Short Circuit

Newsletter of the Canberra Mathematical Association INC

VOLUME 14 NUMBER 10 OCTOBER 2023

NEWS AND COMMENT

The date and the keynote speakers for the 2024 CMA conference have been confirmed. Date: 16th March. See page 7.

Have a look also at what is on offer from reSolve (on page 6) and from the Mathematics Hub (on page 7).

In this edition you will find a pleasing explanation of the famous Monty Hall problem, contributed by Ed Staples.

As well, on page 2, we have an excursion into the extra-mathematical world of music. It is said that music and mathematics are connected and one might think of various ways in which this could be true. (Some might also wish to refute the claim.)

What can be said with certainty is that ideas and discoveries from non-mathematical realms often motivate mathematical thinking. Abstracting the mathematical essence of an observation can clarify the matter, leading to more questions, predictions and experiments. Applications in the classroom hardly need to be mentioned.

PUZZLE

Ages

This year my age is a multiple of 8. Next year it will be a multiple of 7. I am more than 20 years old but less than 80. What is my age?

What if my age now were a multiple of 7 and next year a multiple of 8?

MEMBERSHIP

Memberships run from 1 Jan to 31 Dec. each year. Membership forms may be downloaded from the CMA website:
http://www.canberramaths.org.au

The several benefits of Membership of CMA may be found on the website.

Coming Events:

AGM: Thursday 16th November 2023.

Conference 16th March 2024.

NEWSLETTER

The CMA newsletter, Short Circuit, is distributed monthly to everyone on our mailing list, free of charge and regardless of membership status.

That you are receiving Short Circuit does not imply that you are a current CMA member.

CMA welcomes all readers.

Inside:

CANBERRA MATHEMATICAL ASSOCIATION

YEAR 12 MATHS MEDALS

The CMA is once again making available to schools and colleges Mathematics medals for the best student in Year 12 Mathematics.

Please contact
Peter McIntyre (Kambah) on 0403 509 952
or Valerie Barker (Aranda) on 0410 151 554
to arrange collection.

THE MONTY HALL PROBLEM

The now well known Monty Hall problem came from the American television game show *Let's Make a Deal* and is named after its original host, Monty Hall.

This explanation is from Ed Staples.

Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a Rolls Royce; behind each of the others, a goat. You pick a door, say door 3. The host, who knows what's behind all doors, says "Look, I'll give you a chance." - and walks over to door 2 and opens it to reveal a goat. He then says to you, "If you like, you can switch to door 1, if you want to, or else stay with door 3 – totally up to you". What should you do?

You might think that it doesn't matter whether you stay with door 3 or switch to door 1. You might argue as follows:

When I picked door 3, I knew that I had a 1/3 chance of winning a Rolls Royce, but the host revealed a goat behind door 2, so now I have a 50-50 chance of winning the car. It wouldn't matter what I did, and he might be trying to entice me away from the car, so I'm sticking with door 3.

As bizarre as it may seem, your argument is incorrect. In fact, you would double your chances of winning the car if you switched to door 1. But why? Try this explanation:

The host would never reveal the car to you, so plainly he has acted strategically when he chose a door to open. As the player, you knew, even before he opened a door, that at least one of the other two doors concealed a goat. Simply knowing that fact (prior to any action by the host) does not affect any of the probabilities, so why would the host's action of confirming what you already know have any effect on the probabilities either? You still have a 1/3 chance of winning by sticking with door 3, implying that you must have a 2/3 chance of winning if you switch to door 1.

Historical note:
The problem was originally posed (and solved) in a letter by Steve Selvin to the publication American Statistician in 1975. However, many readers refused to believe switching was beneficial and rejected the explanation. Even when given explanations, simulations, and formal mathematical proofs, some still do not accept that switching is the best strategy. Paul Erdos, one of the most prolific mathematicians in history, remained unconvinced until he was shown a computer simulation demonstrating the predicted result. (Wikipedia)

THE MUSIC OF TWELVE

A guitar fretboard is designed to allow just twelve distinct pitch classes. We refer to pitch *classes* because we consider as separate entities sets of pitches that blend so well that they sound alike, except for being higher or lower. For example, the lowest and highest guitar strings both produce a note 'E'. The twelve pitch classes are like an axiom of the music with which we tend to be most familiar. Why are there not seven pitches, or ten or thirteen?

We recognise that the sensation of pitch is related to the frequency of a vibration, which in turn can depend on, among other things, the vibrating length of a string. Doubling the frequency of a vibration by halving the string length produces a pitch in the same class as the pitch produced by the whole string.

Guitarists know that by placing a fingertip lightly half-way along a string, a note can be produced that sounds very like the open string note but higher. A similar effect occurs when the finger is placed one third or one quarter of the way along the string. These pitches, called harmonics, parallel an experiment of Pythagoras. He noticed that a string divided into two vibrating parts by a bridge produces a pair of harmonious sounds when the two lengths are in a simple whole number ratio.

We might plausibly fill in the interval between a pitch and the next higher member of its pitch class using intervals corresponding to simple frequency ratios 1:1, 9:8, 5:4, 4:3, 3:2, 5:3, 7:4 and 2:1. Such a scale has seven gaps between eight pitches. Hence, it is said to span an *octave*. While being harmonically pleasant, this scale has unevenly sized intervals between successive pitches.

Another option is to build a scale using only the intervals 3:2 and 4:3, or its reciprocal 3:4. In this scheme it is possible to arrive close to the 2:1 octave interval after a succession of 12 steps, but not quite. We could have 3:2 × 3:4 × 3:2 × 3:4 × 3:2 × 3:4 × 3:4 × 3:2 × 3:4 × 3:2 × 3:4 × 3:2 = 531441:262144, which is slightly bigger than 2:1.

As a compromise, one which has been adopted in western music since about the time of J. S. Bach, we can make the 3:2 intervals too small by an imperceptible amount and the 3:4 intervals slightly too large. In this way we obtain a scale with 12 equal steps from any pitch to its octave.

To understand why the number twelve is linked to such an agreeable result, and to check for other possibilities, consider a pair of pitches whose frequencies are in the ratio 3:2. We hear such pairs of pitches in the opening two notes of the bugle call *The Last Post*, and similarly in the opening notes of the theme music to the movie *2001: A Space Odyssey*. We hear them in what in rock guitarists call a power chord, and the strings of the violin family are tuned in this ratio. It is a recognisable and natural musical interval.

Assuming that we want to keep octaves (2:1) perfectly in tune, we might look for a ratio 2^q:1 that corresponds to the 3:2 interval. The index q must be an irrational number between 0 and 1 but perhaps there are good *rational* approximations to it. The denominator of such an approximation would indicate how many equal steps might be inserted between a pitch and its octave to preserve the 3:2 ratio as nearly as practicable. In effect, we ask for approximate solutions to $2^q = 3/2$. Equivalently,

$q = \log 3 / \log 2 - 1 \approx 0.58496250072\ldots$.

Rounding this decimal produces rational approximations with denominators 10, 100, 1000, and so on. However, by expanding the decimal as a continued fraction we can find much better rational approximations, with smaller denominators. By truncating the continued fraction at successive stages, we obtain the sequence

$\{1, ½, 3/5, 7/12, 24/41, 31/53, 179/306, \ldots\}$.

When $q = 7/12$, we see that 2^q has the value 1.4983… which is very close to 1.5. We could choose a better approximation with a larger denominator, but $q = 7/12$ is good. This number suggests that the octave can be conveniently divided into 12 scale steps, each increasing in frequency by the factor $2^{1/12}$. (These steps, called semitones, correspond to a difference of 1 fret on the guitar.) The seventh step up from the fundamental pitch gives an interval close to the 3:2 ratio.

(Fine differences in pitch are measured in cents, each of which is 1/100 of a semitone. That is, an increase in pitch of one cent corresponds to a frequency increase by a factor of $2^{1/1200}$. Depending on circumstances, humans can detect pitch changes of about 6 cents.)

In the list of rational approximations, 3/5 is next best to 7/12. This suggests that a scale of 10 equal steps would have its sixth step corresponding to the ratio $2^{3/5}$:1. This interval is noticeably sharper than the 3:2 ratio, by about 18 cents, while the $2^{7/12}$:1 interval is flat by less than 2 cents, which is hardly perceptible.

Music based on the axiom of the equally spaced 10-note scale sounds very different from the customary 12-note model. Examples exist but you will have to search for them.

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ABOUT THE CMA

NEWSLETTER OF THE CANBERRA MATHEMATICAL ASSOCIATION INC

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We're on the Web!
http://www.canberramaths.org.au/

The Canberra Mathematical Association (Inc.) is the representative body of professional educators of mathematics in Canberra, Australia.

It was established by, among others, the late Professor Bernhard Neumann in 1963. It continues to run - as it began - purely on a volunteer basis.

60 years ago

Its aims include

* the promotion of mathematical education to government through lobbying,
* the development, application and dissemination of mathematical knowledge within Canberra through in-service opportunities, and
* facilitating effective cooperation and collaboration between mathematics teachers and their colleagues in Canberra.

THE 2023 CMA COMMITTEE

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Theresa Shellshear is CMA's COACTEA representative.

Bruce Ferrington is CMA's AAMT representative.

Joe Williams is the website manager.

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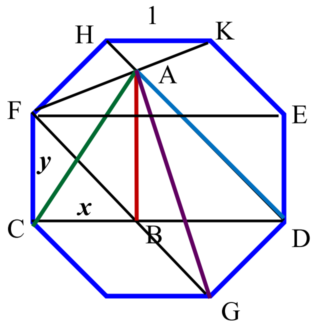
http://www.facebook.com/pages/Canberra-Mathematical-Association/110629419011275

PUZZLE SOLUTION from Vol 14 No 9

Amazing octagon



Labels have been added to some points and segments in the diagram.

We can assume opposite sides of the regular octagon are parallel, and diagonals like CD are parallel to pairs of sides of the octagon. Diagonals like CD have length $1 + \sqrt{2}$, with CB = 1 and BD = $\sqrt{2}$. Each of the three angles at vertex D is 45°.

If we knew that the red segment BA was perpendicular to the diagonal CD, it would follow that triangle ABD is isosceles and the red diagonal AB has length $\sqrt{2}$. Then, the teal diagonal AD has length $\sqrt{4}$; in triangle ABC, the green segment has length $\sqrt{3}$; and in triangle ADG the purple diagonal AG has length $\sqrt{5}$.

To confirm that BA is perpendicular to CD, a plan will be to represent the line segments in the diagram as vectors. They can be expressed as linear combinations of a pair of basis vectors. We take as a basis the orthogonal unit vectors CB = $\boldsymbol{x}$ and CF = $\boldsymbol{y}$, and show that the component of BA parallel to $\boldsymbol{x}$ has coefficient 0, so that BA is parallel to $\boldsymbol{y}$.

Observe that triangles AHK and KEF are similar. Let FA have length a, and let AK have length b. Then, by the similar triangles, $(a + b)/(1 + \sqrt{2}) = b$. Thus, $a = b\sqrt{2}$. Then, $a/(a + b) = 2 - \sqrt{2}$. That is, the vector FA is $2 - \sqrt{2}$ times the vector FK.

In terms of the basis vectors,
FK = $(1 + 1/\sqrt{2})\cdot\boldsymbol{x} + 1/\sqrt{2}\cdot\boldsymbol{y})$. So that
FA = $(2 - \sqrt{2})((1 + 1/\sqrt{2})\cdot\boldsymbol{x} + 1/\sqrt{2}\cdot\boldsymbol{y})$.

Putting the pieces together, we have
BA = $-\boldsymbol{x} + \boldsymbol{y} + (2 - \sqrt{2})((1 + 1/\sqrt{2})\cdot\boldsymbol{x} + 1/\sqrt{2}\cdot\boldsymbol{y})$, and this simplifies to $0\cdot\boldsymbol{x} + \sqrt{2}\cdot\boldsymbol{y}$, as required.

CAREERS AND MATHEMATICS

Careers and Mathematics
Mathematics is in every job - we all know that but do our students? We will explore a different job and the mathematical activities involving this job from the website "On the Job".
Let's have a look at the Feedlot Manager. Detailed information about the Feedlot Manager can be found here [Feedlot Manager - Career Advice - Environments - On The Job]
Context and relevance: To get beef on any Australian table it has to go through a process. One of these procedures is to fatten the beef for slaughter. This is where Feedlots come into play.
On the Job Activities for the Classroom: These activities are to interest students and encourage them to look, in our case, at the mathematics involved in each particular job.

Activity 1: The Feedlot Manager's Working Life: what s/he needs to know: A SWOT Analysis
Students are presented with a detailed article from the Dept of Agriculture & Fisheries looking at the feedlot profitability. They are to analyse the article and write up as many questions as they can after brainstorming. They have to pose at least one mathematical question. As a class, they are to discuss the big question: What does a feedlot manager need to know?

Activity 2: Solid Wastes in the Feedlot – What of it?
Students are given an article from The Meat & Livestock Association Australia about waste management particularly manure in a feedlot. Students are to answer a number of mathematical calculations about the amount of manure produced; how much needs to be taken away; a comparison with the sale of cow manure; and, the overall cost of turning this waste product into a resource.

Careers & Mathematics can be found at https://onthejob.education/teachers_parents/Mathematics_Teachers/Careers_Mathematics_Index.htm
Contact Information
If you are investigating a job or person in that job, please contact me Frances Moore – I would be happy to hear from you.
Frances.Moore@onthejob.education

Mob 0410 540 608

re(Solve)

Designed by the AUSTRALIAN ACADEMY OF SCIENCE

DESIGN FOR LEARNING LAB

Multiplication and Division

LEARN DESIGN TEACH

Join the reSolve team for a free two-day Design For Learning Lab on teaching multiplication and division in the primary and lower secondary classroom.

Multiplication and division are important parts of the mathematics curriculum, but students often struggle to build deep understanding.

In this interactive workshop, participants will unpack the key ideas and skills in multiplication and division, and explore representations central to developing students' understanding and fluency.

Participants will also investigate learning sequences in the domain and contribute to the design of these sequences.

Cost	Free, catering provided
When	**24–25 October; 9am–3pm** *Attendance required both days*
Where	Academy of Future Skills, Caroline Chisholm School

Our Design For Learning (D4L) Labs are a new initiative of Australian Academy of Science Education. Participants will:

LEARN – Take a deep dive into a content area of mathematics and how to best teach it so that students develop a deep connected understanding of the topic.

DESIGN – Work with the reSolve team to design new learning sequences, to be released on the reSolve website in early 2024.

TEACH – Take the learning sequences back to your school, teach them to your class and provide feedback that will shape the new resources.

Participants of our D4L Labs will be acknowledged on the reSolve website and receive a certificate for 15 hours of self-identified professional learning aligned to AITSL standards.

Australian Academy of Science Education

Register now at resolve.edu.au/d4l-Canberra

For further information please email education@science.org.au

FREE PROFESSIONAL LEARNING

(mathematicshub.edu.au)

The Mathematics Hub belongs to Maths in Schools: Teaching and Learning Resources to Support Mathematics. It is funded by the Australian Government Department of Education and managed by Education Services Australia.

The **Maths in Schools** Professional Learning program has a range of free professional learning resources to suit your needs, including **free online courses**, **training events** and **professional learning packs** delivered by The University of Adelaide. The program aims to build confidence and positive mindsets in maths and numeracy, founded on evidence-based research and pedagogy and harnessing the many high-quality resources and programs found on the Mathematics Hub.

The program is available to all teachers but has a particular focus on reaching participants based in schools in lower socio-economic areas, regional, remote locations, schools with higher First Nation student enrolments and out of field teachers looking to increase their confidence in teaching maths and numeracy.

NGUNNAWAL

The Ngunnawal people are the Traditional Custodians of the Canberra region with a continued cultural, spiritual and historical connection to the area.

Evidence of the Ngunnawal's ancestral connection of the Canberra region stretches back over tens of thousands of years where cultural, social, environmental, spiritual and economic connection to these lands and waters has been maintained in a tangible and intangible manner.

Ngunnawal country was defined by the language of the people inhabiting the land, not by lines clearly marked on a map. Generally, the borders covered the area from Yass to Boorowa, towards Coolac, the highlands west of the Shoalhaven and back to Goulburn. This is an area of almost 11 000 square kilometres.

Ngunnawal is bordered by Wiradjuri, Ngarigo, Gundungurra and Yuin lands.

This link (Ngunnawal) connects to a website with a fuller account of the Ngunnawal story, with links to YouTube videos.