CIRCUIT

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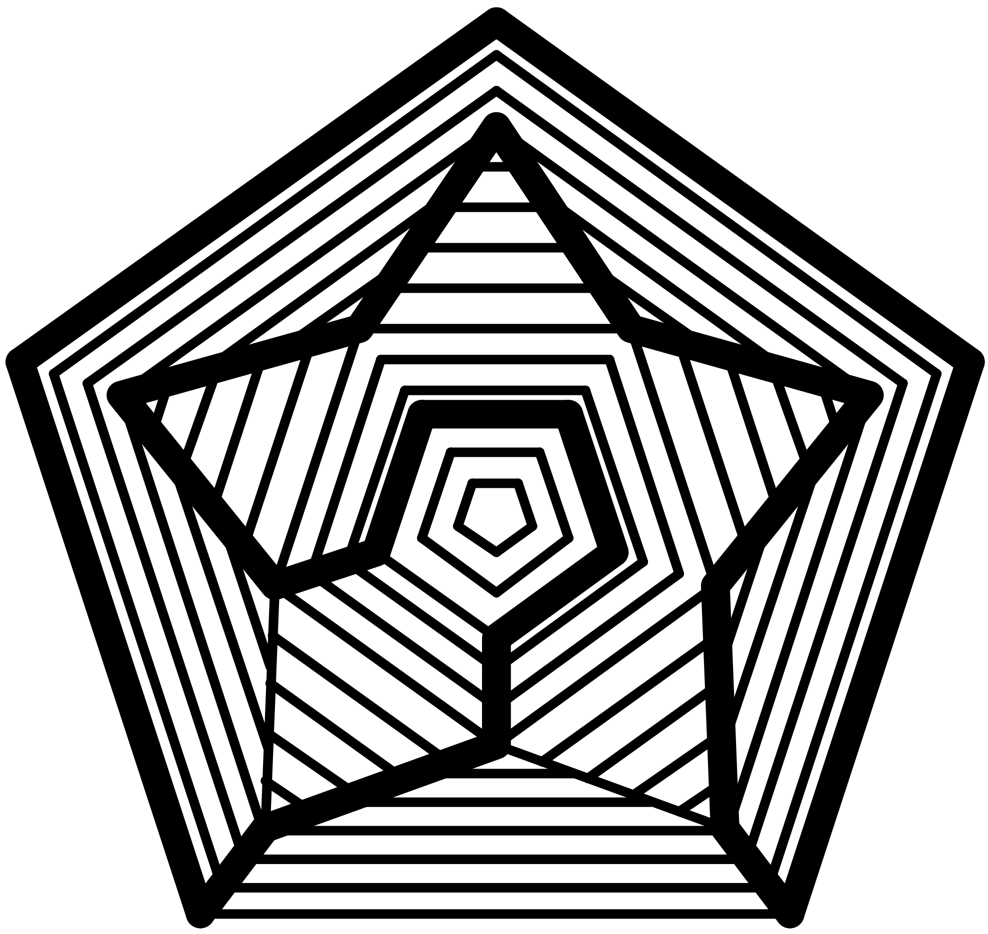

CANBERRA MATHEMATICAL ASSOCIATION

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CMA INTERNET HOME PAGE

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The objects of the Canberra Mathematical Association are to promote interest in mathematics, to encourage improvements in the teaching of mathematics and its applications, to provide means of communication among teachers and students and to advance the views of the Association on any question affecting the study or teaching of mathematics and its applications.

The Canberra Mathematical Association Logo depicts a Hamiltonian Circuit on a dodecahedron.

CONTRIBUTIONS

Circuit is always keen to receive articles, notes, problems, letters and information of interest to members of the CMA. Please contact the Editor if you wish to contribute to Circuit. If possible contributions should be submitted as Word 6 documents.

FROM THE PRESIDENT

Thank you to all those who have contributed to the success of the various undertakings of our association during 1999. In particular thank you to members of the Council who have worked throughout the year to maintain the many functions of the CMA. The ever increasing workload which is the lot of teachers presents challenges to the CMA's ability to respond to the goals of our organisation. It is timely to reflect on those goals and contemplate how we each can contribute to their achievement. CMA goals include;

- to enhance the mathematical confidence of teachers
- to act as a network and forum for the discussion of issues related to the teaching of mathematics
- to provide professional stimulation and resources for mathematics teachers and educators
- to encourage the learning of mathematics by fostering a range of activities for students of all ages
- to act as a lobby for mathematics educators in the ACT
- to assist teacher networks across schools and sectors
- to foster the love of mathematics
- to foster better mathematics teaching and learning

We presented an extremely successful professional development evening in September when we had the Executive of AAMT in Canberra. Over 100 teachers attended and a recurring comment from teachers in their evaluations was that they valued the opportunity to network with peers and felt valued when provided with a quality afternoon tea and evening meal, as well as the quality presentations.

Preparations are well under way for the 18th Biennial Conference of AAMT in January 2001 (15-19 Jan) which, if you are not already aware, is being hosted by the CMA. Steve Thornton, Beth Lee, Margaret Rowlands and Jan MacDonald have spent many hours planning what will be a wonderful opportunity for local teachers to experience excellent professional development. Keep the Conference in mind when you are planning your own personal PD program and that of your school. I encourage members of our mathematics education community to consider being presenters at the Conference.

Paulene Kibble

NOTICE BOARD

Graphics calculators for loan

The Canberra Mathematical Association now has available a set of 30 TI83 graphics calculators, kindly lent to us by Texas Instruments. These are available as a set for schools wishing to try them in their classrooms. They are most suitable for students in years 8 to 12. They can be used in most areas of the mathematics curriculum, but are particularly useful for statistics and functions and graphs.

The following conditions apply:

- The set must be borrowed as a complete set.
- The borrowing period is four weeks, either the first four weeks of each term or weeks 5 to 9 of each term.
- Schools borrowing the calculators should make a commitment to use them in at least four classes, involving at least two teachers.
- Schools borrowing the calculators should make a commitment to document the way in which they have been used, for possible publication in Circuit or other CMA publication.

Professional development can be arranged, and is encouraged. Ideally schools borrowing the calculators should arrange for a professional development session involving both their own staff and some from nearby schools.

For further details, or to talk about how you might use these calculators in your school please contact **Steve Thornton on 6201 2017.**

Writers Wanted for On-line Mathematics Study Guides

The Curriculum Corporation is developing an on-line project with Encyclopaedia Britannica. The writing task is to develop study guides for students in years 7–12 in Science, Mathematics and English to use independently. These study guides will contain key information on a range of topics. Each study guide topic module will comprise approximately 12–20 screens of text with associated web-links, graphics, interactive activities and student self assessment.

Knowledge required.

It is more important to have a solid understanding of the curriculum area than specific syllabus or curriculum framework understanding. Experience in writing for delivery on-line is not essential, but an ability to use the Internet would be an advantage.

Employment of writers

Initially writers will be given one study guide of approximately 12–20 screens. If satisfactory, more work will be offered. Strict deadlines for the completion of the modules will be set. Writers will be paid for satisfactory work within the deadline. Payment will range from between $400–$600 depending on the number of screens required for each module.

A detailed **Writer's brief** will be available.

Completion of Project

It is envisaged that this project will be ongoing and will extend into the full range of curriculum areas and year levels.

If you require further information please don't hesitate to contact Mary Holmes maryh@curriculum.edu.au or telephone 03 9207 9600.

FROM THE EDITOR

To start with, two apologies are in order. The first is for the lateness of this *Circuit* which I can only put down to the fact that it takes a fair amount of time and effort to assemble the material and produce and distribute the journal to members. It is not that this particular issue took any more effort than previous ones, just that end-of-year busyness has precluded me putting in the time required until the last few weeks.

The second apology is to Jill Middleton who contributed *What Chance Do You Have?* to our July issue. My proof reading and editing allowed a couple of gremlins to creep in, so the following two corrections need to be made to what was printed.

Lines -8 to -6 in the left hand column on page 10 should read

The probability of winning in our lifetime is $2600 \times 7 \times P(6) \cong 3.19 \times 10^{-4}$ or about 1 chance in 447.

Also the first word in line -8 of the right hand column on page 10 should read "Remember" in place of "Remembering", since the words as originally printed did not form a sentence. I hope these glitches did not mar the impact of Jill's piece for readers.

Now for some positives. CMA members will have many opportunities over the next twelve months to actively involve themselves in organising the January 2001 Biennial AAMT Conference here in Canberra. It will be 21 years since CMA last hosted this Conference. The byline for that January 1981 Conference was "Mathematics - theory into practice" which resonates nicely in my opinion with "Mathematics: Shaping Australia", the byline for the coming gathering. I think all of us ancients who were around back then fondly remember the energising after effects and positive spin-offs which active participation as a member of a conference team can bring. Remember that many hands make light work and that any contribution you would like to make in the lead up stage, at the Conference itself, or afterwards will be most welcome. So do join in the fun.

In this issue there is a minor focus on Outcomes-based Education, linked in to threads which try to demonstrate how thoughtful and sensitive mathematics teaching has a vital role to play in creating confident and reasonable citizens able to find social unity in diversity. Constructive dialogue in mathematics classrooms where students feel comfortable about expressing their doubts and fears and learning how to explain their line of thinking clearly and logically, ultimately serves as a model for democratic discussion in the wider society.

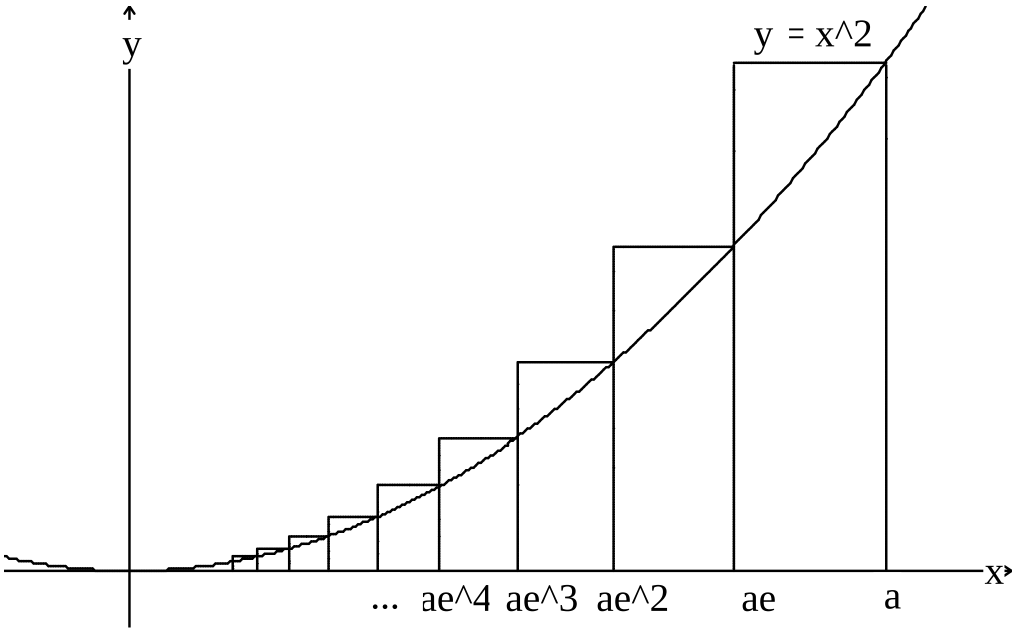
Thank you to all contributors to *Circuit* this year. Without you the journal would not exist and I think CMA would be much the poorer. In particular I would like to thank CMA Councillors Andy Wardrop, Jan MacDonald, Paulene Kibble and Steve Thorton, plus Warren Atkins at the University of Canberra, for their regular input of ideas, support and/or material. As always my wife Rosemary has borne the brunt of transforming raw material into printable copy. I thank her sincerely for all her hard work and encouragement. Finally I wish all of you a peaceful and rewarding year 2000.

Fermat's Technique

Perhaps next time you head towards the fundamental theorem of calculus in your classroom, you may wish to consider Fermat's technique of finding expressions for areas under curves, beautifully outlined in Boyer's *History of Mathematics*. What I like about the method is its simplicity and thus its accessibility to students coming to terms with a higher level of abstraction.

The method is essentially an exercise in geometric series, and I think it is instructive and perhaps refreshing to introduce a little bit of variation from the standard introductions.

Fermat considered that the area between $x = 0$ and $x = a$ below the curve $y = x^n$ and above the x axis, to be approximated by circumscribed rectangles where the widths of each rectangle form a geometric sequence. The partition points of the interval of integration, from the right hand side, are made at $x = a$, $x = ae$, $x = ae^2$, … as shown in the diagram on the next page



for the curve $y = x^2$. The area A contained within the rectangles for this particular curve is given by:

$$A = (a - ae)\, a^2 + (ae - ae^2)\,(ae)^2 + (ae^2 - ae^3)\,(ae^2)^2 + \ldots$$

This is a geometric series with common ratio $r = e^3$, $e < 1$ and first term $(1-e)\, a^3$.

Using the limiting sum formula, we have

$$A = \frac{a^3(1-e)}{1-e^3}$$

$$= \frac{a^3(1 - e)}{(1 - e)(1 + e + e^2)}$$

$$= \frac{a^3}{1 + e + e^2}\ .$$

Now as $e \to 1$ from below the widths of the rectangles decrease and the total area closes in on the area under the curve. That is to say

$$\int_0^a x^2\, dx = \frac{a^3}{3}.$$

We can apply the same logic using any curve of the form $y = x^n$ for n a positive integer. The geometric series formed has the first term $a^{n+1}(1 - e)$ and the common ratio e^{n+1}. This means that the denominator in the limiting sum factorises to $(1 - e)(1 + e + e^2 + \ldots + e^n)$ and the general result becomes clear.

We can even apply the method to the mid-ordinate and the trapezoidal rules to produce two further expressions for A. Considering the curve $y=x^2$, the expression for A using the mid-ordinate rule becomes

$$A = \frac{a^3}{4}(e + 1)^2(1 - e)\,(1 + e^3 + e^6 + e^9 + \ldots)$$

and for the trapezoidal rule,

$$A = \frac{a^3}{2}\,(e^2 + 1)(1 - e)(1 + e^3 + e^6 + e^9 + \ldots)$$

which after determining the limiting sums in both expressions produces the desired result.

Ed Staples
Erindale College

What Teaching and Learning Strategies are the Most Effective with Low and Under Achievers?

This is an edited version of an assignment submitted as part of a Graduate Diploma in Secondary Education at the University of Canberra

Introduction

Low achievement should be distinguished from under achievement. An under achiever is a student who has had access to classroom environments where there is a reasonable expectation of academic success for the members of the group, but has not achieved individual success. By contrast a low achiever has the potential to achieve at a higher level but may not have been encouraged to do so or had the appropriate learning opportunities.

Students in either of these circumstances can be characterised by several behaviours attributable to their educational experiences. When deciding which teaching strategies are most effective with low and under achievers it is imperative to study the learning characteristics of the individual students and try to determine possible contributing causal factors of these characteristics. The aim is to determine strategies which are appropriate to both remedying any learning problems and accommodating their learning needs.

Characteristics of Low and Under Achieving Students

Firstly, low or under achieving mathematics students are likely to possess a fear of failure and consequently a negative attitude towards mathematics (4, p397). Repeated failure fosters their dislike of school and then leads these students to question the value of schooling. It follows that these students often demonstrate low motivation. If they see no reason for learning a topic they will put in no effort to do so, especially if they expect failure.

Ineffective teaching is a major issue in student low or under achievement. Considering that most of the time at least twenty individual students occupy a classroom, it is essential that the teacher practice a diverse set of teaching methods and varies lesson activities to cater for different learning styles. Generally low or under achieving students are not passive learners (4, p398). Instructing these students purely on a verbal level is not only inappropriate but will further discourage them. Hence, their participation in classroom activity is vitally important and will contribute to their present and future ability to learn mathematics.

The approach of catering for different learning styles by varying activities will also address the needs of students with short attention spans. Teachers who do not vary the activities during a class period are not providing the most effective learning environment for low or under achieving students. Productivity levels during sessions where activities are not varied tend to be lower than when there is variety, and student motivation is correspondingly affected.

In addition to not being passive learners, low and under achieving students are commonly deficient in reading ability. Poor general literacy skills tend to be characteristic of such students. For learners with restricted reading comprehension, written directions are most effective when they are short and used sparingly.

Under achieving students are distinguished by their conscious nonchalance towards what seem to be reasonable educational expectations. Students' home environment often plays a crucial role in determining their academic performance at school Some parents may place little value on their children's education while others place intense pressure on their children to attain academic success (2, p326). This can result in students either deliberately under achieving or striving for intellectually unattainable goals. Either way the expectations set by the home environment significantly influence a student's academic achievement.

Teacher expectation of student achievement similarly affects students in a classroom environment. The assumption that particular students are incapable of learning certain concepts has led teachers to expect lower standards and consequently fail to provide appropriately challenging mathematical content (4, p398). Lower teacher expectations about student achieve-ment can lead to the failure to develop and nurture the academic potential of low and under achieving students.

Clearly an inadequate curriculum combined with ineffective teaching methods can severely hinder a student's ability to learn.

Effective Teaching and Learning Strategies

Identification of the particular characteristics of low and under achieving students and the possible causes of their problems, leads to the exploration of the most effective teaching and learning strategies that teachers can adopt to remedy the situation. The possible strategies that will be suggested refer to situations where students are ability grouped in streamed classes. Obviously the main issues differ when low and under achieving students are included in non-streamed classes, although many of these strategies can be adapted to either situation.

One of the most important points to remember when teaching low or under achieving students is to keep the learning problems of the student private (3, p25). Unnecessary publicity about a student's problems is more likely to threaten rather than to reveal and develop their academic potential, as they may feel branded and lose motivation to overcome their problem. A teacher has an obligation to keep such problems private and deal with them in a sensitive manner.

As educators it is our responsibility to promote the self-esteem of every student. Building confidence is a fundamental part of the process of overcoming learning difficulties. Basically, if students do not believe they can achieve a higher academic standard then they most probably will not. This relates to the 'self-fulfilling prophecy', whereby if a student is labelled to be of 'average' academic ability then they will achieve at an average level not believing that they can achieve higher.

Teachers must appreciate that providing opportunities for success may not necessarily lead to successful outcomes in all cases. It needs to be understood that not all students possess the right skills to take advantage of their opportunities. Nevertheless, providing low and under achieving students with the tools for success is an essential element of effective teaching. It is often the case that students are devoid of the skills required for successful learning. They need to learn how to learn in order to be able to succeed at given opportunities (3, p26). Hence combining the ability to acquire knowledge and skills with opportunities for success is a useful strategy to employ when teaching low and under achieving students. Recognising such success and noting the improvement and effort of such students is a further step towards promoting self-esteem.

Very often an inadequate curriculum is the cause of low student achievement. As mentioned earlier, low and under achieving students often have short attention spans, are not passive learners, and have low levels of motivation. To remedy these problems the teacher should provide variety during classes and make learning more appropriate. Implementing several different activities during a class helps capture student's interest and keep them busy, not allowing time for boredom to develop. This strategy ties in to the need for students to be consciously involved in classroom activities in order to maximise their learning. The term 'classroom activities' encompasses such things as experiments, visual aids, games, puzzles, and multi-sensory activities.

Providing a variety of activities on a specific topic will not only keep students on task, but will inculcate the notion that the topic is real and appropriate. Using relevant material significantly affects student's level of motivation to learn. Demonstrating how the topic can be used in a variety of ways by providing different activities reinforces its usefulness and relevance, additionally providing students with a sense of progress and making earlier activities and future classes worthwhile. Teachers can encourage students to make connections between topics by posing questions specifically designed for this purpose.

A common mistake made by teachers of low and under achieving students is to eliminate difficult material from the curriculum (4, p398). As discussed earlier, when teachers have low expectations of student achievement they tend to foster low achievement through supplying a lower standard of mathematical content. However, withdrawing more difficult subject material fails to provide students with appropriately challenging material. The difficulty level of the content is not the problem but rather the lack of significant mathematical concepts to motivate students. Students tend to be less motivated when they are not adequately challenged in class. It is therefore important that teachers ensure that their teaching incorporates a wide variety of appropriately challenging content.

Conclusion

Low and underachieving students are characterised by low motivation, fear of failure and susceptibility to teacher and parent expectations. They need to be involved actively in learning activities.

To remedy learning problems and accommodate the learning needs of low and under achieving students teachers can adopt and implement the following effective teaching strategies:

- provide students with tools for success;
- promote self-esteem and build success;
- make learning real and appropriate;
- get the students involved;
- focus on positives such as improvement and effort;
- effectively adjust the curriculum to provide suitably challenging content.

Although these teaching strategies have been identified as being effective with low and under achieving students, it should be noted that any teacher in almost any classroom environment could adapt them.

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Jane Priestley

Race To Zero

It all began with a visit to Charnwood Primary School. I was waiting to talk to Paulene Kibble who was just finishing off her year 1 and 2 maths lesson. The children were rolling dice and counting backwards on a grid. They were allowed to choose which die to use (4, 6, 8, 12 or 20 faces). Once they had chosen the die they wanted

they could not change it during the game. They played the game by starting from a certain number and counting backwards the number of squares shown on the die. As in all good board games they had to have an exact roll to reach zero to finish the game.

So that started me thinking: **what is the best sized die to use for a given length game?** And of course, why restrict the dice to physical ones, when we have the technology to simulate rolls of dice with any number of sides?

A technological aside:

On a scientific calculator, rolls of an n-faced die numbered 1 to n can be simulated by multiplying a random decimal (usually generated with the RAN# button) by n, discarding the decimal part of the result and adding 1.

On a computer spreadsheet, random integers from 1 to n can be generated using INT(RAND()*n) + 1.

On some graphing calculators the RANDINT(1,n) command will automatically generate random integers from 1 to n.

So which is the best sized die to use for a given game?

Clearly a 1-faced die will guarantee that you reach 0 in exactly the same number of rolls as the length of the race. If you use a bigger die you will get back towards 0 more quickly, but if the die is too big you could well be stuck at the end trying to roll an exact score. So there must be an optimum die size for each given game length, probably somewhere between 1 and the game length.

Young students could play the game a number of times with real dice to practice subtraction and to get a feel for chance events. They may, for example, note that it is much harder to roll a 1 with a 20-faced die than with a 6-faced die. They could record and graph the number of rolls needed with several trials of each die to help them to realise that the numbers vary from game to game, but that in the long run some dice are better than others for a given game length.

Older students could simulate a large number of games with different sized dice, graph the results and find the best-sized die for a given game length. Or, of course, college students could try to analyse the game using some ideas from probability.

My analysis of the probabilities involved will, I hope, appear in a future issue of the Australian Senior Mathematics Journal, as it is too lengthy to be reprinted here. As well as simple ideas of probability and expectation, it involves infinite series, Markov chains and recursion relationships.

Reflection

It is surprising how often an apparently simple activity from a junior primary classroom can lead to some very sophisticated mathematical ideas, or indeed, how some relatively advanced mathematics that happens in high school or college classrooms can be adapted and simplified to introduce very young students to some new mathematics. I think this is particularly important at professional development activities, where we often dismiss a session as not relevant because it is pitched at the wrong level. However if we approach such sessions with an open mind it is remarkable what we can learn from each other, or adapt to use in our classrooms.

Just to hint at the results, for a game of length 10 it is best to use a die with four faces. A 6-faced die is the best one to use for games of length 21 to 27. But let's not spoil your students' fun by giving all the answers!

Steve Thornton
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More Extracts from the AAMT Internet Mailing List Community September 1999: Discussion on Outcomes

First Margaret McLaughlan (St. Francis Xavier High School, Canberra) poses some issues

My biggest problem with implementing outcomes based assessment is that it forces the curriculum to be divided up into 'bits' to be assessed. This makes it difficult to use some of the best tasks for students as assessment items - you know the questions I mean - the ones that use lots of different skills and approaches and a range of knowledge to solve a problem. We try to have specific outcomes reported on, e.g. a student's 'ability to understand and use fractions', and then an outcome that is more general, e.g. 'ability to use mathematical skills in unfamiliar situations,' but I am still not completely happy with this approach either.

Another approach would be to have less specific outcomes, but then the outcomes become so broad as to be too difficult to describe or are a waste of time as they lose any meaning. Does anyone have a better idea?

Then Paul Sutton (Ingham State High School, Qld)

McLaughlan wrote: "My biggest problem with implementing outcomes based assessment is that it forces the curriculum to be divided up into 'bits' to be assessed."

I tend to agree. Outcomes are a little like parts of a motorbike spread over the garage floor. Left alone they will just lie there. Assembled in the right way the thing is capable of motion at considerable speed.

We need to be careful we do not focus on the mathematical bits lying on the classroom floor. Our classrooms must be a place in which all the parts work together so our students fly (if you will forgive the exaggerated analogy). We might have to report on the bits, but we must focus on the integration. Seems a shame that the real value of what we do remains unappreciated because it is hard to describe.

For these reasons, I am sure the outcomes focus is just one more phase education will pass through. Already, the business world is recognising that their long term future is more than today's bottom line and long term relationships with customers are important. Covey (*Seven Habits of Highly Effective People*) also makes the point that you should treat your employees as you would have them treat your best customer.

If he is right, education systems cannot continue to undervalue their staff by focussing only on those outcomes the writers manage to get in print. I am not critical of the writers. They will play their role in history and do their very best but the outcomes focus alone will not be enough.

I cannot yet see the next system we will try but am sure after 27 years of watching them arrive it is out there somewhere. The good news is that I think it gets a little better when we do improve things and when we learn from our mistakes.

There are still plenty of opportunities to do it better for our students.

Next Barry Kissane (Murdoch University, WA)

I haven't really followed the threads of this 'discussion on outcomes', and know only too well that, like other education terms (e.g. problem solving), the term can mean different things in different settings.

But I can't ignore the following remarks:

"My biggest problem with implementing outcomes based assessment is that it forces the curriculum to be divided up into 'bits' to be assessed." (Margaret McLaughlan),

to which Paul responded:

"I tend to agree. Outcomes are a little like parts of a motorbike spread over the garage floor. Left alone they will just lie there. Assembled in the right way the thing is capable of motion at considerable speed."

I have lived most of my professional life over the last 30 years in settings in which the curriculum was divided up into bits in order to be assessed. This has applied in external examination settings (which tend to be based upon big lists of small content things), in internal school settings (where large collections of 'objectives' are described and used), in official syllabi, listing big collections of things, sometimes even on a 'per year' basis. The worst excesses of the 'behavioural objectives' movements were the extreme examples of this problem. To the extent that it is unhelpful to chop things up into too many bits, I am in complete sympathy with the viewpoint expressed.

For me, however, the great potential of outcomes is that they may allow us to see some *bigger* pictures of what we expect students to know, understand and be able to do, INSTEAD of all the minutiae that have plagued us in the past. If that is not happening, then you are not dealing with outcomes; rather, you are dealing with re-named 'objectives'.

I do not deny that some can (and have, and no doubt will) reinterpret 'outcomes' to fit the practices of the past; indeed, this may be precisely why the "outcomes based assessment" problem is described as a problem. (I say 'may be' in ignorance of the particular setting from which this concern came). But the simple response to this problem is that such misinterpretations are missing the point about outcomes.

I wonder if the discussion has been informed by the VC99 thread about outcomes?

Finally, some suggestions from Debbie Scott, Numeracy in Schools Project Officer

Margaret, I actually think that outcomes really OPEN up the potential for assessing meaningful mathematical learning. The focus needs to be on the learning experience though and then identifying the range (often huge) of outcomes demonstrated by the students. This requires a great deal of familiarity with both the scope and development of learning but this comes with time and experience. There are a number of outcomes papers in this year's AAMT Virtual Conference which has INVESTIGATIONS as it's overriding them. Jan Senior's, in particular, looks at these issues. I think you would find the whole thread of the conference interesting.

And the interchange continues ...

Class Talk

Good democrats need a firm foundation in maths.

I had explained to a class of 11-year-olds what they had to do, and they were working silently on their exercise. Suddenly one of the brighter boys announced that he had finished. Marking his book, I wondered how best to keep him occupied for the time being. "Why don't you try to write down how you think about these problems to get them right," I suggested.

His alarm at the thought jerked a reply from him. "No fear!" he blurted. "If I try to interfere like that with my thinking, I'll start to get them wrong."

Now it was my turn to be alarmed – and astonished. This kind of pupil is always likely to succeed, yet he was unable to explain the line of thinking that gave him the right answer. What hope was there, then, for the many youngsters who fail repeatedly in mathematics and can never say why, even to themselves? Too much mathematics is taught without open discussion of what it means.

This is a serious failing, given how vital mathematics is to us. The German poet Hans Magnus Enzensberger has observed that there has never been a civilisation quite like ours today, which is so inescapably dependent on it. Mathematics is uniquely capable or providing us with a common language of influence and respect, and of risk, value, probability and certainty.

Without its discipline, politicians can speak like lawyers with words intended only to convince, not bind. And democracy inevitably becomes a realm of fudged agreements, endless obscurity and spin. Between increasingly frustrated citizens and their increasingly unresponsive representatives is only an ever-increasing bureaucracy, answerable to neither.

So democracy and mathematics are inextricably intertwined. Two thousand years ago the Greeks devised the logic- and evidence-based arguments that we use in today's mathematics. And they did so, as the Cambridge philosopher Geoffrey Lloyd has said, not to create mathematics per se, but to give ordinary citizens the means and confidence to participate in democracy. Deprived on these skills, people are so much political putty.

No country can claim to be democratic if too few of its people are ready and able to participate in its debates, to take on their responsibilities, to vote or think and speak for themselves. As the Oxford historian Theodore Zeldin says, to be a true member of a society, what matters is whether you are willing to think for yourself and to say what you think.

Today, too many people are unwilling to do so, either because they've been told too often they are just ordinary and thus assume they have nothing of importance to say, or because they have received too many knocks in life. And where do we first feel the harsh rap of judgment, or ridicule, most acutely for daring to voice our uncertainties or doubts, or question given ideas? It's among our peers – and especially in the mathematics classroom.

So mathematics teaching can hardly be said to be politically neutral. Taught sensitively and thoughtfully, and mainly through dialogue, as the ancient Greeks intended, it can help to unify societies. It shows the most privileged and disadvantaged that they can share a common discourse, can use a common language and can arrive at an agreement. But taught in an authoritarian, dogmatic, insensitive way, mathematics can have a very different result and highlight fundamental differences between people.

Such ideas can subdue millions. Just as my pupil is likely to become convinced of his right to be a member of a technical elite, so many others will become convinced that they are members of an underclass. This is where the greatest damage is now being done do democracy.

These are worrying thoughts in the face of what is happening to the European Union's bureaucracy, which, like that of the former Soviet Union, just keeps on growing. I have long argued that democracy in the EU is not supported as it should be by its mathematics teaching. Three years ago, the EU agreed to fund a major study, led by the German mathematician Hartmut Köhler into a link between mathematics teaching and democratic education*. One of its basic tenets is that mathematics teaching should never be used, as the French mathematician Didier Norden has protested, "to stab young people through the heart". He claimed that no other discipline was ever so perverted from its original path of social harmony and cohesion to become a tool of social division and calibration.

In mathematics lessons young people should be taught models of simple, complete, logical argument. Through constructive dialogue they should be encouraged to express their doubts and fears, and taught how to explain their line of thinking clearly and logically. Everyone can take part in arriving at decisions, and no question should ever be met with ridicule. This is the foundation of democracy. The classroom is where it must be built.

Colin Hannaford is Director of the Institute for Democracy from Mathematics, Oxford. *Mathematics Teaching and Democratic Education* is available from Hartmut Köhler at the Stuttgart Institute for Education and Teaching, D-70174 Stuttgart

From Opinion Point, New Scientist, 28 August 1999

QUOTABLE NOTES AND NOTABLE QUOTES

More Joys of Maths

"There is no imaginable mental facility more serenely pure than suspended happy absorption in a mathematical problem."

Christopher Morley in his novel *Plum Pudding*

Note: Christopher Morley was the son of the mathematician Frank Morley who discovered the following rather surprising geometry theorem, now called Morley's theorem, about 100 years ago:

> The points of intersection of the adjacent trisectors of the angles of a triangle form an equilateral triangle.

The golden rule for maths teachers

You must tell the truth, and nothing but the truth, but not the whole truth.

From the Web

A model for enhancing the role of the investigative process in the mathematics curriculum

1. Find an interesting/meaningful problem.
2. Undertake an informal, unstructured exploration of the task, which yields data.
3. From the data create theories, hypotheses, conjectures.
4. Invoke problem solving strategies to prove or disprove theories.
5. As part of this, apply known basic skills.
6. Extend and generalise – what else can I/we find?
7. Publish.
8. Go back to Step 1.

adapted from Charles Lovitt's abstract for his paper *Investigations as a central focus for a mathematics curriculum,*
AAMT Virtual Conference, 1999

Toward Statistical Literacy - Citizen Beware!

When Miss Marple, Agatha Christie's famous detective, was asked why she always believed the worst about human nature, she responded that "the worst is so often true." Similarly, statistical reports in the media involve flaws regularly enough that some initial scepticism is well deserved.

The most cautious general course when dealing with reports involving statistical studies is to treat them as public announcements that studies have been done and not as clear guides to the content or reliability of the studies. As well as being on the look out for evidence that researchers, reporters, or advertisers have committed one or more blatant errors in statistical reasoning, we also need to cultivate a general awareness that statistics can yield highly divergent interpretations. When a particular interpretation of the reported data pattern is advanced, have the analysts reasonably excluded other possibilities, or failed even to recognize them?

Ultimately, should the conclusions really matter then there is no avoiding the arduous task of finding the study and reading it. And contacting the author for further details is both wise and legitimate.

For the alert individual, statistical humbug should be no harder to ferret out than other forms of illogical argument. It just takes practice and time.

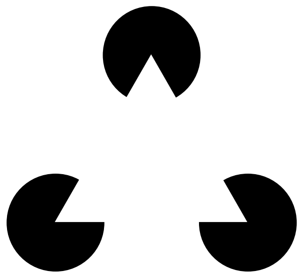
Adapted from *How Numbers are tricking you*, by
Arnold Barnett, MIT,
in Technology Review On-line

Surely providing practice in and time for exposing and dealing appropriately with the statistical humbug, which so often underlies attempts to influence the behaviour of all of us is, a core issue for all mathematics educators.

Now for a relatively trivial but amusing example, courtesy of New Scientist, *Feedback*, 14 March 1998, pointed out by Rodney Hall of Wiltshire, UK, and published in his local newspaper, the *Western Daily Press:*

> "Motorists in Bristol have more chance of coming back to their car and finding it still there than drivers elsewhere in the country.
>
> "New figures from the AA show that, nationally, one car is stolen every 79 seconds, while the figure for Bristol is one car every 35 minutes."

A Triangle That Isn't Really There – Or Is It?



A Limerick to Check Your Arithmetic

A dozen, a gross and a score,
Plus three times the square root of four,
Divided by seven,
Plus five times eleven,
Equals nine squared and not a bit more.

from the Web

Weighing Viruses

A nanobalance, a vibrating carbon nanotube that can directly weigh microscopic organisms and particles piggybacked onto it, has been demonstrated by researchers at Georgia Tech. First discovered in 1991, a carbon nanotube is essentially a sheet of carbon atoms (arranged in a hexagonal pattern) rolled up into a nanometer-diameter, that is 10^{-9} meter diameter, tube capped at both ends with carbon hemispheres.

In the present experiment, the researchers viewed their nanotubes, protruding from a carbon fiber, with an electron microscope. The fiber in turn was attached to a gold wire, mounted on an insulator. This setup allowed the researchers to send an electrical current through the nanotubes. Putting the whole assembly on a special sample holder put it only 5–20 microns away from an oppositely charged electrode. Applying an oscillating electrical voltage to the wire, opposed to this electrode, caused the tubes to vibrate; the vibration was a maximum at the resonant frequency. Attaching a particle to such a nanotube would change this resonance frequency, enabling researchers to deduce the mass of the particle.

With this technique, the researchers measured the mass of a graphite particle to be 22 femtograms, that is 22×10^{-15} grams. In general, this technique can determine the mass of particles with similar dimensions in the femtogram to picogram range, that is the range from 10^{-15} to 10^{-12} gram. This includes viruses.

adapted from *Physics News Update,*
The American Institute of Physics, March 1999

Computer Error Messages - A Zen Alternative

Tokyo, Japan, May 20 - Sony has announced its own computer operating system now available on its hot new portable PC called the Vaio. Instead of producing the cryptic error messages characteristic of Microsoft's Windows and DOS systems, Sony's chairman Asai Tawara said, "We intend to capture the high ground by putting a human, Japanese face on what has been - until now - an operating system that reflects Western cultural hegemony. For example, we have replaced the impersonal and unhelpful Microsoft error messages with our own Japanese haiku poetry." The haiku messages are just as informative as Microsoft's and they make you pause just long enough that you're able to fight the impulse to put a fist through the screen.

The chairman went on to give examples of the error messages:

A file that big?
It might be very useful.
But now it is gone.

You seek a Web site.
It cannot be located.
Countless more exist.

Chaos reigns within.
Stop, reflect, and reboot.
Order shall return.

ABORTED effort:
Close all that you have worked on.
You ask way too much.

Yesterday it worked
Today it is not working
Windows is like that.

First snow, then silence.
This thousand dollar screen dies
So beautifully.

With searching comes loss.
The presence of absence.
"June Sales.doc" not found.

The Tao that is seen
Is not the true Tao
Until you bring fresh toner.

Windows NT crashed.
The Blue Screen of Death.
No one hears your screams.

Stay the patient course.
Of little worth is your ire.
The network is down.

A crash reduces
Your expensive computer
To a simple stone.

Three things are certain:
Death, taxes, and lost data.
Guess which has occurred.

You step in the stream
But the water has moved on.
Page not found.

Out of memory.
We wish to hold the whole sky,
But we never will.

Having been erased,
The document you are seeking
Must now be retyped.

Serious error.
All shortcuts have disappeared.
Screen. Mind. Both are blank.

emanating from the Eastern regions of cyberspace

Circles Versus Squares

People are better at spotting imperfect circles than dodgy squares.

Johannes Zanker at the Australian National University in Canberra asked five people to distinguish perfect circles from minutely stretched ellipsoids, and squares from “near-square” rectangles (*Naturwissen-schaften*, vol 86, p 492). The observers were more sensitive to the deviations in the circles than the distorted squares.

People’s brains judge spatial relationships best when they see whole curved shapes, Zanker believes. “By investigating such problems, we get clues about the strategies and tricks which the brain is using to help us survive in a demanding visual environ-ment,” he says.

In Brief, New Scientist, 16 October 1999

Education as Adding Value?

‘An education enables you to earn more than an educator.’

Anon

PROBLEMS AND ACTIVITIES

Remember that we include a coding system which attempts to indicate in terms of Year levels the suitability range for each item. Thus 6 - 8 suggests an item accessible to students from Year 6 to Year 8.

(1) A Wearing Problem

9 - 12

Suppose that the tyres on the front wheels of a car wear out after 40 000 km, and those on the back wear out after 60 000 km.

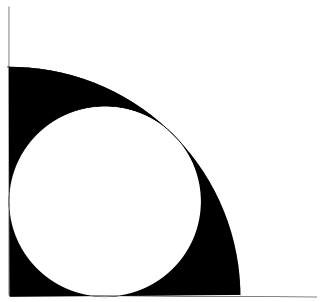
(a) Assuming that the tyres are rotated so that they all wear out at the same moment but the spare is not included in the rotation, how many kilometres can you get out of the four tyres? Give a rotation scheme which achieves this outcome.

(b) Now assume that the spare tyre is included in the rotation while front and rear wheel tyre wear parameters stay the same. How many kilometres can you get out of the five tyres? Give a rotation scheme which achieves this outcome.

(2) Quadcircle

9 - 12

What percentage of the area of the quadrant is taken up by the circle which is tangent to the two radii and the circumference of the quadrant?



(3) Two Results About Numbers, Courtesy of Greater and Lesser Brute Force

9 - 12

(a) If you have access to a programmable calculator or computer and programming language, write a program to find the five positive integers which are equal to the sum of the cubes of their digits.

(b) On the other hand, prove that no two-digit positive integer is equal to the sum of the squares of its digits.

(4) 11 - 12

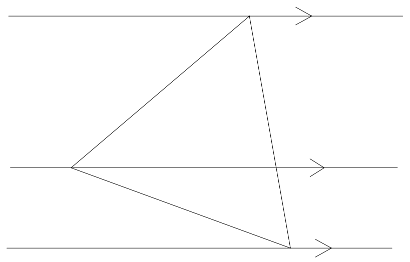
Suppose n is a positive integer and let $f(x) = \frac{x}{1 + n^2 x^2}$. Without using calculus, show that $|f(x)| \le \frac{1}{2n}$.

(Hint: look at $\frac{1}{f(x)}$.)

(5) Equipar

10 - 12

Given any three parallel lines explain how to construct an equilateral triangle which has a vertex on each of the lines.



THUMBNAIL (and longer) REVIEWS

Readers are welcome to contribute to this section. Reviews can cover books, periodicals, videos, software, CD ROMs, calculators, mathematical models and equipment, posters, etc.

The Mathematical Brain
by Brian Butterworth
Published by Macmillan, 1999.

So you think you're bad at maths? Meet Charles, he has a normal IQ and a university degree yet has problems telling whether 5 is bigger than 3. And what about Signora Gaddi, an Italian woman who hears and sees normally but, following a stroke, is deaf and blind to all numbers above 4?

Their stories and others are told by neuropsychologist Brian Butterworth in this book. For Butterworth, they are living evidence that the brain contains a special device for making sense of numbers. It's just a little knot of cells over your left ear, but when it's working properly, this number module doesn't just allow us to see the world in terms of numbers - it compels us to. We can't stop enumerating, says Butterworth, any more than we can avoid seeing in colour. Even as a baby, it was making you notice discrepancies, in, say, how many spoonfuls of food came your way compared with how many came out of the jar.

But if most people have this innate and unstoppable number sense, why do so many numerical skills seem so hard to acquire? And why aren't most of us in the Einstein league of maths brains? Or perhaps we are?

There may be people with extraordinary number abilities, Butterworth says, but these have less to do with the number module than with obsession and hard work. Really? "We don't know how trainable it is," he admits, but he says one intriguing study found career cashiers were just as quick as maths prodigies at multiplying four and five-digit numbers in their heads. His suspicion is that what makes the Einsteins of this world good is what makes everybody good - hard work and practice.

Adapted from Opinion Interview
by **Alison Motluk**, *New Scientist* 3 July 1999

Life by the Numbers
A series of seven video tapes and an accompanying book by Keith Devlin
Produced by WQED, Pittsburgh, USA, 1999.

Keith Devlin, who was a consultant for the *Life by the Numbers* series, has some wise words for those who fear complexity in mathematics: "To most people, mathematics makes the world more complicated. *It doesn't do that!* Math makes the world more simple! Mathematicians are simplistic creatures. We look at the world in the simplest possible way. We look at it in such a simple way that the only way of capturing the simplicity is with symbols, lines, nodes, edges of graphs. We strip away the complexity!" Unfortunately, this wonderful observation is tucked away at the end of the sixth program of this seven part PBS series.

The subtitle of this series, "*math like you've never seen it before*", is a good one, because it applies just as much to those of us in the profession as it does to the average non-specialist viewer. These TV programs are about how mathematics is used all around us, often in conjunction with physics and engineering, in science, medicine, industry, commerce, sports, art and entertainment. The approach taken is fresh, and not all of these manifestations of *applied* mathematics will be familiar to the very people who are entrusted with getting students to learn mathematics - those of us who teach. That's a good enough reason for us to watch the series: too often we lose sight of the fact that we are an exclusive minority, the few survivors of a long, difficult and often unnatural process of learning (in graduate school) and training (on the job), which leaves many talented but bewildered people in its wake, and inevitably has the rest of us convinced that if anybody knows what mathematics is and what it's good for, *we* do. Don't be so sure!

While an increased appreciation for the beauty of mathematics does come with advanced learning, and may serve as a primary motivation for why *we* do what we do, that subtlety is usually lost on the rest of the population. Here, the relevant question is: Do we have a real appreciation of the range of application of mathematics in the world around us today? Do some of us feel that such applications are suspect because they are lacking in "purity", or are not founded on firm theoretical bases? Such reservations would certainly colour our reactions to the content and slant of this series.

One of the principal messages of *Life By The Numbers* is that mathematics permeates virtually all of our lives, and that people can be motivated to learn mathematics via things they find interesting, be that special effects in movies and educational films, the miracle of the internet, the history of art, the wonders of cosmology, the pitfalls of gambling, sports analysis, building better boats, map making, flight simulation, national surveys, wearable computers, modelling international economies, DNA, life insurance, playing chess on the surface of a doughnut, or the chances of being attacked by giant locusts!

The seven parts are titled: "Seeing Is Believing" (special effects), "The Numbers Game" (sports), "Patterns Of Nature" (biology), "Chances Of A Lifetime" (probability), "Shape Of The World" (exploration), "A New Age" (information age) and "Making A Difference" (education). The last, which this writer has not yet seen, is the only one to consider the teaching of mathematics. Unfortunately, many PBS affiliates have chosen to air only the first program, and others haven't aired any. A PBS station in Georgia defended their decision not to bring this series to the state whose students standardized mathematics scores are ranked among the lowest in the nation by claiming that the programs were "too esoteric". So much for education and trying to make a difference.

Throughout the series there is a conscious effort to downplay "the geek factor" and stress the human element and the excitement quotient. Fancy graphics are used, in moderation. Clever production tricks keep the series lively, such as occasional glimpses of personal lives or amusing clips from old movies, and overall the editing is seamless. While some might question the real value of *some* of the applications of mathematics presented here, it's a small price to pay for the overall good to our profession these shows could do. If anything, there is too much material per show, it's hard to absorb it all in one sitting. That's not such a bad complaint, however, and all the more reason to order this series on videotape for your library. This is a series that deserves to be widely seen.

Extracted from a review by **Colm Mulcahy**, Spelman College, Atlanta, Georgia, available at MAA Online.

Achieving Outcomes-based Education Premises, principles and implications for curriculum and assessment

by Sue Willis and Barry Kissane, published by the Australian Curriculum Studies Association, 1997.

This 44 page booklet was originally published as a paper by the Education Department of Western Australia under the title *Outcome-based Education: A Review of the Literature*. For me the basic questions are what is outcome-based education (OBE) and why should educators bother with it? In the sense in which it is used in this booklet "OBE describes an educational process (founded) on trying to achieve certain specified outcomes in terms of individual student learning". OBE incorporates assessment "of individual student progress based on and justified in terms of the outcomes students actually achieve".

Various conceptualisations of OBE are possible. Spady distinguishes transformational, transitional and exit outcomes. Generally advocates of OBE consider that the same outcomes should apply to all students. This raises issues such as recognition of progressive achievement levels and the difficulties of mapping the complex phenomena of learning onto hierarchies of performance levels.

Not surprisingly, proponents of OBE believe that curriculum content and structures should facilitate students' opportunities to achieve the outcomes which the curriculum explicitly articulates. Successful OBE implementations achieve exit outcomes beyond academic learning, incorporating cooperation and collaboration and providing experiences which enhance learner confidence and build respect for others. This requires that schools become supportive learning communities in which all involved grasp the purposes and meanings of what they are doing and are clear about what is expected of them. Thus one characteristic of successful OBE is that students become self-directed learners who expect to be able to demonstrate their evolving capacities. Achievement of outcomes signifies that students can do whatever is required consistently and well. "Falling over the line" on a test or two is not sufficient.

OBE is a process by which school communities collectively take responsibility for making curriculum decisions in line with desired outcomes. The teacher's responsibility is to facilitate the actual learning outcomes and not just cover a specified list of curriculum topics. Clearly this cannot occur unless education systems and the community generally provide the resources, structures and other support to underpin successful outcomes for all.

At the heart of every system for education are the processes used for assessment. In this case the key issue is has the student achieved the outcome and if not, what can be done about it? Given the myriad assessment possibilities, an "evidence-based judgemental model" for OBE assessment seems to be the best choice. This amounts to school-based standard-referencing and combines verbal descriptors with examples of student performance.

In sum, Willis and Kissane present a useful account of the OBE enterprise. Useful references are provided but there is no index. I suspect that one's susceptibility to OBE as a relatively new educational push and one's preparedness to actively engage with it depend critically on one's view of the nature and role of schooling. L. Darling-Hammond puts the issue cogently: "we must rethink the uses of assessment, since we have entered an era where the goal of schooling is to educate all children well, rather than selecting a 'talented tenth' for knowledge work". Whether OBE adds something constructive in the search for improved school education seems to me to depend almost totally on there being a genuine consensus amongst all those involved that "the goal of schooling is to educate all children well".

Peter Enge

SOLUTIONS TO PROBLEMS AND ACTIVITIES

(1) (a) If the car covers a total of d km in wearing out all four tyres then, whatever the rotation pattern, the total road contact distance for front tyres is 2d km and for rear tyres is also 2d km. Since front tyre life is 40 000 km and rear tyre life is 60 000 km, and there are four tyres, we have the equation

$$\frac{2d}{40\,000} + \frac{2d}{60\,000} = 4\,.$$

The solution is d = 48 000.

Possible tyre rotation schemes for achieving this result are to interchange front and back wheels either diagonally or on the same side after 24 000 km, or to rotate the wheels anticlockwise (say) about the car every 12 000 km.

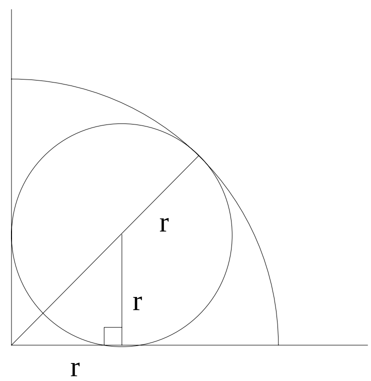
(b) With 5 tyres, the equation becomes

$$\frac{2d}{40\,000} + \frac{2d}{60\,000} = 5$$

with solution d = 60 000.

One possible rotation scheme for achieving this outcome is again to rotate the five wheels (including the spare) anticlockwise about the car every 12 000 km.

(2)



From the diagram, if r is the radius of the circle then the radius of the quadrant is $r(1 + \sqrt{2})$.

$$\therefore \frac{\text{area of circle}}{\text{area of quadrant}} = \frac{\pi r^2}{\pi/4\,(1+\sqrt{2})^2\, r^2}$$

$$= \frac{4}{3 + 2\sqrt{2}}$$

$$= 12 - 8\sqrt{2}$$

$$\cong 0.69.$$

So the area of the circle is about 69% of the area of the quadrant.

---

(3) (a) The five positive integers are 1, 153, 370, 371 and 407. We leave the programming to you. As Paul Halmos points out in his piece Mathematics as a Creative Art, included in *Readings for Calculus* edited by Underwood Dudley and published by the Mathematical Association of American, “most mathematicians couldn’t care less” about results like this. However, the task of **proving** that there are only five positive integers which are equal to the sum of the cubes of their digits is of slightly more mathematical interest. We leave this to you.

(b) Let a and b be digits with $a \neq 0$. Our problem reduces to proving that the equation $a^2 + b^2 = 10a + b$ has no solution.

Rearranging this equation in the equivalent form

$$(a - 5)^2 = 26 - b - (b - 1)^2 ,$$

we observe that since b is a digit and

$$(a - 5)^2 \geq 0,$$

we can only have $0 \leq b \leq 5$. (why?)

So if our equation has a solution, then $(a - 5)^2$ takes one of the values 5, 13, 19, 23 or 25, but none of these cases gives a solution in which a is a non-zero digit and our proof is done.

---

(4) Using the hint we have for $x \neq 0$

$$\frac{1}{f(x)} = \frac{1 + n^2 x^2}{x}$$

Hence for $x > 0$ we can write

$$\frac{1}{f(x)} = \frac{(nx - 1)^2 + 2nx}{x}$$

$$= x\left(n - \frac{1}{x}\right)^2 + 2n.$$

So for $x > 0$, $\frac{1}{f(x)} \geq 2n$, with equality occurring when $x = \frac{1}{n}$.

Now when $x > 0$, $f(x) > 0$

and so $f(x) \leq \frac{1}{2n}$ ... (1)

But f(x) is an odd function, so

for $x < 0$, $f(x) = -f(-x) \geq -\frac{1}{2n}$ using (1).

Noting that $f(0) = 0$, we can conclude that

$$-\frac{1}{2n} \leq f(x) \leq \frac{1}{2n},$$

or equivalently $|f(x)| \leq \frac{1}{2n}$.

Of course a graph of the function $y = f(x)$ confirms that f(x) has a local (and global) minimum at $x = -\frac{1}{2n}$ and a local (and global) maximum at $x = \frac{1}{2n}$.

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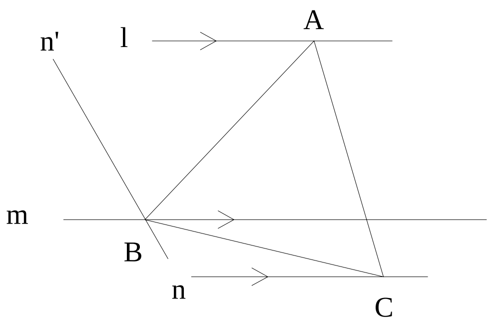
(5)



Given the three parallel lines l, m and n as shown, choose an arbitrary point A on l. Assuming that the required equilateral triangle ABC exists, if C were rotated clockwise through 60° as in our diagram (another solution results if we go anticlockwise) it would land on l at B.

This gives the clue to our method of solution, since we can of course rotate all points on the line n clockwise through 60° about A, obtaining the line n'. (To visualise the rotation of a line about an external point I find it helpful to imagine the perpendicular arm from the point to the line swinging through the angle of rotation). The point B where n' and m intersect is a vertex of the required triangle, and knowing the length AB we easily locate the third vertex C of our equilateral triangle on n.