Examples of optimisation in applied and pure mathematics

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0. Why do we optimise?
   - most models of reality have
     too many or
     too few solutions
   - ultimately all optimisations are very local
   - so we can do things we could not otherwise have done

1. Warwick's approach to optimising
   - have the courage to discard a solution

2. Minimising polygons
   - origin of problem
     http://www.ralphthetriangle.com/trailers/sps
   - The square, the triangle, and the hexagon
     http://www.ralphthetriangle.com/papers/sth.pdf
   - pseudo-tilings of pentagons.
     http://www.ralphthetriangle.com/papers/edgemin.pdf

3. Zoltan's approach to optimising
   - do we really need to do this at all?

4. Optimising without calculus
   - AGM inequality
   - 2-dim picture of RMS >= AM >= GM >= HM
   - E.g. Minimise f(x,y) = 12/x + 18/y + xy
                  g(x,y) = 4x + x/y^2 + 4y/x
     for x,y > 0
   https://artofproblemsolving.com/wiki

5. Miniature dough mixer
   - 1 metre diameter to 30mm
   - expanding circles to find optimal pin location
   http://www.ralphthetriangle.com/trailers/m8

6. Universally optimal sphere packing (pure & applied)
   - Maryna Viazovzka
   - sphere packing in 8-dim and 24-dim
   - Leech lattice (from cannonball vector)

7. Perfect pyramids
   - an 8-dim simplex (hyper-tetrahedron) has both
     its hypervolume and all hyper-faces integral.
   http://www.ralphthetriangle.com/papers/perfectpyramids.pdf

8. Bacteriophage phi-X174
   - 9 known genes in 5375
   - re-use of gene segments.
   - Moebius band
   http://www.ralphthetriangle.com/papers/bio_topology.pdf

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Edge Minimisation Series

You have an unlimited supply of unit edges (imagine them to be matchsticks) from which you can construct various polygonal shapes..

---SQUARES---

Q1) What is the largest number of unit squares you can construct using 12 unit edges if no sharing of edges is allowed?

Q2) What is the largest number of unit squares you can construct using 12 unit edges if sharing of edges is allowed?

Q3) What is the fewest number of edges required to construct 8 unit squares if sharing of edges is allowed?

Q4) Can you find more than one configuration of 8 unit squares constructed with the minimum number of unit edges?

Q5) Make a list of the minimal number of edges required to construct n squares where n runs from 1 to 25.

Since squares can tile the plane it makes sense to ask the following.

Q6) Can you form a conjecture that might provide you with a pattern to always produce a minimal configuration for each n?

---TRIANGLES---

Since equilateral triangles also tile the plane it makes sense to ask the following.

Q7) Make a list of the minimal number of edges required to construct n equilateral triangles where n runs from 1 to 24.

Q8) Can you form a conjecture that might provide you with a pattern to always produce a minimal configuration for each n?

---HEXAGONS---

Since regular hexagons tile the plane it makes sense to ask the following.

Q9) Make a list of the minimal number of edges required to construct n regular hexagons where n runs from 1 to 19.

10) Can you form a conjecture that might provide you with a pattern to always produce a minimal configuration for each n?

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