SHORT CIRCUIT

Newsletter of the Canberra Mathematical Association INC

VOLUME 14 NUMBER 2 FEBRUARY 2023

NEWS AND COMMENT

School is back! ...well almost, as this edition of Short Circuit slides into your inbox. May this year be a rewarding one for teachers and students alike.

Events over the past few years have forced changes on the practice of teaching and it seems likely that adaptations to evolving conditions will necessarily continue. It has always been so, as the 60-year history of the CMA will surely recall. CMA turns 60 this year. Expect a celebration and some reflection.

One brave change in the CMA calendar is the relocation of the annual conference to March. We hope those who usually attend the conference will approve of the change and find the new time in the year convenient. There is more information on page 5 and more to follow.

The CMA council has two new members this year: Roisin Boadle from Erindale College and Matthew Millikin from Marist College. Your professional association needs continual renewal and must benefit from the adaptations that new councillors can initiate.

As is sometimes the case, there are some white spaces left in this newsletter. These are reserved for your feedback, short articles, news items and other comments. Your contribution will be most welcome.

CANBERRA MATHEMATICAL ASSOCIATION

Coming Events:

CMA Conference 2023

Saturday March 18 at ADFA.

Since CMA is turning 60 in 2023, the conference theme will be ***All About Sixty.***

If you would like to be a presenter, send us an email.

canberramaths@gmail.com

See page 5

MEMBERSHIP

Memberships run from 1 Jan to 31 Dec. each year. Membership forms are on the CMA website: http://www.canberramaths.org.au

Membership of CMA includes membership of the Australian Association of Mathematics Teachers and a subscription to one of two AAMT journals.

Members receive a one-third discount for the CMA conference and attractive rates for CMA professional development events.

CMA members may attend conferences of the AAMT affiliates in other states, at member rates.

Inside:

PUZZLES

1 Heterosquares

A heterosquare is defined as a $n \times n$ square array using n^2 consecutive numbers starting from 1, such that every sum of a row, column or diagonal is different. It is like an extremely 'unmagic' square.

Explain why there cannot be a 2×2 heterosquare.

Can you find a 3×3 example?

2. Typos

In a test I was asked to multiply 34 by 86, but being dyslexic, I accidentally multiplied 43 by 68 to get 2312. When my test paper came back, my answer had been marked correct.

Why?

What is the condition that must be satisfied if the product of two 2-digit numbers is to remain unchanged when the positions of the digits in both numbers is reversed?

NMSS 2023

The ANU-AAMT National Mathematics Summer School 2023 took place over the two weeks from 8 to 21 January.

Associate professor Norm Do of Monash University, director of the NMSS, reports that there were three *main-group* attendees from the ACT. They were: **Nicholas Shuttleworth** of Narrabundah College, **Thomas Lin** of Canberra Grammar School, and **Alexis Au** from Canberra Girls Grammar School.

In addition, ACT student **Oscar Brown**, who attended the 2022 NMSS, was invited back as a member of the *experienced* group for this year.

The 2023 NMSS ran successfully, and Professor Do says it was particularly pleasing to return to the ANU campus for the first time since 2019.

Professor Do has thanked CMA for its part in promoting the summer school and in the student selection process, both of which he says are critical to what the NMSS does. The student cohort that is assembled plays a large part in dictating the culture of the school.

Teachers of Year 11 students are encouraged to be on the lookout for promising students whose names they might put forward at the appropriate time for the 2024 NMSS.

FACTORS IN THEIR PRIME

From Peter Fox

This is a preview of a talk Peter is planning for the CMA conference in March. The conference theme *All about sixty* is reflected in the historically significant factoring properties of the number 60.

Prime factorisation can be an extremely *powerful* representation of a number, unfortunately it is often glossed over as students generate the corresponding factor trees before moving onto the next topic. A selection of interesting activities and investigations are provided here, with some left for the reader to explore.

National curriculum (Version 9.0) references include:

> Represent natural numbers as products of powers of prime numbers using exponent notation. [AC9M7N02]
>
> Extend their understanding of the integer and rational number systems, strengthen their fluency with mental calculation, written algorithms and digital tools; and routinely consider the reasonableness of results in context. [Elaboration]
>
> Use exponents and exponent notation to consolidate and formalise their understanding of representations of natural numbers, and use these to make conjectures involving natural numbers by experiment with the assistance of digital tools. [Elaboration]

Factor Sum: Determine the sum of the factors of 40.

The factors of 40 are: {1, 2, 4, 5, 8, 10, 20, 40}. The sum of these factors is equal to 90. Now consider the prime factorisation $40 = 2^3 \times 5^1$. To determine the sum of all the factors, calculate: $(2^0 + 2^1 + 2^2 + 2^3) \times (5^0 + 5^1) = 90$. We arrive at the same answer. Is this a coincidence?

For small numbers you might argue that it is just as quick to write down all the factors and sum them rather than determine the prime factorisation and then express them as a combination of sums and products.

Try calculating the sum of the factors of 3600. There are 45 factors, so it is easy to miss one. Instead, we can write $3600 = 2^4 \times 3^2 \times 5^2$. Our alternative approach to finding the sum of the factors is simple: $(2^0 + 2^1 + 2^2 + 2^3 + 2^4) \times (3^0 + 3^1 + 3^2) \times (5^0 + 5^1 + 5^2) = 12{,}493$.

This is a nice algorithm for year 7 students to use and explore. An even better opportunity exists for students in years 9 and 10, a delightful way to show students that 'factorising' a number or expression can reveal important details. Consider the first example involving the sum of the factors of 40 = $(2^0 + 2^1 + 2^2 + 2^3) \times (5^0 + 5^1)$. Start by evaluating each term:

(1 + 2 + 4 + 8) (1 + 5).

Now expand the brackets, term by term:

1 + 2 + 4 + 8 + 5 + 10 + 20 + 40

Our algorithm now makes sense!

Counting Factors: How many factors does 36 have?

This is a relatively simple problem to solve without resorting to the prime factorisation, simply list all the factors: {1, 2, 3, 4, 6, 9, 12, 18, 36}, so 36 has 9 factors. We can however deduce the quantity of factors by looking at the prime factorisation: $2^2 \times 3^2$. Another number that has exactly 9 factors is 100. We can express 100 as $2^2 \times 5^2$. The number 441 = $3^2 \times 7^2$ also has 9 factors. Look at the differences and similarities in the prime factorisation of these three numbers. Develop and test your own hypothesis for generating other numbers that have exactly 9 factors.

Now try: 200. The factors of 200 are: {1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 200}, a total of 12 factors. We can express 200 as a product of its prime factors: $2^3 \times 5^2$. What about 1323? The number 1323 can be expressed as a product of its prime factors: $3^3 \times 7^2$. Notice the similarities in the prime factorisation? Can you think of another number that would have exactly 12 factors?

The 'rule' for determining the quantity of factors is not provided here, however the free "Factors that Count" activity on the Texas Instruments Australia website includes a student worksheet, teacher notes and answers for a complete investigation that includes the opportunity to introduce some simple coding.

Euler Totient Function: Sound's scary, it's not!

The Euler Totient Function returns the quantity of numbers for any whole number that are co-prime. By example, we can consider the number 12. Looking at all the whole numbers less than or equal to 12; the numbers {1, 5, 7, 11} have no factors in common with 12 (other than 1) and are therefore referred to as co-prime to 12. The Euler Totient Function for the number 12 therefore returns the value 4. Written as: j(12) = 4.

We can determine this quantity from the prime factorisation of 12. The prime factorisation is $2^2 \times 3$.

The calculation: $12 \times (1 - ½) \times (1 - 1/3) = 4$.

Once again, this seems a little trivial for a small number, but try 100. Count all the numbers from 1 to 100 that are co-prime with 100. Already this task is somewhat onerous. However, the calculation is very quick when using the prime factorisation:
$100 = 2^2 \times 5^2$.

The calculation: $100 \times (1 - ½) \times (1 - 1/5) = 40$.

We could write down all the numbers less than 100 that are co-prime to 100: {1, 3, 7, 9, 11, 13, 17, 19, 21, 23 ... 99}. Try it for yourself, using the long-hand approach and compare it to the short-cut using prime factorisation.

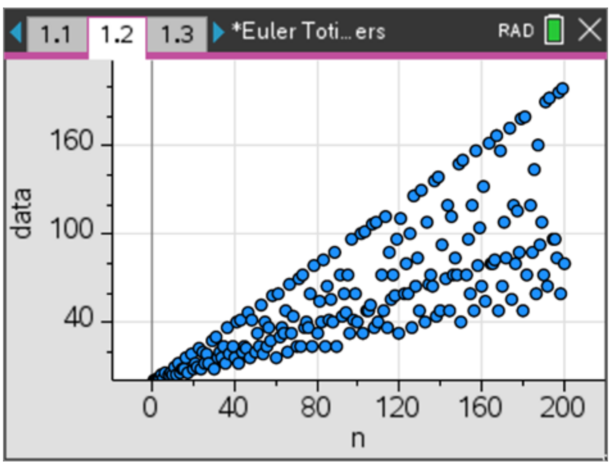
The graph shown below is a scatterplot of the Euler Totient Function for the first 200 numbers. The first characteristic that one might notice are the limiting values. What would be the equation to this line? There also appear to be a couple of trends within the data and also below. Typical values along the bottom of the graph include:

4, 6, 8, 12, 18 ...

What is significant about these numbers?

Conclusion

There are many other uses for prime factorisation, lowest common multiple and highest common factor are two that are commonly used, each of these highlights the use of Venn diagrams! The famous mathematician Ramanujan explored highly composite numbers by consideration of prime factorisation, this exploration is not overwhelming, particularly if calculators are used!



NEWSLETTER OF THE CANBERRA MATHEMATICAL ASSOCIATION INC

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We're on the Web!
http://www.canberramaths.org.au/

ABOUT THE CMA

The Canberra Mathematical Association (Inc.) is the representative body of professional educators of mathematics in Canberra, Australia.

It was established by, among others, the late Professor Bernhard Neumann in 1963. It continues to run - as it began - purely on a volunteer basis.

Sixty years ago

Its aims include

* the promotion of mathematical education to government through lobbying,
* the development, application and dissemination of mathematical knowledge within Canberra through in-service opportunities, and
* facilitating effective cooperation and collaboration between mathematics teachers and their colleagues in Canberra.

THE 2023 CMA COMMITTEE

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Theresa Shellshear is CMA's COACTEA representative.

Sue Wilson is CMA's AAMT representative.

Joe Wilson is the website manager.

Short Circuit is edited by Paul Turner. **ISSN 2207-5755**

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AN ALGEBRA MOTIVATOR

A recent Quanta magazine article explains how the essence of a method of data encryption, called Reed-Solomon codes, can be understood with high school algebra.

Data, these days, whether it is intended to be interpreted as text messages, numerical measurements, sound files, visual imagery, or anything else, is routinely transmitted as numbers. Accordingly, in an encryption process, we can think of both the ciphertext and the plaintext as numbers.

In a simple example of the method described in the article, a message is split into a sequence of two-number words. The words are thought of as points in 2-dimensional space.

An agreed upon fixed private key represents the x-coordinates of two points that constitute a word. The ciphertext message, broadcast publicly, is the y-coordinates of the points. The two points define a line in the plane, $y = Ax + B$.

Using the secret x and the public y values for the two points, we can form two equations that can be solved to find the coefficients A and B. The coefficients are the plaintext.

For example, suppose the private key is 3,7 and the ciphertext is 11,19. The two equations are then
$11 = 3A + B$
$19 = 7A + B$
Hence, the plaintext is $A = 2$ and $B = 5$.

The reader will easily see how the encryption is carried out in the first place.

The message is hidden to everyone who does not have the secret x-coordinates, although a clever automated guessing and checking algorithm would no doubt crack the code in a feasible amount of time.

The method can be made more complicated by using polynomials of degree n to represent words of length $n + 1$. This requires solving systems of n equations to get the coefficients, but this is not hard to do by computer. Other types of mathematical curves on which to locate the points might further complicate matters.

Security may not be the important consideration in data encryption. The method described in the article includes a useful error-correcting possibility.

Suppose, in addition to the two points that constitute a word, one or more check numbers are broadcast. A corresponding number of further x-coordinates is agreed upon. The check points are known to lie on the line for the intended word so that if the points are found not to belong to a line, a transmission error has occurred. It may then be possible to use a line of best fit in order to estimate the correct y-coordinates and so to retrieve the plaintext.

Finding a line (or curve) of best fit is another nifty piece of linear algebra. It can be done nicely with matrices.

PT

CMA conference 2023

All about 60

Saturday March 18
Australian Defence Force Academy (UNSW)

The keynote speaker for the Neville de Mestre Memorial Lecture will be Dr Merrilyn Goos (University of the Sunshine Coast)

Registration Fees: members \$100, non-members \$150, concession \$40.

Fees should be deposited directly to the CMA bank account as no money will be collected at the door. Invoices will be supplied on request.

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Now is a good time to renew your CMA membership in order to benefit from the member discount rate. Go to http://www.canberramaths.org.au/

PUZZLE SOLUTIONS from Vol 14 No 1

1. Possible?

$\frac{5^x}{1+5^x} = \frac{2}{5}$ The numerator on the left cannot be a multiple of 2, and the denominator cannot be a multiple of 5. So, x is not an integer. It is possible to rearrange things to see that $5^x = \frac{2}{3}$

This can be solved for x by taking logarithms, but this step is unnecessary since

$\frac{125^x}{1+125^x}$ is just $\frac{5^{3x}}{1+5^{3x}}$ or $\frac{8}{35}$

2. Impossible?

$$x^{x^{x^{x^{\cdots}}}} = a$$

If the infinite tower of powers does represent a real number, then it is equivalent to

$x^a = a$ and so, $\log x = \frac{\log a}{a}$

By calculus or with the help of a graphing calculator, we find that the function given by $\frac{\log a}{a}$ attains its maximum value when $a = e = 2.718\ldots$.

So, the statement $x^{x^{x^{x^{\cdots}}}} = a$ makes sense provided $0 < a < e$.

In the case $a = 2$, we have $x = \sqrt{2}$.

3. Conveyor belt

[There was an error in the second triple. It should have read (5, 12, 13).]

In the array,

a	*b*
d	*c*

the letters *a, b, c, d* are consecutive Fibonacci numbers. According to the Fibonacci sequence the consecutive numbers are equivalently $a, b, a+b, a+2b$. We form the triple $\{ad, 2bc, ac + bd\}$. So, the triples have the form $\{a(a+2b), 2b(a+b), a(a+b)+b(a+2b)\}$.

After some manipulation, it can be shown that the sum of the squares of the first two terms is the same as the square of the third term.

Note that this relation is true for any Fibonacci-like sequence. That is, a and b can be chosen arbitrarily.