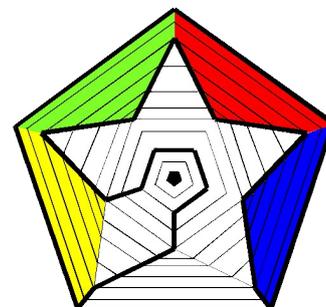


SHORT CIRCUIT

Newsletter of the Canberra Mathematical Association INC

VOLUME 13 NUMBER 10

OCTOBER 2022



NEWS AND COMMENT

The CMA dinner, briefly interrupted by the Annual General Meeting, will take place on Thursday 17th November, 6:00 p.m. at the CIT restaurant, Reid campus. Come along and enjoy a convivial meal and possibly put yourself forward as a CMA committee member.

At the Special General Meeting of the AAMT most attendees voted to adopt the new constitution.

CEO Allan Dougan reports that there will be no change to AAMT's operations.

Paperwork will now be submitted to the SA government to change AAMT to a company limited by guarantee.

At the September meeting of the CMA council, the decision was made to hold the **2023 conference** in March rather than in August, follow-

ing a proposal from vice president, Bruce Ferrington.

This is a significant change from long established practice and will require immediate work from the incoming 2023 Council. The meeting was very positive in its desire to make this a success.

Entries for the National Mathematics Talent Quest were received from students in Years 4 and 9 at Radford College and Melrose High School respectively.

Only 61 entries (in 39 categories) were received nationally and all states have noted the impact of COVID and consequent teacher shortages on the capacity of schools and students to present work.

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Coming Events:

Dinner and AGM Thursday 17th November, CIT Reid, 6:00 p.m.

Wednesday Workshops:

Check for notices sent separately.



MEMBERSHIP

Memberships run from 1 Jan to 31 Dec. each year. Membership forms can be accessed from the CMA website: <http://www.canberramaths.org.au>

Membership of CMA includes affiliation with the Australian Association of Mathematics Teachers and a subscription to one of two AAMT journals.

As a member, you are entitled to attractive rates for the CMA annual conference and CMA professional development events.

CMA members may attend conferences of the AAMT affiliates in other states, MAV, MANSW, etc. at member rates.

CANBERRA
MATHEMATICAL
ASSOCIATION

PUZZLES

1. Near enough

The following irrational numbers are all quite close to integers: $e + 2\pi$, $\pi + 92\phi$, $10\phi^e$, where e is the base of natural logarithms, π is the circle constant, and ϕ is the golden ratio $(1 + \sqrt{5})/2$.

Which of these best approximates its nearby integer?

2. Could do better

The irrational number ϕ is near the rational number $809/500$, but it is also not far from $233/144$. Make a case for choosing one approximation over the other.

3. Exponential x

There are three values of x that satisfy the following equation.

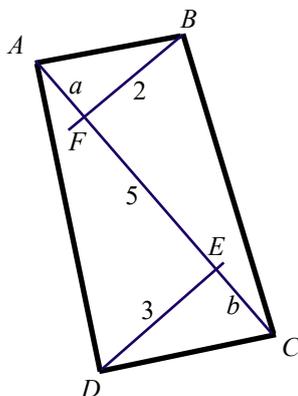
$$4^x + 7^x + 11^x = 1^x + 2^x + 9^x + 10^x$$

If the equation is modified to

$$4^x + 7^x + 11^x = 2^x + 9^x + 10^x$$

there are still three solutions, but they are much harder to find and may be impossible to write down.

4. Survey



The angles at A , C , E and F are right angles. The surveyor has not recorded the distances a and b . However, it is still possible to find the area of the quadrilateral, given the right angles at A and C .

A MYSTERIOUS FIBONACCI PROPERTY

The Fibonacci sequence begins 1, 1, 2, 3, 5, 8, 21, ...

Consider the ordered triples of three consecutive Fibonacci numbers F_{2n} , F_{2n+1} , F_{2n+2} , starting on even numbered terms.

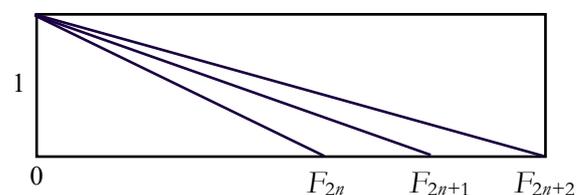
For example, we could form the ordered triples $\{1,2,3\}$, $\{3,5,6\}$, $\{8,13,21\}$ etc.

Now, we might form the reciprocals of the elements in each triple. $\{1, 1/2, 1/3\}$, $\{1/3, 1/5, 1/8\}$, $\{1/8, 1/13, 1/21\}$, ...

If these fractions are considered to be tangent ratios of acute angles, we find that the angle associated with each first element of the inverted triple was equal to the sum of the angles associated with the other two numbers.

That is, $\tan^{-1} 1 = \tan^{-1} 1/2 + \tan^{-1} 1/3$, and $\tan^{-1} 1/3 = \tan^{-1} 1/5 + \tan^{-1} 1/8$, and $\tan^{-1} 1/8 = \tan^{-1} 1/13 + \tan^{-1} 1/21$ etc.

The angle sums can be represented as in the following diagram.



As n increases the angles become perilously small. Nevertheless, the pattern of angle sums continues indefinitely. [Proof omitted]

We notice that the results can be linked together like a chain of sums since, for any angle sum equation, the last angle is also the first angle of the next angle sum.

This means, for example, that we could write

$$\begin{aligned}\tan^{-1} 1 &= \tan^{-1} 1/2 + \tan^{-1} 1/3 \\ &= \tan^{-1} 1/2 + \tan^{-1} 1/5 + \tan^{-1} 1/8.\end{aligned}$$

But $\tan^{-1} 1/8 = \tan^{-1} 1/13 + \tan^{-1} 1/21$,
so we can form the longer chain $\tan^{-1} 1 =$
 $\tan^{-1} 1/2 + \tan^{-1} 1/5 + \tan^{-1} 1/13 + \tan^{-1} 1/21$.

We could continue doing this and ultimately write

$$\begin{aligned}\pi/4 &= \tan^{-1} 1/2 + \tan^{-1} 1/5 + \tan^{-1} 1/13 + \\ &\tan^{-1} 1/34 + \tan^{-1} 1/89 + \dots\end{aligned}$$

What a striking result this is. ... a limiting sum of
angles from the Fibonacci sequence.

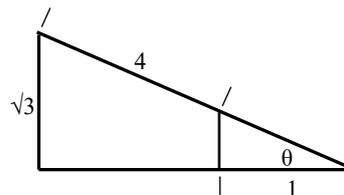
Very mysterious indeed!

ES

PUZZLE SOLUTIONS from [Vol 13 No 9](#)

1. Mischievous

Find θ .



Remarkably, in radian measure the angle is a simple fraction of π and in degrees it is an integer.

The hypotenuse of the small triangle is $\sec \theta$. Thus,

$$4 + \sec \theta = \sqrt{3} \operatorname{cosec} \theta. \text{ Then,}$$

$$4 = \sqrt{3}/\sin \theta - 1/\cos \theta. \text{ Adding the fractions gives}$$

$$2\sin \theta \cos \theta = \sqrt{3}/2 \cos \theta - 1/2 \sin \theta. \text{ Then,}$$

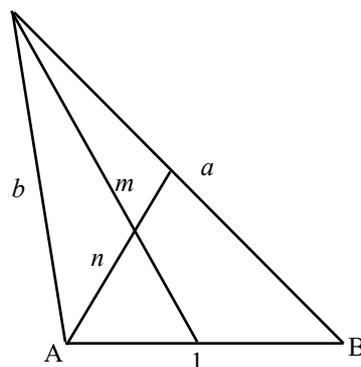
$$\sin 2\theta = \sin \pi/3 \cos \theta - \cos \pi/3 \sin \theta$$

$$= \sin (\pi/3 + \theta)$$

So, $2\theta = \pi/3 + \theta$ and it follows that $\theta = \pi/9$ or 20° .

2. Fiendish

The lines m and n inside the triangle are medians. They are in
the ratio 2:1, as are the angles A and B. Find a and b .



By the sine rule and the fact that $A = 2B$, we have

$$\cos B = a/2b. \quad (1)$$

Then, using the cosine rule and (1),

$$m^2 = a^2 + 1/4 - a^2/2b \quad (2)$$

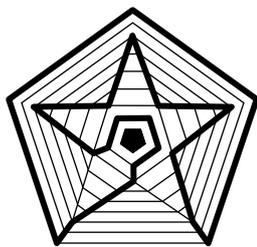
$$n^2 = (a/2)^2 + 1 - a^2/2b \quad (3)$$

Since $m^2 = 4n^2$, we use (2) and (3) to obtain

$$a^2 = 5b/2.$$

Using $b^2 = a^2 + 1 - 2a \cos B$, and (1), we deduce

$$b = 3/2 \text{ and } a = \sqrt{15}/2.$$



**NEWSLETTER OF THE CANBERRA
MATHEMATICAL ASSOCIATION
INC**

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We're on the Web!

<http://www.canberramaths.org.au/>

THE 2022 CMA COMMITTEE

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	Joe Williams	

Theresa Shellshear is CMA's COACTEA representative.

Sue Wilson is CMA's AAMT representative.

Joe Wilson is the website manager.

Short Circuit is edited by Paul Turner.

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ABOUT THE CMA

The Canberra Mathematical Association (Inc.) is the representative body of professional educators of mathematics in Canberra, Australia.

It was established by, among others, the late Professor Bernhard Neumann in 1963. It continues to run - as it began - purely on a volunteer basis.

Its aims include

- * the promotion of mathematical education to government through lobbying,
- * the development, application and dissemination of mathematical knowledge within Canberra through in-service opportunities, and
- * facilitating effective cooperation and collaboration between mathematics teachers and their colleagues in Canberra.



Find us on Facebook

CAREERS AND MATHEMATICS

Wind Turbine Technician.

For details of the activities listed below, refer to the [website](https://onthejob.education/classhome_activities/environments/wind_turbine_technician.htm):

https://onthejob.education/classhome_activities/environments/wind_turbine_technician.htm

Context and relevance:

Climate change and renewable energy can be upper most for the modern student. These activities show the conversion of kinetic wind energy to electrical energy.

Activities for the Classroom:

Activity 1:

The Science behind frozen wind turbines – Retrieval Chart Strategy

Secondary

In this activity, students are to investigate the science and mathematics behind problems with frozen wind turbines and carry out a critical analysis.

Activity 2:

Mathematical Calculations and Wind Turbines

This is a collection of mathematical calculations from New Zealand, UK, China, Australia and Try Engineering and include:

Primary

2 Activities around Graphing and Wind Power

Primary, Middle, Secondary

Students are to design their own Windmill and test it out.

Secondary

These three activities are based on Wind Turbine Power calculations and were created by the Royal

Academy of Engineering UK.

Middle

Wind Energy Math Calculations: Calculating the Tip Speed Ratio of Your Wind Turbine. Imperial measurements used. 9 Sample problems.

Primary

Hydro Australia presents activities for students in Years 6 & 7. The unit for Year 7s: This inquiry-based unit helps students discover the basic fundamentals of wind power technology by building and testing wind turbines. The challenge is to generate the greatest amount of electricity by varying the numbers, angles, sizes and shapes of turbine blades.

Middle, Secondary

A third activity is found here: “So how can Australia transform into a renewable energy powerhouse without leaving anyone behind? 6 Thinking Hats”.

While there is mathematics involved it is not specifically directed at mathematics.

Careers & Mathematics can be found at

https://onthejob.education/teachers_parents/Mathematics_Teachers/Careers_Mathematics_Index.htm

Contact Information

If you are investigating an aspect of mathematics or would like information about a person in that job, please contact me Frances Moore – I would be happy to hear from you.

Frances.Moore@onthejob.education

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