CIRCUIT

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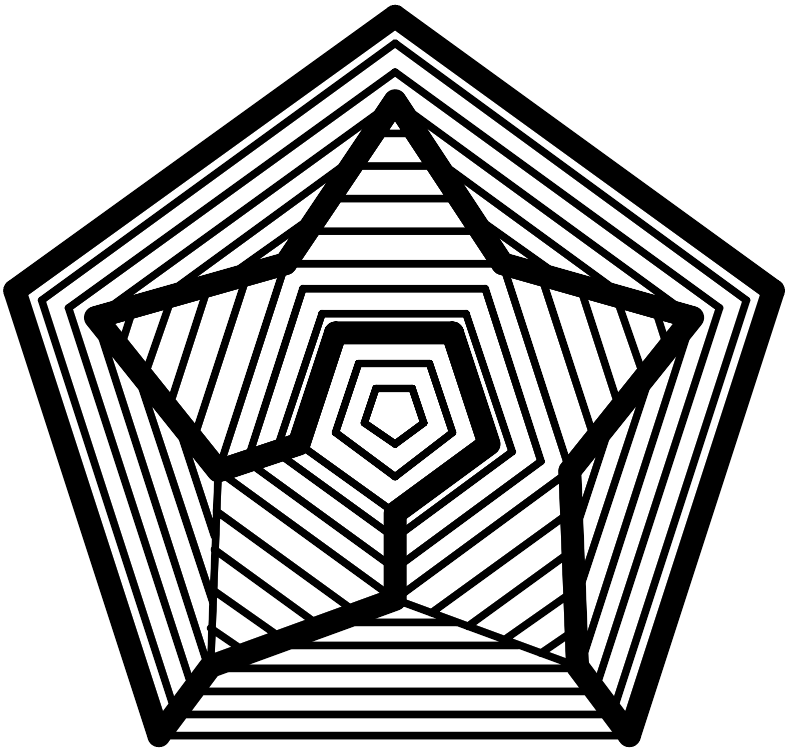

CANBERRA MATHEMATICAL ASSOCIATION

OFFICE BEARERS 1998

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CMA INTERNET HOME PAGE

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CONTENTS

Page

The objects of the Canberra Mathematical Association are to promote interest in mathematics, to encourage improvements in the teaching of mathematics and its applications, to provide means of communication among teachers and students and to advance the views of the Association on any question affecting the study or teaching of mathematics and its applications.

The Canberra Mathematical Association Logo depicts a Hamiltonian Circuit on a dodecahedron.

CONTRIBUTIONS

The Circuit publishing team is always keen to receive articles, notes, problems, letters and information of interest to members of the CMA.   Please contact a member of the publishing team if you wish to contribute to Circuit.

FROM THE PRESIDENT

With the last term for 1998 now well under way, my thanks again to those on CMA Council who have given their time and expertise this year. It is quite a commitment to be on CMA Council as many of you will know from experience.

The election of officers for 1999 will be held at the AGM on October 30 just before the 1998 Dinner. Please consider nominating for a position on executive or on Council. With CMA hosting the national AAMT conference in January 2001 it will be an interesting time to be involved, if not on CMA Council then as one of the team contributing to the conference. AAMT folk from around Australia still mention the 1980 Canberra conference as a wonderful and friendly conference. Take the chance to be part of the next one in 2001.

I would particularly like to thank Peter Enge for his leadership of the Circuit team of Margaret Rowlands and Kevin Taylor (and Rosemary Enge for her time and efforts), Heather and Andy Wardrop for their great efforts in setting up the four August Canberra Times articles on mathematics, and Annabelle Cassells for her part in keeping schools informed about coming events. Annabelle also plays a vital role in keeping CMA Council in touch with ACT Department of Education activities.

Steve Thornton has been a very valuable addition to the ACT mathematics community and to CMA over the last two years. We look forward to Steve's continued involvement over the next few years leading up to the AAMT conference. Steve keeps us in close touch with AAMT as a member of the AAMT Executive.

Margaret Rowlands has again made an important contribution to the professional development of her colleagues, and been a wonderful support to me in getting things done. Betty Growder as Secretary, and Jan Macdonald as Treasurer, have played important roles on Council, and my thanks go to them. Bob Bryce's quiet support, wisdom and encouragement are crucial to CMA links not only to ANU but to our other tertiary institutions. Paulene Kibble has been a key addition to Council this year as she has contributed her general as well as her special primary perspectives to Council. Paulene has organise a CMA banner to be launched at the AGM and Dinner. We encourage other primary teachers of mathematics to join Paulene on Council. A thank you also to Margaret McLaughlin for her insights and support on Council.

Recently, Dr Peter McIntyre organised an excellent evening for Year 10, 11 and 12 students at ADFA. Peter initiated three presentations by Professor Colin Pask, Dr Geoff Aldours, and Dr Rodney Weber. Their talks covered the role of mathematics in electronic networks, ultrasound and bushfire control. Thanks too, to Joan Robson who presented a short talk on Signadou courses. It is regrettable that more teachers and students did not take the opportunity to attend these interesting and challenging presentations.

We have taken on board comments about the repetition of menu at last year's dinner. Margaret McLaughlin has organised the CIT again as venue for us for Friday October 30, but this time we will have a buffet with three courses. Make it a night to meet up with "old" friends and colleagues. Lucky door prizes again!

Steve and Annabelle have organised quite a different evening for us on Monday 26 October, "Accessing mathematics through Asia, and accessing Asia through mathematics". The evening will begin with a guided tour of the Chinese Embassy from 4.30 to 5.30 pm, and then there will be a follow up workshop on teaching and learning strategies at Yarralumla Primary from 5.30 to 6.30. The cost will be $5 for members and $10 for non-members. Then we plan a banquet meal to follow at the Yarralumla Court Chinese Restaurant. If interested please let Annabelle know on Tel: 6205 9337 ASAP as numbers are limited.

Planning for the 2001 conference is underway. We will keep you informed as the program evolves.

Hope the remainder of Term 4 goes smoothly for you all.

Beth Lee, October 1998

NOTICE BOARD

FOR SALE:
Papers from the
1997 MANSW Conference

Treasurer Jan MacDonald has the collected papers from last year's MANSW Conference here in Canberra available for $10.

Contact her at Stromlo High School on 6205 6166 if you want a set.

FROM THE PUBLISHING TEAM

Apologies for the lateness of this *Circuit*. No, it hasn't been a three-term year, but yes this is only our third *Circuit* for the year. Unless something extraordinary happens, and in spite of our undertaking at the start of the year to put out one issue per term, this is probably our final effort for 1998.

We hope you have had a stimulating and fruitful year, and that whatever your situation you have occasionally found something of interest in *Circuit*. Producing a publication like this takes time and effort and next year we hope to be able to make the winkling out of material, editing and publishing more of a team effort. It would be particularly useful to have a couple of people on the team who would take responsibility for ferreting out contributions which reflect the diverse nature of mathematical activities and institutions in Canberra. Unfortunately this year we have not managed to reflect this wide range of activities quite as well as we had hoped.

As we head towards summer and the holiday season, perhaps some among you may even be inspired to reflect on your teaching or some aspect of mathematics to the point where a contribution to next year's *Circuit* is the result. Even a small trend in this direction would result in *Circuit* better reflecting the diversity of CMA members.

Once again Peter Enge would like to acknowledge the unstinting efforts of his wife Rosemary in typing up, editing and formatting *Circuit*. The publishing team also expresses grateful thanks to all members of CMA Council, and Jan MacDonald in particular, who assisted with the printing and mailing out of *Circuit*.

Have a peaceful, safe and relaxing summer break, and maintain your mathematical edge.

Reflections on the Third World Federation of National Mathematics Competitions Congress

Five Australians, Peter Taylor, Warren Atkins, Graham Pollard, Hans Lausch and Steve Thornton attended the third WFNMC Congress in Zhong Shan, China from 22 to 27 July 1998.
Here Steve Thornton and Warren Atkins reflect briefly on their experiences.

The conference was attended by approximately sixty delegates from some thirty countries, each delegate having a particular interest in promoting the study of mathematics via competitions. There were six topic areas, including resources, geometry, recreational mathematics, research, local competitions and national competitions, as well as keynote talks.

A short ferry journey from Hong Kong, Zhong Shan is a small (by Chinese standards!) city of about 1.4 million. It is part of a Special Economic Zone, and is relatively wealthy and visitor friendly. We enjoyed a tour of the local area, including visits to the birthplace of Dr Sun Yat Sen, and the site of the opium wars.

Part of the conference was held in an extremely impressive local high school. The school had been specially equipped as a demonstration school, each classroom boasting a high technology teacher console consisting of video, tape, computer with projection display, and overhead/photographic unit that could project solid objects as well as transparencies.

Steve Thornton:

It was interesting to me to hear a number of different perspectives on mathematics education, and to realise that the issues facing teachers in Australia are not unique. It was very interesting to note the similarities between Australia and Canada, both in terms of the status of teachers and current trends in mathematics education. The Canadians, in particular, share our desire to make mathematics competitions as inclusive as possible, rather than being the preserve of the elite. It was also quite challenging for me, personally, to listen to people who set and solve Olympiad style mathematical problems, at a level far beyond anything seen in schools.

The most rewarding part of the conference for me was the small group problem-solving. I opted, naturally, for the pre-Olympiad group, and joined a Canadian, Englishman and Latvian to solve, evaluate and improve a set of about fifty problems, aimed at students in about Years 6 to 10, that had been submitted for our consideration. Working with other teachers and mathematicians, sharing ideas for tackling problems and eventually arriving at some neat solutions (at least in a few cases) was a most rewarding experience. It highlighted to me the need for all teachers of mathematics at all levels to maintain an interest in mathematics, and particularly in using mathematics to solve problems.

Warren Atkins:

One of the things that comes out of such a conference for me is the relatively large number of mathematicians (university types) who are interested in promoting mathematics to school kids and who believe that an effective way to do this is through competitions and challenges. While there are a few who are interested only in the elite kids (and put their efforts in there), most seem to be interested in promoting mathematics at all levels.

It is also evident that there is a wealth of published resources for competitions and enrichment mathematics around the world, particularly in English, if we know where to get it and can afford it. It seems to me that there was a general feeling against the decline of geometry in schools, though I cannot understand why this has come to pass if mathematicians and teachers did not want the decline.

Some of the papers in the sessions I attended were excellent; some not so. Nevertheless it was a rewarding experience over all, though getting yourself back home to the grind and seeing some of the difficulties confronting teachers in our high schools dampens enthusiasm somewhat.

The next WFNMC Congress will be held in 2002, in Melbourne. We are particularly keen to involve classroom teachers in the conference, as it provides a unique opportunity to focus on problem-solving and to share ideas with, and learn from people from around the world. The WFNMC also publishes a journal, *Mathematics Competitions*, containing articles relating to the promotion of mathematics and problem-solving via competitions. Further information about the WFNMC can be obtained from the Australian Mathematics Trust.

Steve Thornton and Warren Atkins, Australian Mathematics Trust.

Group Theory

This is the transcript of an Ockham's Razor talk by Tom Petsinis broadcast by ABC Radio National on Sunday 8 March 1998, and introduced by Robyn Williams. It is gratefully reprinted here with permission from both Tom Petsinis and the ABC.

Summary

Tom Petsinis has written a novel about the mathematical genius Evariste Galois, who helped invent Group Theory.

Transcript

Robyn Williams: There have been some odd choices of subject for great literature: gloomy princes with parent problems in darkest Denmark; unrequited love on the Yorkshire Moors in midwinter; or how about this? The short life of an obscure Frenchman who helped invent Group Theory, a branch of maths that can curdle milk at twelve paces.

Well Tom Petsinis has done it: written a superb novel about just such a mathematical genius. He remembered the name from a lecture way back at the University of Melbourne, and then, when himself teaching at the Victoria University of Technology, colleagues encouraged Tom to try his ambition, to put his science and his art together in the form of a story. It's called The French Mathematician and its terrific.

Tom Petsinis: Evariste Galois was born on the 25th October, 1811 in Bourg-la-Reine, a town which at the time was just outside Paris. He was the second of three children born to Nicholas-Gabriel and Adelaide-Marie. His father, a liberal Republican, was Mayor of the town and owner of a private boarding school.

Galois' early education was supervised by his mother, a devout Catholic. At the age of twelve he was sent to Paris to study at the College of Louis-le-Grand. He showed no particular talent in his first three years at school. Small for his age, with a tendency to play the fool, he was demoted. He enrolled in a course of mathematics (not a compulsory subject at the school) simply for the sake of doing something different. This was to prove the decisive event in his life. Within a few months he was reading the works of Lagrange and exploring advanced mathematics.

In 1828 he attempted to enter the Polytechnic. Unprepared, he failed the examination and returned to Louis-le-Grand.

A year later he published his first mathematical paper and was already working on his Theory of Equations. In May and June of 1829, he submitted several papers on the Solubility of Equations to the French Academy, but was greatly disappointed by the Academy's lack of response. On the 2nd July his father committed suicide. A week later, Galois failed in his second and final attempt to enter the Polytechnic. He now began to engage in politics.

The following year he entered the Ecole Normale, a school that trained young men to become teachers. He published three more papers, but there was still no word from the Academy. Revolution swept through Paris in late July. Charles X was deposed and replaced on the throne by Louis-Philippe. Now an ardent Republican, Galois was expelled in December for accusing the school's director of cowardice during the Revolution. He joined the Artillery of the National Guard, a pro-Republican unit, only to see it disbanded a few weeks later by a Royal decree.

In January of 1831 he gave a few poorly-attended public lectures. In a desperate bid to make the Academy take note of his work, he submitted a third version of his paper on equations. As with the previous papers, nothing came of this. He became a member of the prominent Republican group called The Society of the Friends of the People. In May he was arrested on a charge of threatening the life of Louis-Philippe. He spent a month in prison before being acquitted. Another arrest followed in July, this time for being illegally armed and wearing the uniform of the disbanded unit. He spent nine months in prison.

In March of 1832 his poor health forced the authorities to transfer him to a sanatorium. He was released on the 29th of April. A month later he was challenged to a duel in somewhat mysterious circumstances. Fearing the worst, he spent the evening writing his famous last letter, in which he summarised his discoveries. He addressed the letter to Chevalier, entreating his friend to find a mathematician who might understand its contents. The following morning he was seriously wounded and taken to the Cochin Hospital, where he died a day later, the 31st May, 1832.

This biographical sketch makes clear that Galois' tempestuous life was caught between the interplay of history and the individual, creativity and tradition, order and chaos, genius and self-destructiveness. Novels, plays and films have been based on the artists, musicians and poets of the Romantic age, yet there is very little on its scientists and mathematicians. Quantum physics proposes the possibility of multiple universes existing parallel with the 'real'; or 'visible'; universe determined by the presence of an observer. This is a good analogy for the approach I took with my subject. It could be agued that multiple personalities of historical figures exist in parallel within historical texts, and that in writing about Galois the person, as opposed to the mathematical work, one particular Galois is realised from countless possibilities.

For me then, the task of recreating Galois was very much like trying to solve a mathematical equation. Given the accepted facts, the dichotomies in his personality, the speculations about his death, and a host of unknowns, his life became an equation with many variables. But as I thought more about the accepted differences between mathematics and literature, I soon realised that they weren't so different after all. As the presence of an experimenter is said to affect the outcome of an experiment, so the reader's subjectivity infuses the text with life and validity. My re-creation of Galois, therefore, would be valid to the extent that readers could raise him from the text and body him forth into the present.

My work would not be a conventional third person narrative in the past tense; I wanted to get close to my character, explore him in ways others had not done, see and hear things that were beyond the range of biographers, and for this I needed a first person voice in the present tense.

The book that made Galois known to a wide circle of English readers is *Men of Mathematics*, written by E.T. Bell in 1937. In a chapter entitled 'Genius and Stupidity', Bell portrays his subject as a misunderstood genius surrounded by spite, envy and foolishness. Bell saw mathematics as a new religion, with God as the primal geometer and pure mathematicians as His prophets. But as Bell rightly knew, a religion needed its heroes and martyrs. What better martyr than one killed at the age of 20 at the hands of ignorance and stupidity? As he was not a creative mathematician, Bell served mathematics by raising statues and making legends. Unfortunately, in his zeal to spread the faith, he twisted and rearranged facts. He created the legend that Galois discovered his findings in a fit of inspiration on the eve of the duel. Yet we know from the dates of manuscripts and their submissions to journals that Galois' discoveries were made over a period of several years.

The only book-length biography of Galois in English is Leopold Infeld's *Whom the Gods Love,* published in 1948. This is a curious work, a mixture of fact and fiction, accurate scholarly research and wild speculation. Like Bell, he raises his subject on a pedestal with the words: "Galois is perhaps the greatest mathematical genius who ever lived." Infeld's work contains two serious flaws. He has Galois seduced by a woman who, acting as a police agent, succeeds in luring the young revolutionary into a fatal duel. He calls this woman Eve Sorel, a prostitute. The facts are quite different: research has revealed that she was Stephanie Dumotel, the daughter of the resident doctor at the sanatorium where Galois spent the last months of his life. The plot thickens: Infeld makes Galois' adversary a police agent. Yet in his *Memoirs* Alexandre Dumas states quite clearly that Galois was killed by Pecheaux d'Herbinville, a loyal Republican. In several letters to friends written on the eve of the duel, Galois mentioned that he was sworn to secrecy concerning his opponent's identity, adding that the fellow was 'of good faith', meaning a good Republican.

In asking myself what had driven Infeld to write his book, the question turned on me. In the foreword to his book, Infeld wrote that he used Galois to allay his own fears and insecurities, to exorcise his demons through a kind of identification with Galois. Is this perhaps always the relationship between subject and writer? Does the subject offer the writer something lacking in his or her life? Why else would a writer sacrifice flesh and blood to paper and ink? What were my motives in writing about Galois? How many people had capitalised on the unfortunate life? How many had profited precisely because he had suffered? Was I about to exploit him for my own ends? No, I thought. In recreating Galois, I would reclaim his life and do justice to his complex character. Perhaps the relationship between writer and subject was more symbiotic than parasitic, a relationship based on trust and mutual betterment.

In order to understand my subject and his work, I studied and researched the mathematical, political, social and cultural aspects of Galois' world. Post-Napoleonic Paris was in a state of chaos. Bourbons, Legitimatists, Bonapartists, Republicans, Socialists, Utopians, Saint Simonists, Anarchists, Jesuits, these and many other groups were competing to win hearts and minds. It's ironical that Galois' work on mathematical groups and their structure arose from this social and political turmoil. Perhaps his longing for order and harmony was a kind of reaction to a world in chaos.

As political reaction to conservatism turned to revolution, Classicism in the arts was being challenged by Romanticism. Artists saw new possibilities for content and form. The theatre in Paris had enormous influence, and Victor Hugo's plays *Cromwell* and *Hernani* were influential in spreading the new ideas. Eugene Delacroix expressed the new spirit through paintings 'Liberty leading the People'. In mathe-matics, challenges were made on the Classical temple of Euclidean geometry. The axioms and postulates in Euclid's text, *The Elements* had been the irrefutable basis of geometry for 2,000 years. Suddenly, his fifth postulate, often called the parallel line postulate, came under attack. Quite independently of each other, the Russian Lobachevsky and the Hungarian Bolyai proposed a non-Euclidean geometry based on curved surfaces, in which the sum of the angles of a triangle could be more or less than 180-degrees. Galois' work was also imbued with the Romantic spirit. The radical nature of his approach to equations, together with his terse exposition, which often contained intuitive leaps, produced a new algebra.

By abstracting equations and looking at their solutions in terms of groups, Galois was able to determine their solvability. He wasn't concerned with finding particular solutions, but in the more fundamental question of determining *a priori* whether or not solutions existed.

Overlooked in his lifetime, Galois' discoveries were eventually brought to the attention of the mathematician Joseph Louisville, who edited his papers in 1843 and announced a new, fertile area of mathematics. The importance of Galois' work is clear from the tribute paid to him by Camille Jourdan, who, in 1870, published a 600-page mathematical text which he considered nothing more than a footnote to Galois' work.

In this century, Galois' seminal discoveries have continued to inspire researchers. A branch of mathematics, Galois Theory, has been named after him. His Group Theory approach to algebra has led to the recognition of structure as an important aspect of contemporary mathematics. Like so much of pure mathematics, his abstract theories have been used to make advances in applied areas such as crystallography, genetics and nuclear physics. A few years ago Andrew Wiles put forward a solution to Fermat's *Last Theorem* which had baffled mathematicians for almost 400 years. The 200-page proof would not have been possible without Galois Representations.

Of the material I researched in order to recreate Galois, two books deserve special mention. D.E. Smith's *A Source Book in Mathematics*, which contains a translation of Galois' famous last letter; and photocopies of Galois' original papers gathered and edited by Bourgne and Azra. From the latter, I sensed Galois' personality emerging, not so much from the mathematics (indeed, it may be argued that mathematics and personality are mutually exclusive), as from the words, scribbles and sketches in the margins of these papers. There are even transcriptions of the letters sent to him by the woman responsible for the duel. It is in these hastily written asides, in the sketches of faces, in the words, "I have no time" that we perhaps best glimpse the youth of 20, writing feverishly in order to overcome the fear of death and to save his soul through mathematics.

Robyn Williams: The book is called *The French Mathematician* and it's highly recommended. Published by Penguin. You've been listening to the author, Tom Petsinis, lecturer in mathematics at the Victoria University of Technology.

Tom Petsinis
Lecturer in Mathematics,
Victoria University of
Technology, Melbourne

If readers of *The French Mathematician* would like to submit written responses or reviews, *Circuit* would be only too happy to print them in the next issue.

Accelerating Students in Years 8 and 9

In late August Rex Boggs from Glenmore High School, Rockhampton, initiated a vigorous discussion in the AAMT Internet Mailing List Community. In brief, Rex wondered whether it was reasonable to propose an accelerated mathematics programme for capable high school students. This stimulated a flurry of interesting emails. The following contribution from Steve Thornton deftly rounded off the debate.

To me there are two key issues. One is what we value as mathematical knowledge. In most cases it seems to be procedural competency rather than understanding. For the high achievers, conceptual understanding often develops in spite of what we do, rather than because of it. The focus on procedures is a significant factor in reducing the time available to study worthwhile and challenging mathematics, and is part of the reason why high achievers are bored and uninterested. I wondered whether acceleration is just introducing a greater number of procedures, rather than helping to deepen understanding. If so, is there really any value? Quality, rather than quantity, seems to me to be the key.

The second key issue relates to quality versus quantity. How effective is the time we currently spend? It seems to me that, in junior secondary school, we spend a huge amount of time revisiting the same topics year after year, almost always in the same way. The first topic in Year 7 is operations with whole numbers (presumably because we don't expect them to remember what they learnt in primary school). The first topic in Year 8 is fractions and decimals (because they forget during the holidays what they learnt in Year 7). And so on. For students who struggled the first time around, repetition is usually unproductive, although maturity does play a part for a few; for those who succeeded the first time around, repetition is boring and pointless. The repetition also conveys the message to students that it doesn't really matter how well they learn something because they will do it again the next year anyway.

The so-called 'spiral curriculum' seems to be an unquestioned basis for most of what happens in school mathematics. For example, if you look at the number section at the beginning of most, say, Year 8 texts, it tends to focus on computational skills and, maybe, listing factors and multiples. But why re-visit computation through some process of repetition of Year 7? There is an almost endless supply of really challenging problems concerned with primes, factors and multiples that only need some elementary algebra and number facility. They give the lower achieving students plenty of practice and revision, if that's what's wanted, but at the same time develop a much richer and deeper understanding of number for those who can see the bigger picture.

The same argument can be used about almost every topic in the junior secondary curriculum. By asking students to repeat topics in the same way as in previous years, which is what happens a lot now, we perpetuate a curriculum model which is based on knowing procedures rather than understanding mathematics, and, at the same time, waste a huge amount of time.

So I think when we look at any issue such as grouping, we should first look at the basis of our curriculum, and ask a more fundamental question about what we really value in mathematics. If it's conceptual understanding, then depth, time and, most importantly, problem-solving are the keys.

The difficulty with a curriculum which is based on conceptual understanding is that it is unfamiliar to teachers, and we don't really know how to go about it. I would really like to hear from people who have looked at a curriculum model in which conceptual understanding of mathematics is developed through problem-solving. Note: this is not the same as having some problem solving lessons in mathematics lessons!

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QUOTABLE NOTES AND NOTABLE QUOTES

Mathematics in Kindergarten

"A small boy of five came into the kindergarten one morning with radiant face and sparkling eyes, crying out in joyful tones: "I have something for you! It's hard and long and has four edges and two ends!" The precious object was held behind him, while he danced around in fond anticipation of the pleasure he was about to give his teacher, of whom he was very fond. "What can it be?" she answered, entering sympathetically into his pleasure. "Do show it to me." In proud triumph the hand which held the treasure was extended, and in the palm lay a burnt match. And the kindergarten teacher accepted it as a gift of value, for had it not helped to unlock the great world of form and its elements - faces, corners, and edges?"

From a nineteenth century kindergarten teacher's report via page 226 of *The Penguin Book of Curious and Interesting Puzzles* by David Wells

Sylvester's Tribute to Mathematics and Selected Mathematicians

J. J. Sylvester (1814-1897) was an eminent and prolific mathematician into his eighties. He worked in both England and America, founding the American Journal of Mathematics at Johns Hopkins University in 1878. He was appointed Savilian Professor of Geometry at Oxford University in 1885.

"... there is no study in the world which brings into more harmonious action all the faculties of the mind than the one of which I stand here as the humble representative and advocate. There is none other which prepares so many agreeable surprises for its followers, more wonderful than the transformation scene of a pantomime, or, like this, seems to raise them, by successive steps of initiation, to higher and higher states of conscious intellectual being.

This accounts, I believe, in part for the extraordinary longevity of all the greatest masters of the Analytical Art, the Dii Majores of the mathematical Pantheon. Leibnitz lived to the age of 70; Euler to 76; Lagrange to 77; Laplace to 78; Gauss to 78; Plato, the supposed inventor of the conic sections, who made mathematics his study and delight, who called them the handles or aids to philosophy, the medicine of the soul, and is said never to have let a day go by without inventing some new theorems, lived to 82; Newton, the crown and glory of his race, to 85; Archimedes, the nearest akin, probably, to Newton in genius, to 75, and might have lived on to be 100, for aught we can guess to the contrary, when he was slain by the impatient and ill-mannered sergeant sent to bring him before the Roman General, in the full vigour of his faculties, and in the very act of working out a problem; Pythagoras, in whose school, I believe, the word mathematician (used, however, in a somewhat wider than its present sense) originated, the second founder of geometry, the inventor of the matchless theorem which goes by his name, the precognizer of undoubtedly the miscalled Copernican theory, the discoverer of the regular solids and the musical canon (who stands at the very apex of this pyramid of fame), if we may accept the tradition, after spending 22 years studying in Egypt and 12 in Babylon, opened school when 56 or 57 years old in Magna Graecia, married a young wife when past 60, and died, carrying on his work with energy unspent to the last, at the age of 99. The mathematician lives long and lives young; "the wings of his soul do not early drop off, nor do its pores become clogged with the earthy particles blown from the dusty highways of vulgar life."

Some people have been found to regard all mathematics, after the 47th proposition of Euclid, as a sort of morbid secretion, to be compared only with the pearl said to be generated in the diseased oyster, or, as I have heard it described, 'une excroissance maladive de l'esprit humain.' "

J. J. Sylvester in his Presidential Address to the British Association for the Advancement of Science, 6 January 1870.

An Unusual Mathematical Motto

"Another roof, another proof."

Paul Erdös (1913-1996)

Erdös was the peripatetic mathematician who travelled around the world from one mathematics department to another, living out of the contents of two small suitcases, solving problems and writing papers with about 300 co-authors in the course of his life.

The Persistence of Weather

Although there seems to be nothing reliable about the weather, especially when planning a picnic, there is actually a (very) long range consistency at work. Armin Bunde at the University of Giessen, and his colleagues from Milan, Potsdam, and the Bar-Lan University (Israel) have now conducted the most thorough multi-year study yet of correlations in daily temperature records. What they did in effect is to ask: if the weather is sunny and warm today, what will be the likelihood that it will be sunny and warm tomorrow, and the next day, and after x days?

Choosing randomly 14 meteorological stations worldwide - ranging from Pendleton, Colorado (with 57 years of data) to Prague (with the longest daily temperature record, 218 years) - and factoring out seasonal effects by comparing not temperatures but departures from the average daily temperature, they are able to tease out the persistent rhythms of temperature. As expected, they observed that after x days, the weather is less and less likely to be similar to that on day 1.

In particular, they find:

- that the falloff in correlation is **not** exponential in nature (proportional to e^{-x}) as many had thought; rather it obeys a power-law rule (i.e., the correlation is actually proportional to x^k);
- the exponent, k, with a value of -0.65, is roughly the same for all the cities;
- the persistence of this behavior seems to hold over at least a decade and maybe as long as a century or more.

From *Physics News Update*,
American Institute of Physics Bulletin of
Physics News, Number 383, July 24, 1998.

A Necessary and Sufficient Condition for Ensuring Orderly Discussion in Meetings, Classrooms, etc.

Allow only one interruption at a time.

It's a Dangerous Existence in Australia Too

Circuit reader Yvonne Wisbey of Flynn has enclosed a couple of tables from the *ABS 1997 Year Book Australia* in response to our plea in the last issue. Yvonne says she was amazed by the data on tooth loss, and we are sure that readers will find the 1995 Australian figures on external causes of death a useful trigger for discussion and investigation with their students.

Not many of the categories for Australian and British death rates are directly comparable but the risk of death from motor vehicle traffic accidents is quite similar in both countries (about 1 in 8000), while in Australian the risk from death by homicide (1.8 per 100 000) is nearly twice the corresponding British rate (1 per 100 000). Of course we are assuming common definitions of terms in these comparisons, always an issue in statistical comparisons.

EXTERNAL CAUSES OF DEATH - 1995

Cause of death	no.	%	Crude death rate(a)
MALES			
Suicide	1 872	36.3	20.8
Motor vehicle traffic accidents	1 398	27.1	15.6
Accidental falls	457	8.9	5.1
Homicide	204	4.0	2.3
Drowning and submersion	190	3.7	2.1
Poisoning by drugs/medications	201	3.9	2.2
Other	831	16.1	9.2
All external causes	5 153	100.0	57.3
FEMALES			
Suicide	495	21.9	5.5
Motor vehicle traffic accidents	631	27.9	7.0
Accidental falls	538	23.8	5.9
Homicide	129	5.7	1.4
Drowning and submersion	69	3.1	0.8
Poisoning by drugs/medications	97	4.3	1.1
Other	301	13.3	3.3
All external causes	2 260	100.0	24.9
PERSONS			
Suicide	2 367	31.9	13.1
Motor vehicle traffic accidents	2 029	27.4	11.2
Accidental falls	995	13.4	5.5
Homicide	333	4.5	1.8
Drowning and submersion	259	3.5	1.4
Poisoning by drugs/medications	298	4.0	1.7
Other	1 132	15.3	6.3
All external causes	7 413	100.0	41.1

(a) Deaths per 100 000 mid-year population.
Source: Derived from ABS Causes of Death data 1995.

PREVALENCE OF TOOTH LOSS

		Age group						
Year	Unit	15-24 years	25-34 years	34-44 years	45-54 years	55-64 years	65-74 years	>74 years
Edentulous(a)								
1979	%	1.3	5.4	14.0	26.5	40.2	60.7	78.6
1989-90	%	0.6	1.4	5.7	14.9	28.9	43.2	63.4
1994	%	0.1	0.5	4.0	10.9	20.6	32.5	52.2
1996	%	0.1	0.6	2.0	8.1	19.9	32.7	48.7
Mean number of missing teeth(b)								
1994	no.	1.8	2.6	4.3	7.1	9.7	12.5	15.4
1996	no.	2.0	2.6	4.2	6.2	8.7	11.9	14.7

(a) Percentage of persons edentulous (i.e. having no natural teeth).
(b) Mean number of missing teeth in dentate persons.

Source:
1979 Special Supplementary Survey; 1989-90 National Health Survey, Health Related Actions, Australia (4375.0); 1994 National Dental Telephone Interview Survey, Australian Institute of Health and Welfare Dental Statistics and Research Unit; 1996 National Dental Telephone Interview Survey, Australian Institute of Health and Welfare Dental Statistics and Research Unit.

Thank you Yvonne!

An Application of Circle Geometry: Obtaining More Wood from the Trees.

An ingenious new way to saw up logs produces stronger timber and reduces wastage by a third, according to a Swedish logging company.

Using a technique patented by the Royal College of Technology in Stockholm, the timber company SCA plans to "star-cut" logs perpendicular to their growth rings to produce six radial planks and six triangular sections.

Normally, logs are cut to a "rectilinear geometry". In traditional sawmills, parallel cuts made along the length of a log produce planks of variable breadth, which are then cut down to standard sizes. More sophisticated sawmills can make both parallel and perpendicular cuts to produce the largest, most valuable standard timber sizes from any given log.

But because logs have a circular profile, cutting them into rectangular sections produces a lot of waste. Typically, only 50 per cent by volume of a long emerges from the mill as timber. The remainder is sawdust and unusable parts from the outside of the log, which are used as firewood or turned into paper or chipboard.

By contrast, says Anders Ek, SCA's head of marketing, 65 per cent of a star-cut log is useful timber. Only the outside of the log and the low-quality "pith" at its centre are wasted. "This represents a significant increase in efficiency of log usage", he says. However, the system is only suitable for larger logs, 27 centimetres or more in diameter.

Another problem with standard cutting systems is that most boards are sliced off the side of the log, so each face has a different number of growth rings lying at different angles. This leads to warping, twisting and cracking as the timber dries and with subsequent variations in temperature and humidity.

But the main faces of star-cut boards are perpendicular to the rings, and their sides are parallel. So although the boards still expand and contract, they don't distort or crack. Such boards are ideal for high-quality products such as musical instruments, window frames and furniture, and sell for between 50 and 75 per cent more than ordinary timber of the same dimensions.

The remaining triangular sections are then glued together using strong industrial adhesives to make wood panels which, like the boards, do not distort with changing temperature and humidity. By removing the knots and by finger-jointing the ends of the triangular sections, it is possible to make large, stable, defect-free panels that look like solid wood.

A star-cutting sawmill is due to open in Junsele in northern Sweden early next year. The new plant will use adapted versions of existing sawmill machinery. It will be operated by Nova Wood, a company that is part-owned by SCA, and will process an estimated 14 000 cubic metres of pine a year. "We expect to generate two or three times the usual value from each log", says Ek.

"In principle this sounds like a very good idea", says Lucinda Leech, an award-winning furniture maker from Oxford. "The main advantage of this approach would be the stability of the timber because you get three times as many vertical-grain boards out of each log. The system could also be applied to hardwoods and produce similar benefits."

From This Week, page 10, **New Scientist**, written by Oliver Tickell, 4 July 1998

An Anachronistic Parable:
Mathematics, Snakes, Trees and Noah

The Flood is over and the Ark has landed. Noah lets all the animals out and says,
"Go forth and multiply."
A few months later, Noah decides to take a stroll and see how the animals are doing. Everywhere he looks he finds baby animals. Everyone is doing fine except for one pair of little snakes.
"What's the problem?" says Noah.
"Cut down some trees and let us live there", say the snakes.
Noah follows their advice. Several more weeks pass. Noah checks on the snakes again. Lots of little snakes, everybody is happy.
Noah asks, "Want to tell me how the trees helped?"
"Certainly", say the snakes. "We're adders, so we need logs to multiply."

Maths humour on the Web

PROBLEMS AND ACTIVITIES

Remember that we include a coding system which attempts to indicate in terms of Year levels the suitability range for each item. Thus 6 - 8 suggests an item accessible to students from Year 6 to Year 8.

(1) A Knot that is Not

4 - 12

Take a piece of cord or string at least 30 cm long and tie a loose overhand knot in it by passing the right-hand end first over and then under the left-hand end. This forms the bottom left part of the knot. Now pass the (original) right-hand end over and then under the left-hand end to make an upper-left overhand knot.

Then thread the right-hand end through the lower and the upper loop, each time passing the cord away from you.

What do you think will happen when you pull both ends of this apparently rather complicated "knot"? Why?

(2) A Question of Logic

5 - 12

A painter named White, a guitarist named Black and a carpenter named Red meet in a cafe. One of the three says: "I have black hair, and you two have red hair and white hair, respectively, but none of us has a hair colour which matches our name. White responds: "You are quite correct." What colour is the carpenter's hair?

(3) Another Prize Problem

9 - 12

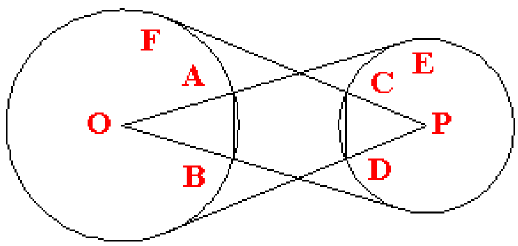
This is another problem from the 1997 M L Urquhart Prize Problem Solving Competition for senior secondary students, named in honour of one of the founders of the Mathematical Association of Tasmania.

Prove that there do not exist integers x, y such that $x^3 - x = 3y + 2$.

(4) Surprisingly Equal Chords

9 - 12

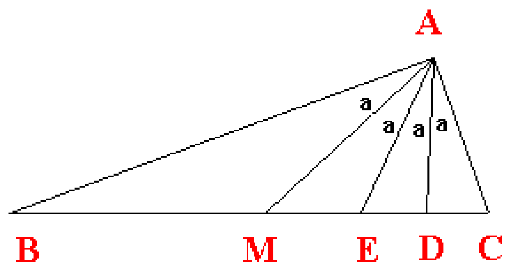
From the centres of two coplanar non-intersecting circles the tangents to the other circle are drawn as shown in the figure. Show that chords AB and CD have equal lengths.



(5) A special Triangle

10 - 12

If the median AM, the angle bisector AE and the altitude AD of ΔABC divide angle A into four equal parts, what is the size of angle A?



(6) A Lotto Problem

11 - 12

The problem, part (c) below was posed to the AAMT Mailing List Community in June. It generated several solutions there. Our solution makes use of two interesting results, (a) and (b) below, which occur in Problems 39 and 40 on pages 50 and 51 of *Basic Techniques of Combinatorial Theory* by Daniel Cohen.

(a) Show that the number of ways of arranging x 1s and y 0s in a line such that no two 1s are adjacent is

$${}^{y+1}C_x$$

(b) Show that the number of r-subsets of {1,2,...,n} that do not contain a pair of consecutive integers is

$${}^{n-r+1}C_r$$

(c) In lotto, 6 numbers are drawn from 45 (without replacement). What is the probability that at least two consecutive numbers are drawn?

THUMBNAIL (and longer) REVIEWS

Readers are welcome to contribute to this section. Reviews can cover books, periodicals, videos, software, CD ROMs, calculators, mathematical models and equipment, posters, etc.

Adults count too.
Mathematics for empowerment
Roseanne Benn
NIACE: The National Organisation for Adult Learning, Leicester, 1997

This review is reprinted with permission of its author and the editor of the *Australian Journal of Adult and Community Education*. It appeared in Vol. 38, No. 2, July 1998.

The state of mathematical competence is a grave concern of governments and education systems. That nations periodically compare mathematical ability levels of their schoolchildren and pressurise schools to address and improve students' basic arithmetical skills is evidence of this concern. Tacitly embodied within this pressure is the concept that accurate, proficient and technologically unaided computations are the most functionally useful mathematical skills for both individuals and the world of business and industry. Within this paradigm, the teaching of mathematics emphasises imparting rules that are known solutions for known problems, providing opportunities for using these rules in increasingly novel and complex situations and assessing competence. Benn designates this functional paradigm of mathematics education as 'a Peek into the Mind of God', in which the aim of the study of mathematics is to discover the already determined set of mathematical rules that are 'written in the sand' and that govern the universe. Competence in making these discoveries from the perspective of this paradigm is located in student ability, teaching by highly numerate tutors and frequent opportunities for practice. Benn identifies the problems and fallacies embodied in this predominant paradigm and provides a contrasting perspective in which she outlines the conditions needed for an empowering mathematics education for adult learners.

Benn resoundingly and cogently challenges the ‘value free, culture free' notions of mathematics education and makes a careful examination of the socially determined conditions that underlie the nationally poor levels of numeracy. Critical literacy theorists would not find Benn's argument surprising and would suggest that questions of mathematical literacy have always been subsumed in the emancipatory intentions within the arguments for an holistic, empowering adult education. Critical theorists have used the term literacy to encompass reading and interpreting of text, social and political awareness as well as mathematical competency. Within the same paradigm, Benn subjects mathematical literacy within adult education frameworks to socially critical scrutiny to explain what it means to be mathematically literate and how mathematics education structurally excludes predictable cohorts in society.

Chapter Four outlines a useful framework for understanding the factors that shape adult mathematics education experience. In this chapter the significant influential factors are identified in a matrix that illustrates the interactions between them. To fully understand what is happening in mathematics education and to understand what has contributed to the poor levels of mathematics in society, Benn argues that the cultural, political and educational forces acting on three important factors need to be considered. These are the goals and experiences of the learner, the tutor and the curriculum. If change in the status quo is to occur, tutors and adult education planners need to subject this matrix to critical scrutiny.

Benn attributes much of the cause of low levels of literacy to the failure of schools to account for the personal values and life experiences of the learner. Furthermore, Benn argues that a critical factor mediating the learners' constructions of themselves as mathematically competent are the tutors' own values and biases that have been entrenched in their own mathematical education. This latter factor, namely, the tutors and their own mathematical education, is especially difficult to address in adult education. Benn believes that mathematics tutors in adult education rarely regard themselves as educators, but rather as mathematicians. As a result of their own education, tutors regard mathematics as an objective, value neutral, culture free content domain and that the purpose of mathematics education is grounded in a functionalist perspective, namely, to 'fix up' those who have failed rather than to fix the curriculum or the teaching approach. By contrast, Benn suggests that adult educators of mathematics need to be taught to use content knowledge dynamically, to be reflective practitioners who are able to 'reflect in action' and to select creatively from a repertoire of strategies to make mathematics learning more accessible for their students.

The third critical factor according to Benn is the curriculum itself. She argues that success oriented approaches to improving mathematical education acknowledge that 'mathematics is a fallible social construct'. Benn claims that modern mathematics has its origins in the 'product of human inventiveness' and, because of this, as a discipline it is culture bound in Eurocentrism. To redress this bias, tutors of adults and mathematics curriculum designers ought to critique mathematics discourse for its racist, sexist and elitist biases both in its content and its teaching. Benn argues that the failure to do so in schools and in the education of mathematics teachers is one of the causes of the wide, predictable divisions in society in mathematical competence.

Benn argues that insufficient levels of mathematical competence limits the life chances of individuals and their capacity to operate fully in society as consumers and to contribute as intelligent, critical citizens. The purpose of mathematics education in education, according to Benn, is to contribute to democracy and active citizenship so that individuals can access infornation sources and understand how they have been produced and interpreted. One needs mathematics to manage one's finances, to cope with domestic tasks, to engage in leisure and voluntary activities, to operate in a modern technological world and to make and remember mental notes.

Benn has not avoided suggesting solutions to the problems of mathematics education. Unfortunately, these solutions are often embedded in her critique of the status quo, so that the reader has to seek them out. She does, however, provide a final chapter in which she summarises the conditions she regards as important for an emancipatory mathematics education. Grounded in constructivist notions of learning, she argues that the culture and life experiences of learners should be the starting point of the curriculum, but should not stop there, for if they did it would maintain the marginal social bargaining position of adult learners. She argues that the curriculum ought to be developed through a public process of identifying a set of mathematical standards and teaching approaches that have been critically scrutinised by tutors for Eurocentric biases.

The methods of teaching need to be interactive, engaging and questioning of the status quo and the content relevant to the learners' needs and interests and brief examples of successful attempts to do so are provided. To enable this approach to adult mathematics education, Benn also suggests a more focused approach to the preparation of adult educators of mathematics is needed The preparation of adult education tutors needs to address their own capacity for critical thinking which may be lacking based on their own early mathematics education. They also need to be supported in developing a critically reflective disposition to themselves and their own personal values and knowledge.

This book has as its basis well substantiated explanations for the current poor state of mathematical competence in society and for its contributions to the predictable social divisions and exclusions in educational success. The arguments are not new, but are retold clearly and creatively with a specific goal in mind, namely, to contribute to making mathematics education for adults more inclusive and as a result more emancipatory. Every student of mathematics education ought to read this to develop a critical view of the discipline. Experienced teachers of mathematics and mathematics education in universities would do well to consider Benn's arguments.

If the book has limitations they can only be described in terms of failing to provide for 'intellectual raiders of texts' who might wish to access quickly a set of guiding principles for carrying out the respective tasks curriculum design and mathematics education. Guides to the way forward are included, but are largely embodied in the discourse of the final chapter. I do look forward to a sequel that elaborates on the final chapter, starting with a set of principles that frame strategies and examples. In addition, the issue of the racism in mathematics gets meagre attention. Chapter Fourteen, 'Adding race to the equation', is the least elaborated argument.

The contribution of this book is its systematic deconstruction of the false premises upon which modern mathematics flourishes and fails. It is a careful uncovering of the elitist biases found in both the content and process of mathematics education. These arguments are carefully made and highly accessible for those not steeped in education jargon. The construction of the book allows the reader to sample separately arguments for non-elitist, non-sexist and non-racist education. Taken as a whole, the book is a carefully sustained and comprehensive constructivist view of emancipatory adult mathematics education.

Dr Janice Orrell
Academic Coordinator,
Staff Development and Training Unit,
Flinders University of SA

A Mathematical Dictionary for Schools
by Brian Bolt and David Hobbs
Published by Cambridge University Press, 1998, $19.97

This compact dictionary uses layout and two colour printing (black and red) to advantage. Its mathematical definitions, examples and diagrams extend from the primary years to about Year 11. The use of language successfully manages to balance correctness, economy with words, clarity and intelligibility relative to the intended audience, as required of a good school dictionary.

There are over 500 headwords. The focus is specifically on the words and phrases which students meet in learning mathematics at school. Thus there is an entry under Euler's formula but not under Euler. Presumably, so as not to over complicate their task, the authors decided not to explicitly cover the history of mathematics in this dictionary. Historical references can hopefully be pursued in larger dictionariess. To give the flavour, there are entries for Angle, Function, Index, LOGO, Relation, Spreadsheet, Topology and Venn diagram. Interestingly, there is no entry for Logarithm but there are entries for Exponent and Power. A sign of the times? This dictionary passes my "significant figures test" with flying colours, that is it manages to define and explain significant figures by means of a few clear sentences and examples. This test has been for me the downfall of several other dictionaries at this and higher levels.

I thoroughly recommend this little dictionary for student and teacher use. At the very least it will help when new topics are broached, when revision is under way or reminders are needed, and it will motivate reference to other sources of information.

Peter Enge

Essential Analysis Exercises for Middle School Mathematics
edited by Brian Woolacott, Bruce Henry, Michael Evans.
Published by Cambridge University Press, 1998, $11.66.

This is a collection of exercises plus hints and answers aimed at students in Years 9 to 11 who need practice tackling problems situated in an extended context. Problem contexts typically consist of an information digesting, comprehension phase followed by a sequence of mathematical activities which exercises a variety of mathematical capabilities and require students to make sense of what they are doing.

The problems are classified by major content focus into Arithmetic, Algebra, Geometry and Measurement, Trigonometry and Probability. A separate Full Solutions Manual is also available. Some questions relate to a graphics calculator.

This is a low-priced collection of problems which serves its stated purpose. It would make a handy resource in high schools and colleges.

Peter Enge

SOLUTIONS TO PROBLEMS AND ACTIVITIES

(1) It is not too hard to visualize (using the diagram or the actual "knot") that when the ends of the cord are pulled the twists which formed the "knot" are untwisted and so the "knot" disappears.

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(2) Since the first speaker has black hair, White's statement coupled with the first statement implies that White's hair is red. Hence carpenter Red's hair must be black.

---

(3) We are given that

$$x(x - 1)(x + 1) = 3y + 2$$

For x,y integers, the LHS is always a multiple of 3 since x - 1, x, x + 1 are 3 consecutive integers if x is an integer; meanwhile the RHS leaves a remainder of 2 when divided by 3, so it is impossible for the two expressions to be equal.

---

(4) Let the circle with centre O have radius R and the circle with centre P have radius r.

Then $\sin\angle FPO = \frac{R}{OP}$ and $\sin\angle EOP = \frac{r}{OP}$.

Bissecting isosceles ΔOAB, we have

$$AB = 2R\sin\angle AOP = 2R\sin\angle EOP = \frac{2Rr}{OP}$$

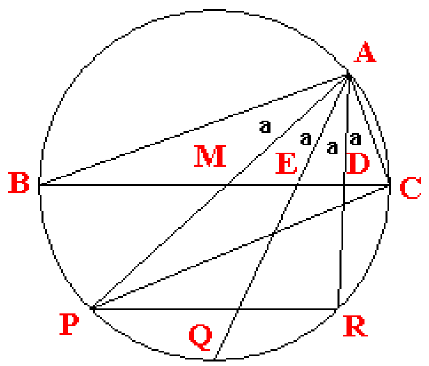
Similarly, bisecting isosceles ΔPCD, gives

$$CD = 2r\sin\angle CPO = 2r\sin\angle FPO = \frac{2rR}{OP},$$

that is, AB = CD.

---

(5) The arms of the four equal angles at A cut off four equal arcs on the circumcircle of ΔABC and give rise to the points P, Q and R in the diagram.



Join PC. Then ∠BCP = ∠CPR since they are each subtended at the circumference by arcs of equal length. Hence BC // PR. Since altitude ADR is perpendicular to BC, it is also perpendicular to PR, making AMP a diameter.

Since M is the midpoint of chord BC and Q is the midpoint of arc BQC, line QM must pass through the centre of the circle. But QM meets diameter AMP at M, implying that M is the centre of the circle and BC is a diameter. Hence angle A is a right angle.

---

(6) (a) To arrange the 1s and 0s as required, imagine using the y 0s as separators between slots (denoted by s) in the list below, each slot holding a single 1 at most

s 0 s 0 s 0 ... s 0 s 0 s (y 0s).

Notice that there are slots at both ends of the list, since y 0s can separate up to y + 1 1s.

Every choice of x particular slots from the available y + 1 slots in the list corresponds to an arrangement with no adjacent 1s. Equally, every arrangement of x 1s and y 0s, which has no two 1s adjacent, corresponds to a particular choice of x slots from the y + 1 available in such list. For example, with x = 3 and y = 5 the list

00101001 corresponds to selecting slots 3, 4 and 6 from the six available.

We conclude that the number of ways of arranging x 1s and y 0s in a line such that no two 1s are adjacent is

$${}^{y+1}C_x$$

(b) We can represent each r-subset of {1,2,...,n} by a unique n-bit binary integer, in which a 1 in the kth bit from the right indicates that k is in the subset and a 0 indicates that k is not in that subset. Thus for n = 3, the 2-subset {1,3} of {1,2,3} can be represented by the binary integer 101.

Now observe that r-subsets containing consecutive integers correspond to binary integers with adjacent 1s and vice versa, so we have a direct connection with part (a). The number of r-subsets of {1,2,...,n} which do not contain a pair of consecutive integers is precisely the number of these n-bit binary integers which contain exactly r non-adjacent 1s and n-r 0s. But this is just the number of arrangements of r 1s and n-r 0s in a list, so that no two 1s are adjacent.

Hence, by (a), the number of r-subsets of {1,2,...,n} which do not contain a pair of consecutive integers is

$${}^{n-r+1}C_r$$

(c) The probability of drawing at least two consecutive numbers when drawing six numbers from 1,2,...,45

$$= \quad 1 - \text{probability of drawing no consecutive integers}$$

$$= \quad 1 - \frac{{}^{40}C_6}{{}^{45}C_6}$$

$$\approx \quad 0.529$$ rounding correct to 3 decimal places.