SHORT CIRCUIT

Newsletter of the Canberra Mathematical Association INC

VOLUME 13 NUMBER 1 JANUARY 2022

NEWS AND COMMENT

Inside:

Coming Events:

2022 CMA conference

Wednesday Workshops:

A happy 2022 to all our readers.

MEMBERSHIP

Memberships run from 1 Jan to 31 Dec. each year. Membership forms can be accessed from the CMA website: http://www.canberramaths.org.au

Membership of CMA includes affiliation with the Australian Association of Mathematics Teachers and a subscription to one of two AAMT journals.

As a member, you are entitled to attractive rates for the CMA annual conference and CMA professional development events.

CMA members may attend conferences of the AAMT affiliates in other states, MAV, MANSW, etc. at member rates.

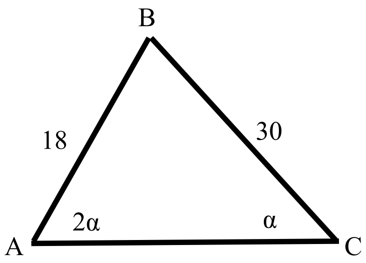
CANBERRA MATHEMATICAL ASSOCIATION

PUZZLES

1. Days and dates

Christmas day fell on Friday in 2020 and on Saturday in 2021. In what year will Christmas fall on a Monday?

2. Bothersome triangle



What area is enclosed by the triangle?

SINES AND THE CIRCUMCIRCLE

This is the second part of a piece contributed by Ed Staples. The first part was in the previous edition, [Vol 12 No 12].

The sine rule

As Euclid pointed out, *Things equal to the same thing are equal to each other*. We may have written down the Sine Rule $a/\sin A = b/\sin B = c/\sin C$ on a white board many times without a moment's thought about what quantity these ratios might all be equal to.

Consider the following triangle where each of the ratios $\sqrt{2}/\sin 45°$, $\sqrt{3}/\sin 60°$, and $((\sqrt{3}+1)/\sqrt{2})/\sin 75°$ is equal to 2 (which can be verified using the well-known exact values).

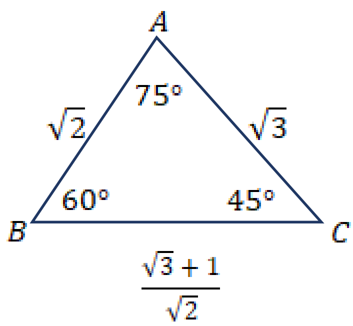


Diagram 1

We might reason that by consideration of the size of any one of the triangle's three angles and the lengths of the sides opposite them, that there may well be other triangles that could have ratios equal to 2. The explanation is shown in the diagram below with the circumcircle shown here for the above triangle.

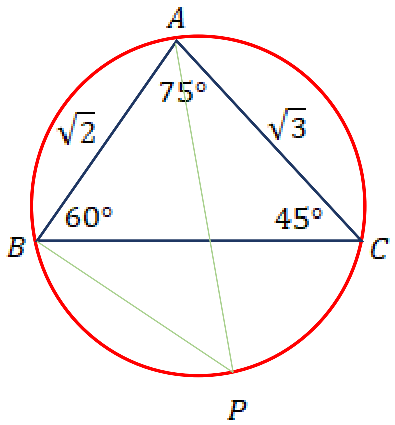


Diagram 2

If we select any point P on the circumference lying on the same side of the chord AB as the point C, then the angle APB in triangle ABP will be the same size as the angle ACB. Hence the ratio $|AB| / \sin APB = 2$, the same value as the ratio in the original triangle.

Further, if we make AP a diameter, the angle ABP is a right angle, so that

$|AP| / \sin ABP = |AP| / 1 = |AP|$.

Thus, the ratio is revealed as the length of the diameter of the circumscribed circle.

So next time you teach the Sine Rule, instead of writing $a/\sin A = b/\sin B = c/\sin C$ why not write $a/\sin A = b/\sin B = c/\sin C = d$, where d is the diameter of the circumscribed circle. Then go on to prove it.

E.S.

THE CANBERRA MATHEMATICS TALENT QUEST 2021

The Canberra Mathematical Association council was delighted with the response from local participants in the first-in-a-long-time Canberra Mathematics Talent Quest.

Students from two schools went on to be successful in the National competition. Shown below is the presentation of the NMTQ trophy to acting principal Michelle Maier at Arawang Primary School. The year 6 class won the award.

The other winning school was North Ainslie Primary.

Look out for CMTQ 2022.

JOB VACANCY

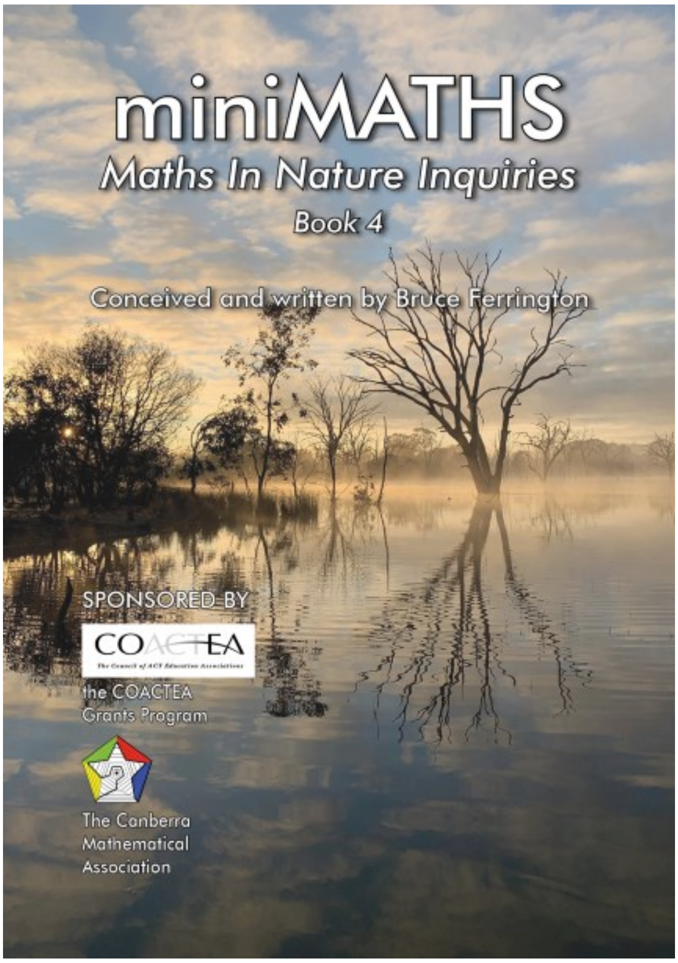
Lyndall Muller, Human Resources Manager at Burgmann Anglican School writes,

Good afternoon,

We are looking for a Maths/Science teacher to work in our Middle School (6-8) in 2022.

Contact: lyndall.muller@burgmann.act.edu.au

NATURE PLAY



Coming soon!

Bruce says,

Take your preschool learners outside to do some maths.

www.minimaths.com.au

NEWSLETTER OF THE CANBERRA MATHEMATICAL ASSOCIATION INC

PO Box 3572
Weston ACT 2611
Australia

E-mail: canberramaths@gmail.com

We're on the Web!
http://www.canberramaths.org.au/

ABOUT THE CMA

The Canberra Mathematical Association (Inc.) is the representative body of professional educators of mathematics in Canberra, Australia.
It was established by, among others, the late Professor Bernhard Neumann in 1963. It continues to run - as it began - purely on a volunteer basis.

Its aims include

* the promotion of mathematical education to government through lobbying,
* the development, application and dissemination of mathematical knowledge within Canberra through in-service opportunities, and
* facilitating effective cooperation and collaboration between mathematics teachers and their colleagues in Canberra.

THE 2022 CMA COMMITTEE

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Theresa Shellshear is CMA's COACTEA representative.

Sue Wilson is CMA's AAMT representative.

Joe Wilson is the website manager.

Short Circuit is edited by Paul Turner. **ISSN 2207-5755**

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http://www.facebook.com/pages/Canberra-Mathematical-Association/110629419011275

PUZZLE SOLUTIONS from Vol 12 No 12

1. Inside or out

Begin by choosing two points randomly on the circumference of a circle. Construct the diameter that is parallel to the line joining the two points. A third randomly selected point is equally likely to be on the same or the opposite side of the diameter as the first two points. In only the first of these cases will the tangents drawn at the three points intersect to form a triangle that encloses the circle. Thus, the probability of this occurring is 0.5.

We have ignored the possibility that the first two points could themselves be at the ends of a diameter. But that event has zero probability.

2. Pascal's triangle

It's complicated!

Let the product of the numbers in row n be P_n . The numbers in row n are just the binomial coefficients $\binom{n}{0}, \binom{n}{1}, \ldots, \binom{n}{n}$

and so, the product can be written

$$\prod_{k=0}^{n} \frac{n!}{k!\,(n-k)!}$$

Then,

$$\frac{P_{n+1}}{P_n} = \frac{\prod_{k=0}^{n+1} \frac{(n+1)!}{k!\,(n+1-k)!}}{\prod_{k=0}^{n} \frac{n!}{k!\,(n-k)!}}$$

$$= \frac{(n+1)!^{n+2}}{n!^{n+1}} \cdot \frac{\prod_{k=0}^{n} k!\,(n-k)!}{\prod_{k=0}^{n+1} k!\,(n+1-k)!}$$

$$= \frac{(n+1)!^{n+2}}{n!^{n+1}} \cdot \frac{\prod_{k=0}^{n} k!\,(n-k)!}{\prod_{k=0}^{n} k!\,(n+1-k)!} \cdot \frac{1}{(n+1)!}$$

$$= \frac{(n+1)!^{n+1}}{n!^{n+1}} \cdot \prod_{k=0}^{n} \frac{(n-k)!}{(n-k+1)!}$$

$$= (n+1)^{n+1} \cdot \prod_{k=0}^{n} \frac{1}{n-k+1}$$

$$= (n+1)^{n+1} \frac{1}{n+1} \cdot \frac{1}{n} \cdot \frac{1}{n-1} \cdots \frac{1}{1}$$

$$= \frac{(n+1)^n}{n!}$$

It follows that

$$\frac{P_{n-1}}{P_n} = \frac{(n-1)!}{n^{n-1}}$$

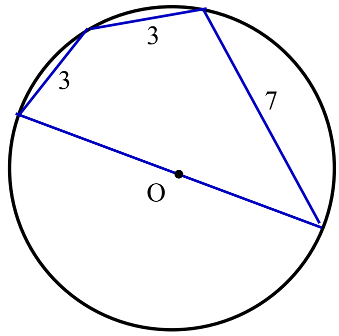
And therefore,

$$\frac{P_{n+1}P_{n-1}}{{P_n}^2} = \frac{(n+1)^n}{n!} \frac{(n-1)!}{n^{n-1}} = \left(1+\frac{1}{n}\right)^n$$

To complete the proof, we recognise that

$$\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e$$

3. Cyclic quadrilateral



Using Ptolemy's theorem, the angle-in-a-semicircle theorem and Pythagoras, we have

$$3D + 3 \times 7 = \sqrt{D^2 - 3^2}\sqrt{D^2 - 7^2}$$

where D is the diameter. And so,

$$3^2(D+7)^2 = (D^2 - 9)(D+7)(D-7)$$

Eventually, we find $D = 9$ and hence, the radius is 4.5.

4. Ptolemy's Theorem

In a cyclic quadrilateral, the sum of the products of the opposite sides is equal to the product of the diagonals.

There are various modern proofs, but Ptolemy would likely have used the idea of similar triangles. No one knows for certain.