would like to be a presenter at the conference, please refer to the information for presenters on our website.

Contact program coordinator Va-
lerie Barker, vnwb@internode.on.net, or conference coordinator Peter McIntyre, p.mcintyre@adfa.edu.au, by Friday 5th June to offer a presentation.

## National Mathematics Summer School

AAMT is going ahead with plans for the National Mathematics Summer School, 10-23 January, 2021 at ANU.
High achieving students going into year 12 who have a passion for mathematics can apply via the website https://nmss.edu.au/apply/.
Detailed information can be found at the websites for AAMT and NMSS. Students from ACT schools are selected by a CMA subcommittee, and some financial assistance is offered for attendees. There is a flyer on the CMA website.

## CANBERRA MATHEMATICAL ASSOCIATION

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## Coming Events:

CMA conference: 15 August 2020.
CMA AGM: 11 November, 2020.

Wednesday Workshop:

## MEMBERSHIP

Memberships run from 1 Jan to 31 Dec. each year. Membership forms can be accessed from the CMA website: http://www.canberramaths.org.au
Membership of CMA includes affiliation with the Australian Association of Mathematics Teachers and a subscription to one of two AAMT journals.
As a member, you are entitled to attractive rates for the CMA annual conference and CMA professional development events.
CMA members may attend conferences of the AAMT affiliates in other states, MAV, MANSW, etc. at member rates.

## PUZZLES

## Tricky

1. Three distinct numbers $a<b<c$ are in arithmetic progression. Their squares $a^{2}, b^{2}, c^{2}$ are in geometric progression. Their sum is $a+b+c=3 / 2$. Find $a$.

## Logical

2. Crack this three-digit numerical key given the following information:
427-One digit is right but in the wrong place
490-One digit is right and in its place
032-Two digits are right but both are in the wrong place
156-All digits are wrong
593-One digit is right but in the wrong place.

## Chess

3. Consider a $3 \times 3$ chessboard. Two black and two white knights are placed on the corner squares of the board, white and black on opposite edges. How many moves does it take for the white knights to swap positions with the black knights?

## Arithmetic

4. Add the two numbers
$0.1234512345 \ldots$ and $0.9876598765 \ldots$

## Literary

5. If someone on a boat on the Mississippi River calls "Mark Twain!" how deep is the water at that point?

## Number pattern

6. If the array shown below is continued, what numbers appear directly above and below 2020?

## 1

234
56789
10111213141516

## Edible

7. (From John Mack, Sydney) A rectangular chocolate bar is shaped as $m \times n$ individual equal sized squares. You wish to break it up into its $m n$ individual squares by a sequence of horizontal and vertical snaps. What is the minimum number of snaps needed?

## Athletic

8. In a 100 m race, A beats B by 10 metres, and B beats $C$ by 10 metres. If they sprinted at uniform speeds, by how many metres did A beat C?

## AMUSING BLACKBOARD

‘The Courier' from Ballarat published an article commenting that 'Covid-19 has changed almost everything ... the way we work ... [and] the way kids are educated'. Accompanying the article is the following posed, possibly stock, photograph.


We remark that teachers can and do make mistakes, and in the classroom these are often pointed out by the more attentive students. However, in an online context lacking the possibility of adequate two-way conversations, there is a risk that such errors will slip by and achieve the status of alternative facts.

## AAMT

The professional associations of mathematics teachers in each Australian state and territory are affiliated with the Australian Association of Mathematics Teachers.

Membership of AAMT is automatic on joining one of the local associations, such as CMA.

Benefits for AAMT members include resources accessible via the AAMT website. The journals Australian Primary Mathematics Classroom and Australian Mathematics Education Journal are published by AAMT and are distributed to members.

Download a complete PDF catalogue of resources sold by AAMT from www.aamt.edu.au/Webshop/Catalogue.
CMA members can order items from the AAMT catalogue at a discount.
AAMT has five Connect with Maths online communities teachers can join.
Also visit http://topdrawer.aamt.edu.au AAMT Top Drawer Teachers website.

## A READER WRITES

## Welcome to 'Mathsland'

Should we be teaching Year 8 mathematics students in a way that suggests to them that maths comprises a collection of rules, tricks, rote procedures, and 'my favourite way of doing it', where the experience and experimentation that were the basis for early years learning, are no longer important? Welcome to 'Mathsland', where this is typical teaching practice.

I have just experienced the fourth in a weekly series of online revision classes for Year 8 maths students. The first session was interesting - I was keen to see how the teacher approached the task, used technology, interacted with the students, presented resources, and how the students responded to this learning environment. Subsequent sessions have been less than riveting, indeed rather disturbing.

Today's lesson was on elementary algebraic manipulation - defining 'like terms', and then working with expressions involving addition, multiplication, and division of mixed algebraic terms. A puzzle was used as a motivator, and was revisited a couple of times later in the lesson - you know the kind of thing: a graphic image shown in four rows, with Row 1 showing 3 chickens said to be 'equal' to 60 ; Row 2 showing 1 chicken plus 2 nests (containing eggs) said to be equal to 26 ; Row 3 with 1 chicken plus 1 nest with eggs plus 1 bunch of bananas, together said to equal another number, and finally, Row 4 showing an expression (using images) to evaluate:
chicken + nest (with eggs) $\times$ partial bunch of bananas.
I can appreciate these types of puzzles, but the whole scenario is unrealistic. The relationship between the objects used in the puzzle and the mathematics to be used is non-existent. Solving the problem depends, first, on a complete suspension of all sense of reality (that is, we have entered Mathsland, where strange things happen for no apparent reason and I am expected to decipher obscure clues to come up with an answer that will be deemed 'correct' or 'incorrect'). Second, solving the problems depends on application of a mathematical rule, or convention, which also bears absolutely no connection with the objects of the puzzle. By the way, this Mathsland is the same place where people buy 69 watermelons and nobody wonders why. A colleague recently gave me another example of the extremes possible in Mathsland with questions like this: If a person weighing 100 kg needs a chair with four legs, how many legs should a chair for a 50 kg person have?

The lesson I watched was filled with 'algebraic expressions' that related to nothing in particular, yet students were expected to decode the information, apply some rules or conventions, or to proceed using 'the way I [the teacher] usually like to do these problems'. The language was loose. For example, in establishing that the expression $p q$ is equivalent to the
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NEWSLETTER OF THE CANBERRA MATHEMATICAL ASSOCIATION INC

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## ABOUT THE CMA

The Canberra Mathematical Association (Inc.) is the representative body of professional educators of mathematics in Canberra, Australia.
It was established by, among others, the late Professor Bernhard Neumann in 1963. It continues to run - as it began - purely on a volunteer basis.

Its aims include

* the promotion of mathematical education to government through lobbying,
* the development, application and dissemination of mathematical knowledge within Canberra through in-service opportunities, and
* facilitating effective cooperation and collaboration between mathematics teachers and their colleagues in Canberra.


## the 2020 CMA COMMITTEE

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expression $q$, the commutativity of multiplication was invoked and explained by saying that the letters in these expressions represent numbers. If you worked out a similar problem with numbers, such as $4 \times 3$, you realise it has the same value as $3 \times 4$, so because the numbers are commutative it is safe to assume the letters are also commutative. However, I note that it is the operation that is commutative, not the letters or numbers. (For example, $4-3$ is not the same as $3-4$.) This imprecision with language can so easily cause confusion and almost always needs to be repaired later.

If I was a student who loved engaging with abstract systems that worked by applying a set of rules and procedures with their own internal logic, perhaps I would get through Year 8 maths easily and look forward eagerly to Year 9, when the teacher has promised that I will be allowed to look at an expanded set of expressions that also include 'powers'. This is the 'lock-step' approach to curriculum typically seen in Mathsland.

Unfortunately, many students in Year 8 are not immediately excited by abstraction. The fact that they are all expected to go through the same set of learning steps that assume they all relate fulsomely to these types of abstractions, is likely why so many of them come to hate maths, think it makes little sense and does not connect to anything they understand about their lives. Dropping out of maths at the earliest opportunity is the consequence of this disaffection. Even worse, the students may be receiving the message that they are not capable of learning the stuff that everyone says is so important.

When I was a student (I went to school in Mathsland), I was taught the mnemonic BODMAS (standing for Brackets, Of, Division, Multiplication, Addition, Subtraction) that was intended to help me remember the mathematical convention for the correct order in which multiple arithmetic operations should be performed. For example, BODMAS would tell you to evaluate the expression $5+2 \times 3$ by doing the multiplication first $(2 \times 3)$, and then
the addition $(5+6)$, giving a result of 11 . If you did the addition first, you would get 21, and would probably be marked incorrect.

Nowadays, I think of calculations like this a bit differently. The experiences people have can sometimes lead to the need to carry out a sequence of calculations that would produce a different result depending on the order in which those calculations were performed. For example, a taxi fare typically involves a flat charge (the 'flag fall', let's say it is \$5) and a charge for each kilometre travelled (let's say I want to go three km , and the rate is $\$ 2$ per km ). If one wanted to formulate an expression to calculate the cost, the formulation would be firmly based on the problem situation being considered. The cost (C) of the fare would be $\$ 5$ plus two lots of $\$ 3$. Written as a formula,

$$
C=5+2 \times 3 .
$$

Each element of the mathematical formulation would relate to some feature of the problem being explored. The way in which the mathematical formulation would be processed to solve the problem would be clear from the demands and nature of the problem. No BODMAS needed.

The issues I highlight here are not just restricted to Year 8 nor Year 8 maths teachers. They are relevant tight through schooling. Abstraction is a very important part of mathematics - you could say an essential and defining feature. However, the ability to abstract and to use abstraction is built on the same fundamental experiential learning that helps young children learn to count. First, they notice the separation of objects in their world, which is an essential precondition for countability ('mum' and 'dog' are different objects). They learn the names and symbols for numeration, and that there is meaning in the ordering of those names and symbols (the numbers displayed in an elevator are ordered, and all elevators use the same ordering). Young children develop an idea of correspondence between objects and their labels (this is 1 apple;
here are 2 apples). They learn that the last number of a count represents the number of objects counted ( $1,2,3$ : there are 3 apples). They learn that the number of objects in a set is an abstract feature not fundamentally related to the characteristics of the objects counted (the colour of the apples doesn't matter). And they learn that when counting a set of objects they can start with any of them - the order in which the things are counted doesn't matter.

These are fundamental aspects of counting, an abstract activity that is firmly rooted in experience and experimentation. None of us learned that in Mathsland.

Perhaps maths instruction right through school can continue to emphasise experiential learning, and development of mathematical understanding based on underlying meaning, thereby building abstract principles from a firmer conceptual base?

Ross Turner
ACER, April 2020

## National mathematics talent Quest

The National Mathematics Talent Quest is now open for entries. All submissions should be in digital format this year. ACT schools should contact the Maths Association of Victoria for further details. Also, see here.

## PUZZLE SOLUTIONS

1. Without the condition $a<b<c$ the solution $a=b=c=1 / 2$ is not hard to find. But, to satisfy all conditions, careful attention to the geometric sequence reveals, $a=(1-\sqrt{ } 2) / 2, b=1 / 2, c=(1+\sqrt{ } 2) / 2$.
2. Key: 290
3. 16 moves
4. 1.1111...
5. About 3.6 metres. Author Samuel Clemens took the nom de plume Mark Twain from the calls made in depth soundings on the Mississippi riverboats. So, 2 fathoms.
6. The numbers of entries in the rows are consecutive odd numbers, so each row must end with a square number. The number 2020 is 5 less than $45^{2}$. So, the number above it is 4 less than $44^{2}$ and the number below it is 6 less than $46^{2}$. Thus, 1932 and 2110.
7. There are $m n-1$ breaks, whichever way they are done. A contributor to the AAMT email list likened this problem to the opposite one of assembling a jigsaw puzzle row by row. If it is true that a $p$ by $q$ block needs $p q-1$ joins, then a $p+1$ th row can be assembled and added to the block with $q$ new joins. Thus, there will be $p q-1+q=q(p+1)-1$ joins for the $p+1$ by $q$ block and this expression has the same form as for the $p$ by $q$ block. Since the rule works for a 1 by 1 block, and then any $m$ by $n$ block is built up from it by adding rows and columns, it must work for all $m$ and $n$.
8. 19 metres

Work out the speeds of B and then C in terms of A's time for the 100 m . From C's speed calculate the distance C covers in the time it takes for A to finish.

