
AUSTRALIAN MATHEMATICS TRUST

# Algorithmic Thinking

# Structure of Session

- What is an algorithm?
- Why is it important to promote algorithmic thinking in the mathematics classroom?
- Constructing some simple algorithms.
- Checking and testing algorithms.
- Incremental algorithmic design
- Resources for developing algorithmic thinking – overview.
- The CAT competition as a resource – types of questions and how to use them.
- Moving from algorithmic thinking to programming.
- Programming in the Maths classroom.

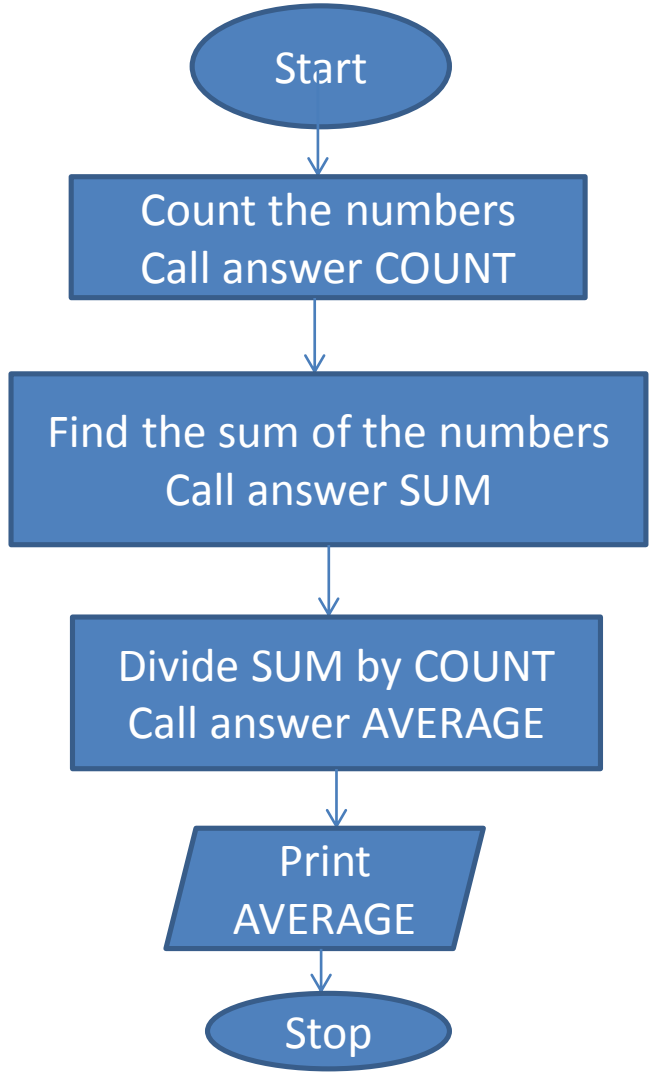
# What is an algorithm?

- An algorithm is a finite set of steps or operations which solve a particular problem or class of problems.
- eg To find the average of a set of numbers.
  - Count how many numbers in the list (COUNT)
  - Add up the numbers in the list (SUM)
  - Divide the SUM by the COUNT (AVERAGE)
  - Print AVERAGE

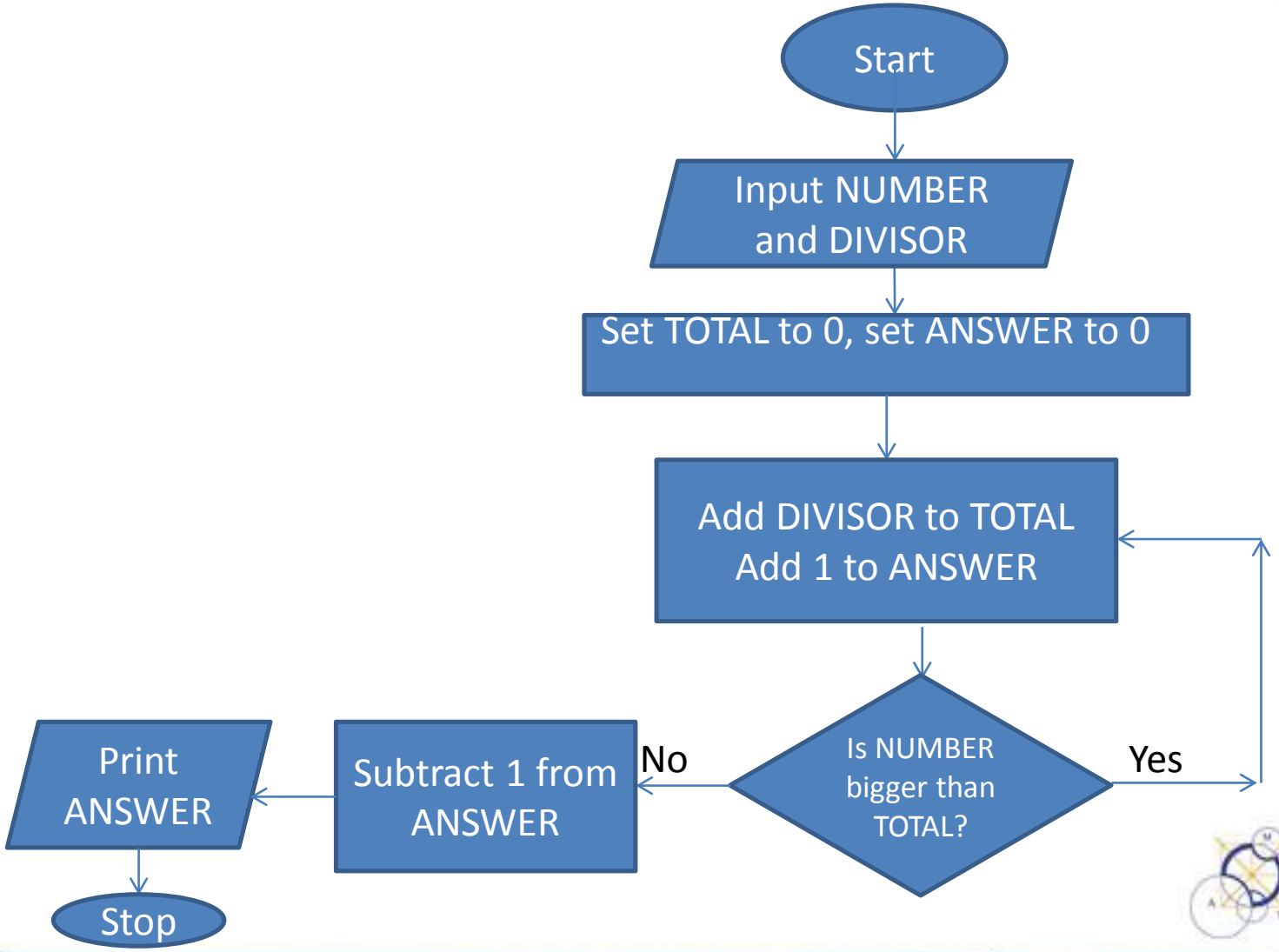
# How can we represent algorithms?

- Everyday language
- Flowcharts
- Pseudocode
- Programming language

# Flowcharts


Start
Count the numbers
Call answer COUNT
Find the sum of the numbers
Call answer SUM
Divide SUM by COUNT
Call answer AVERAGE
Print
AVERAGE
Stop

# Making decisions



# Making decisions

| NUMBER | DIVISOR | TOTAL | ANSWER |
|---|---|---|---|
| 20 | 6 | 0 | 0 |
| | | | |
| | | | |
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# Why is algorithmic thinking important?

- Teaching algorithms alone can be very damaging
- Teaching students how to think algorithmically can be very powerful
- Algorithmic thinking is now a part of the Australian Curriculum

# Australian Curriculum – Digital Technologies (F – 2)

| Knowledge and Understanding | Processes and Production Skills |
|---|---|
| Recognise and explore digital systems for a purpose | Collect, explore and sort data, and use digital systems to present the data creatively |
| Recognise and explore patterns in data and represent data as pictures, symbols and diagrams | Follow, describe and represent a sequence of steps and decisions (algorithms) needed to solve simple problems |
| | Explore how people safely use common information systems to meet information, communication and recreation needs |
| | Create and organise ideas and information using information systems independently and with others, and share these with known people in safe online environment |

# Australian Curriculum – Digital Technologies (3 - 4)

| Knowledge and Understanding | Processes and Production Skills |
|---|---|
| Identify and explore a range of digital systems with peripheral devices for different purposes, and transmit different types of data | Collect, access and present different types of data using simple software to create information and solve problems |
| Recognise different types of data and explore how the same data can be represented in different ways | Define simple problems, and describe and follow a sequence of steps and decisions (algorithms) needed to solve them |
| | Implement simple digital solutions as visual programs with algorithms involving branching (decisions) and user input |

# Australian Curriculum – Digital Technologies (3 - 4) – cont.

| Knowledge and Understanding | Processes and Production Skills |
|---|---|
| | Create and organise ideas and information using information systems independently and with others, and share these with known people in safe online environments |
| | Plan, create and communicate ideas and information independently and with others, applying agreed ethical and social protocols |

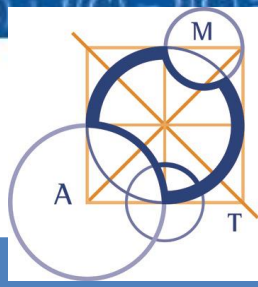
# Australian Curriculum – Digital Technologies (5 - 6 )

| Knowledge and Understanding | Processes and Production Skills |
|---|---|
| Examine the main components of common digital systems and how they connect together to form networks | Acquire, store and validate different types of data, and use a range of software to interpret and visualise data to create information |
| Examine how whole numbers are used to represent all data in digital systems | Define problems in terms of data and functional requirements drawing on previously solved problems |
| | Design a user interface for a digital system |
| | Design, modify and follow simple algorithms involving sequences of steps, branching and iteration (repetition) |

# Australian Curriculum – Digital Technologies (5 - 6 )

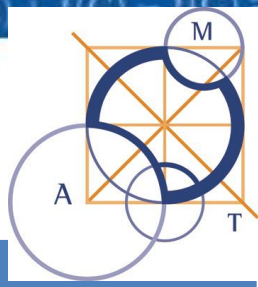
| Knowledge and Understanding | Processes and Production Skills |
|---|---|
| | Implement digital solutions as simple visual programs involving branching, iteration, and user input |
| | Explain how student solutions and existing information systems are sustainable and meet current and future local community needs |
| | Plan, create and communicate ideas and information, including collaboratively online, applying agreed ethical, social and technical protocols |

# Australian Curriculum – Digital Technologies (7 – 8)




| Knowledge and Understanding | Processes and Production Skills |
|---|---|
| Investigate how data is transmitted and secured in wired, wireless and mobile networks, and how the specifications affect performance | Acquire data from a range of sources and evaluate authenticity, accuracy and timeliness |
| Investigate how digital systems represent text, image and audio data in binary | Analyse and visualise data using a range of software to create information, and use structured data to model objects or events |
| | Define and decompose real-world problems taking into account functional requirements and economic, environmental, social, technical and usability constraints |
| | Design the user experience of a digital system, generating, evaluating and communicating alternative design |

# Australian Curriculum – Digital Technologies (7 – 8)




| Knowledge and Understanding | Processes and Production Skills |
|---|---|
| | Design algorithms represented diagrammatically and in English, and trace algorithms to predict output for a given input and to identify errors |
| | Implement and modify programs with user interfaces involving branching, iteration and functions in a general-purpose programming language |
| | Evaluate how student solutions and existing information systems meet needs, are innovative, and take account of future risks and sustainability |
| | Plan and manage projects that create and communicate ideas and information collaboratively online, taking safety and social contexts into account |

# Australian Curriculum – Digital Technologies (9 – 10)

| Knowledge and Understanding | Processes and Production Skills |
|---|---|
| Investigate the role of hardware and software in managing, controlling and securing the movement of and access to data in networked digital systems | Develop techniques for acquiring, storing and validating quantitative and qualitative data from a range of sources, considering privacy and security requirements |
| Analyse simple compression of data and how content data are separated from presentation | Analyse and visualise data to create information and address complex problems, and model processes, entities and their relationships using structured data |
| | Define and decompose real-world problems precisely, taking into account functional and non-functional requirements and including interviewing stakeholders to identify needs |
| | Design the user experience of a digital system by evaluating alternative designs against criteria including functionality, accessibility, usability, and aesthetics |

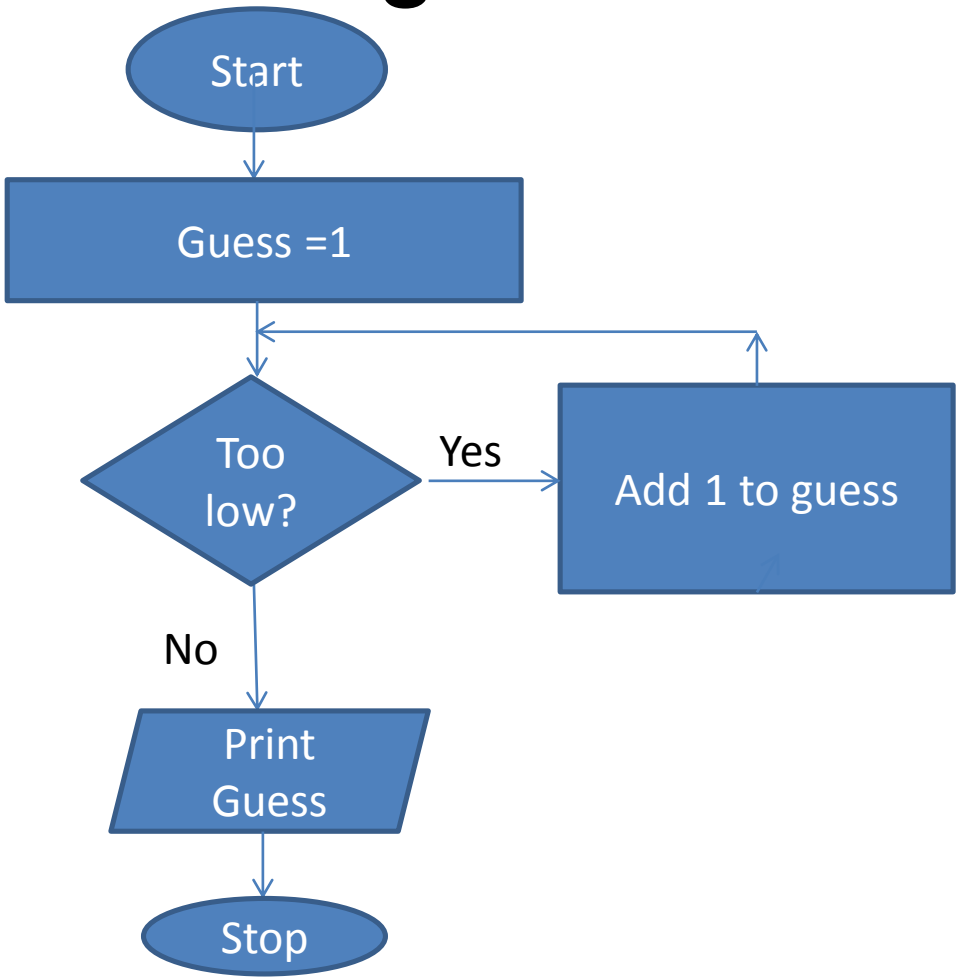
# Australian Curriculum – Digital Technologies (9 – 10)

| Knowledge and Understanding | Processes and Production Skills |
|---|---|
| | Design algorithms represented diagrammatically and in structured English and validate algorithms and programs through tracing and test cases |
| | Implement modular programs, applying selected algorithms and data structures including using an object-oriented programming language |
| | Evaluate critically how student solutions and existing information systems and policies, take account of future risks and sustainability and provide opportunities for innovation and enterprise |
| | Create interactive solutions for sharing ideas and information online, taking into account safety, social contexts and legal responsibilities |
| | Plan and manage projects using an iterative and collaborative approach, identifying risks and considering safety and sustainability |

# Constructing some simple algorithms

- Arithmetic processes (work from high level to low level)
- Search algorithms
- Sort algorithms

# Algorithmic efficiency - a simple search algorithm



# Checking and testing algorithms

- Does it give the results you want?
- Trace the value of each variable using a table
- Carefully design test data to test each branch
- What about special cases?

# Incremental algorithmic design

- When designing more complex algorithms:
  - Work from high level design to low level design
  - Consider solving a simpler problem
  - Build the algorithm incrementally
  - Get someone else (preferably fiendish) to test your algorithm

# Resources for teaching algorithmic thinking

- Yourself
- Code.org: Anybody can learn
- Code Club Australia
- Australian Mathematics Trust: AMT
- Scratch - Imagine, Program, Share
- Codecademy
- mbots
- Bebras Australia | Digital Careers Computational Thinking

# AMT Informatics Program

- Designed to encourage algorithmic and logical thinking and to identify and encourage potential programmers in all students:
  1. CAT (in March) – a non-programming competition in algorithmic thinking, on-line or paper, Years 5 – 12
  2. Programming training module through AMT website
  3. AIO (September) – open programming competition
  4. AIIO (February) – invitational programming competition
  5. Selection schools and further invitational competitions leading to IOI

# CAT (Computational and Algorithmic Thinking Competition)

- Late March – 1 hour, 15 questions
- Emphasises algorithmic thinking.
- Will appeal to some students who do not shine in conventional mathematics.
- Will identify potential programmers.
- Online competition is free in 2016
- Upper Primary, Junior, Intermediate, Senior
- Practice material available

# Structure of Paper

- First six questions – 3 marks each – traditional multiple-choice (5 options)
- Next nine questions - three-stage tasks – 2 marks per stage
  - A three-stage task consists of a small problem to solve where there are three sets of data
  - The first data set is small or simple enough to be susceptible to ad-hoc techniques, but hopefully provides a basis for students to get a feeling for the problem and to develop an algorithm to be used in the remaining data sets.
  - The answers are numbers in the range 0-999.

# Question Types

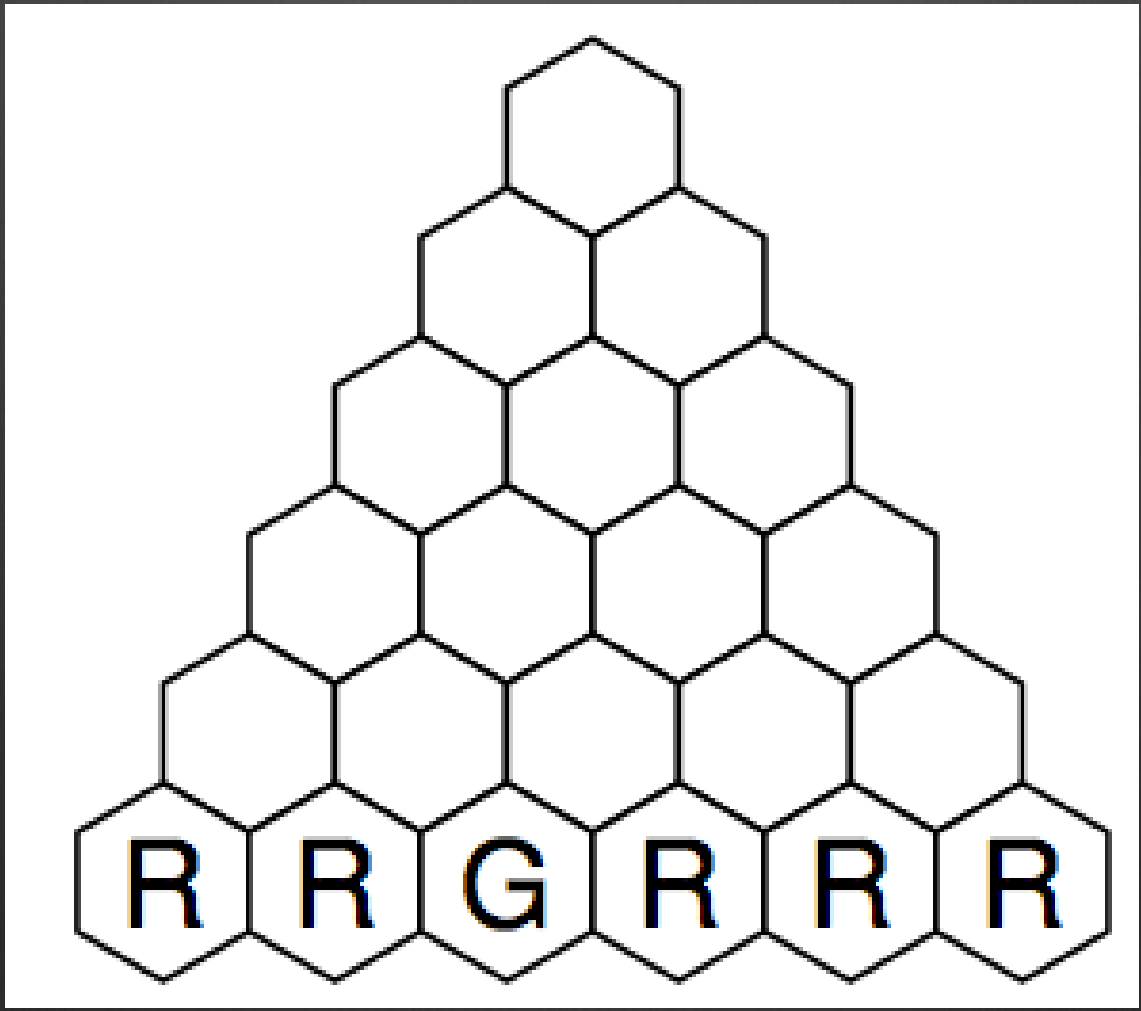
- Rules – testing the ability to follow a simple rule or algorithm
- Logic – testing the ability to logically interpret the meaning of a situation or data set. This may include 'debugging' an algorithm
- Analysis – testing the ability to analyse the application of a given algorithm. This may include searching, sorting, analysing patterns or counting routes
- Algorithm Design – there are a variety of types

# Sample questions

| | |
|---|---|
| **Junior 1** | **Hrossan Quilts** |
| **Junior 5** | **Emu** |
| **Senior 3** | **Frog** |
| **Senior 4** | **Musca** |

# Junior 1. Hrossan Quilts

Each hexagon and the two beneath it must be the same colour (R,G or B) or different colours.
How many blue patches are there in the quilt below?



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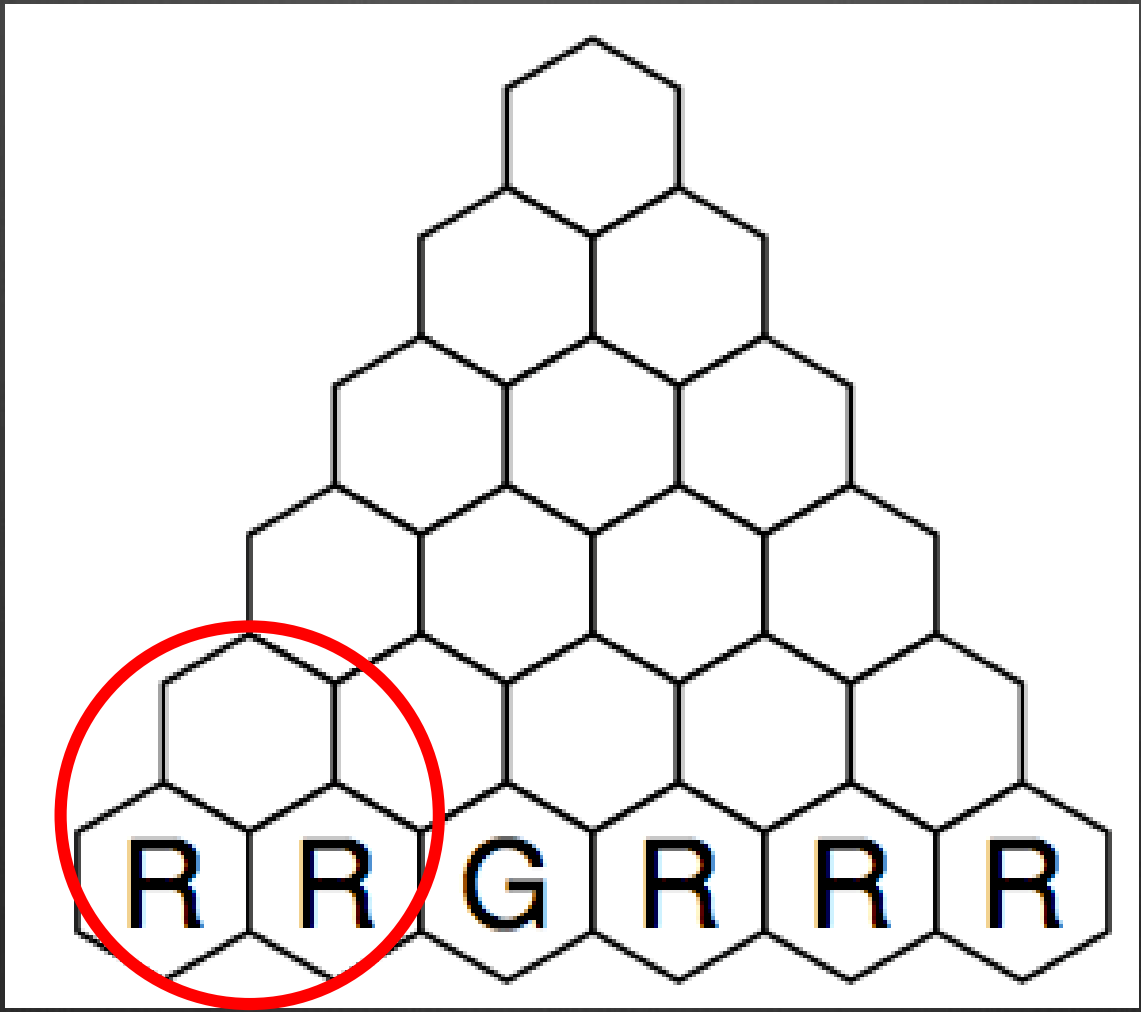

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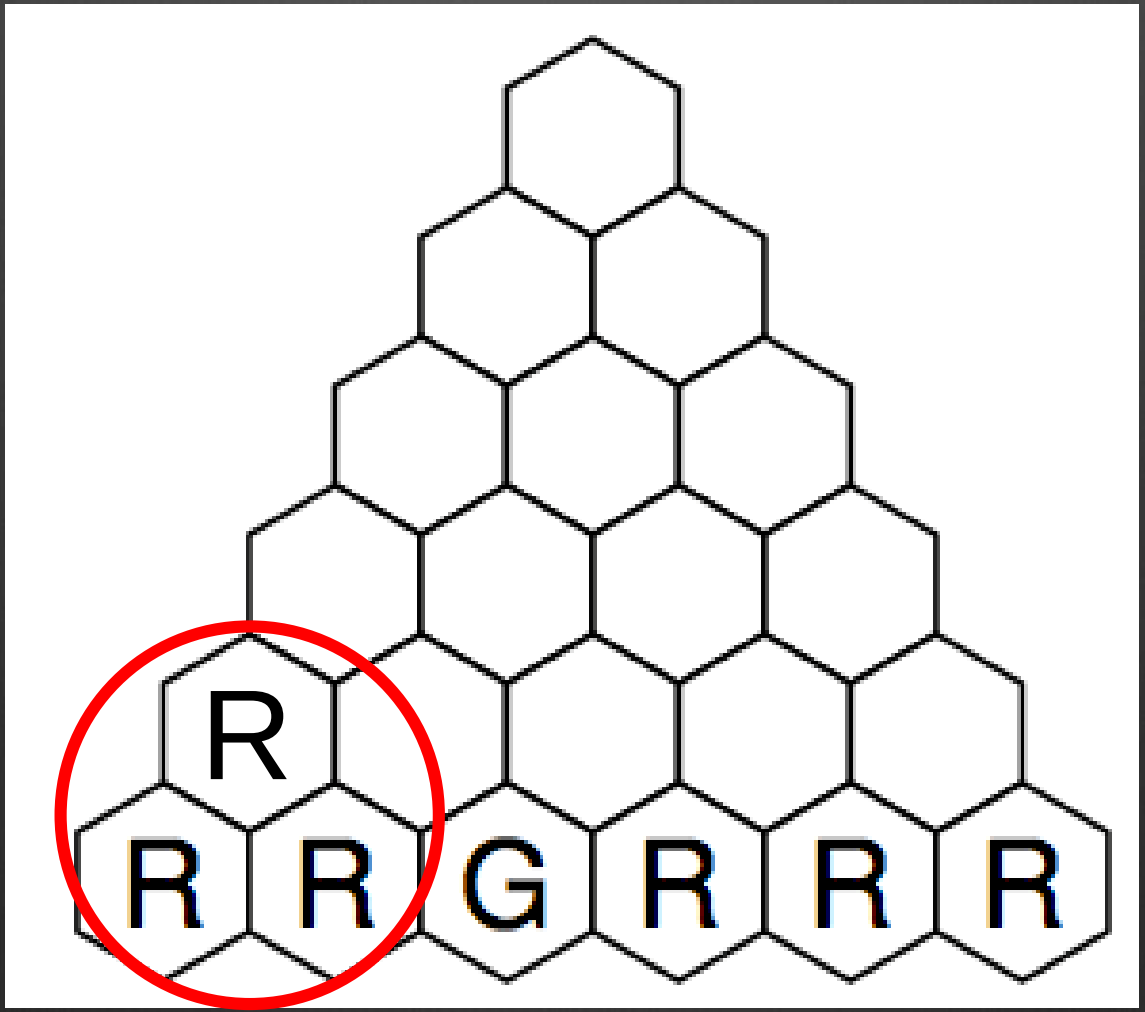

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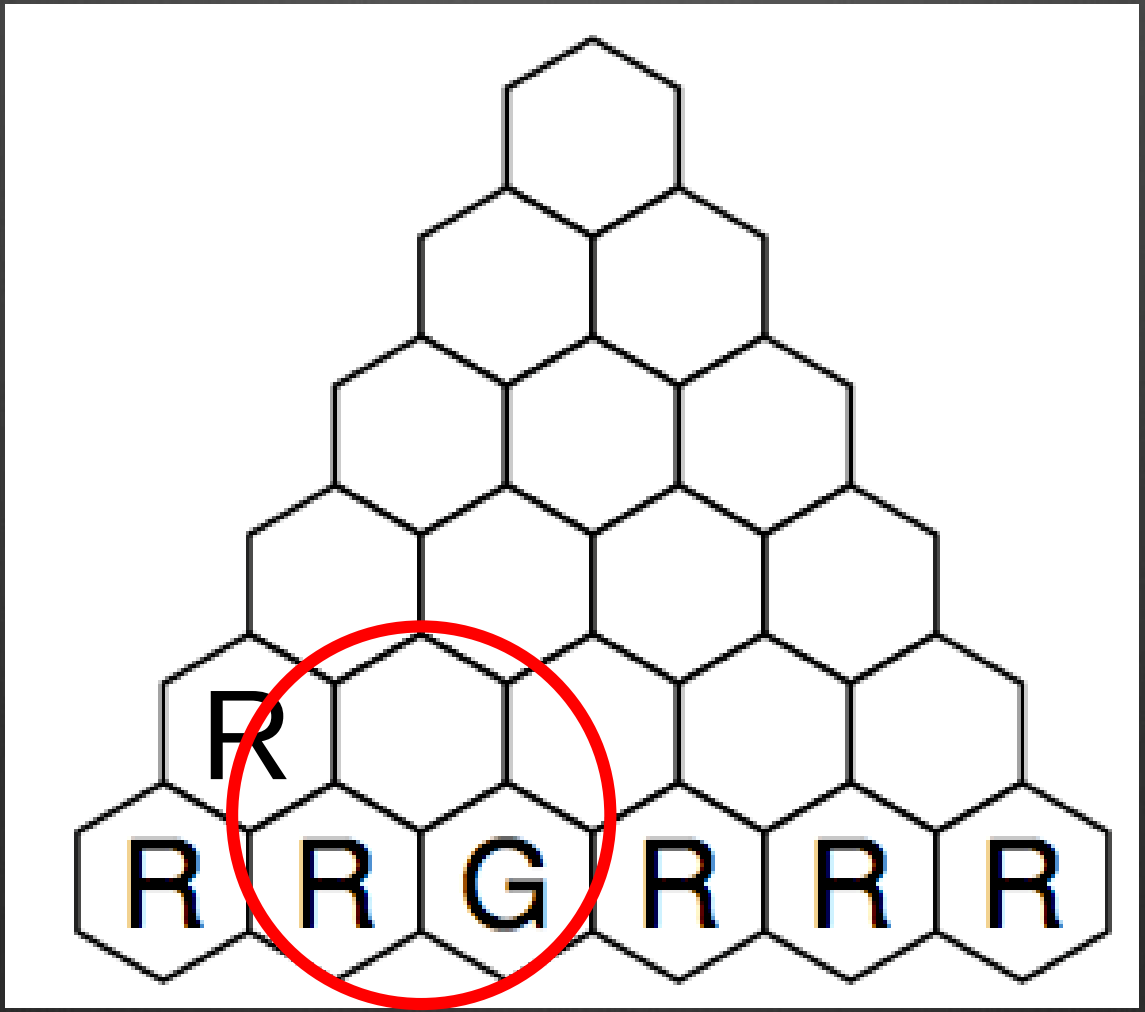

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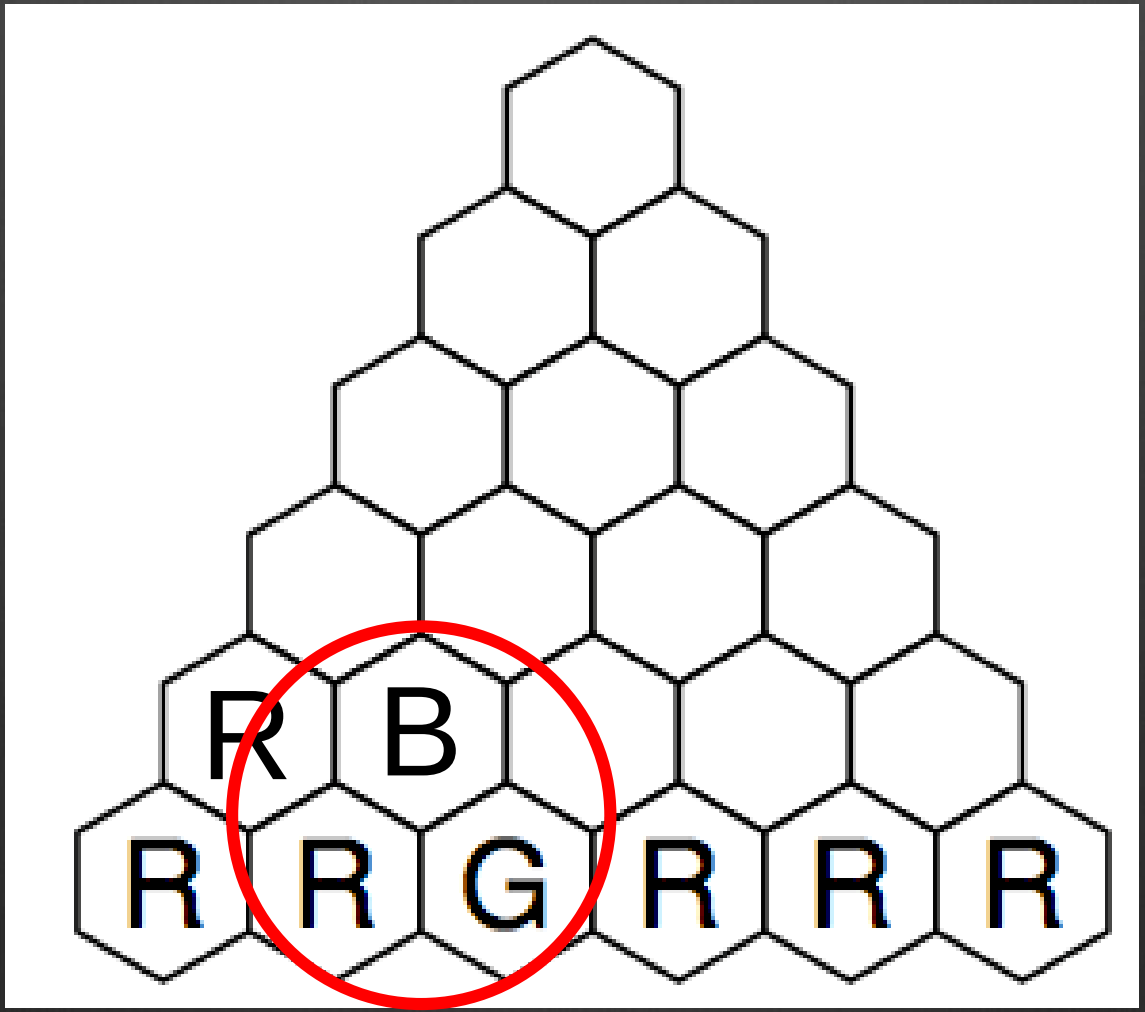

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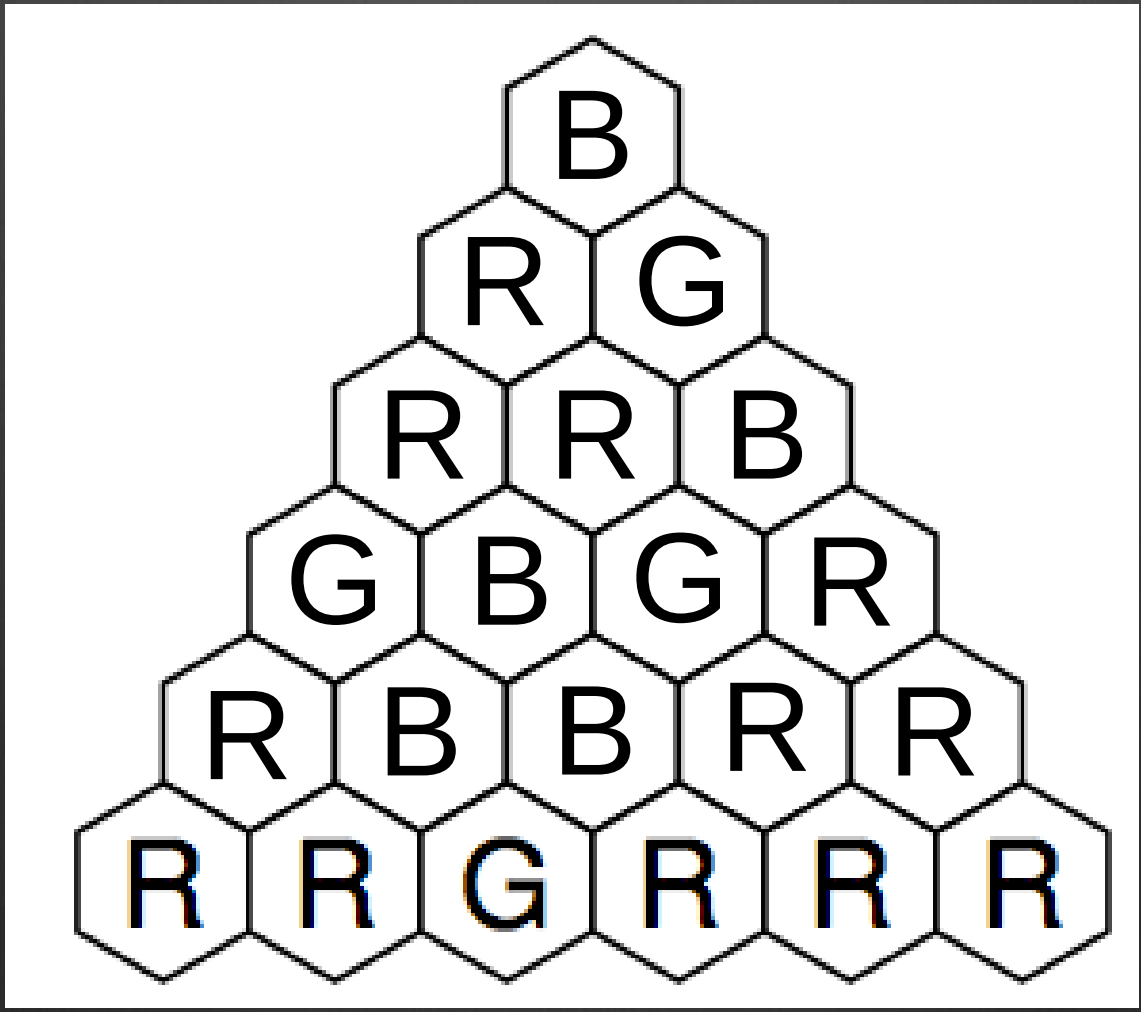

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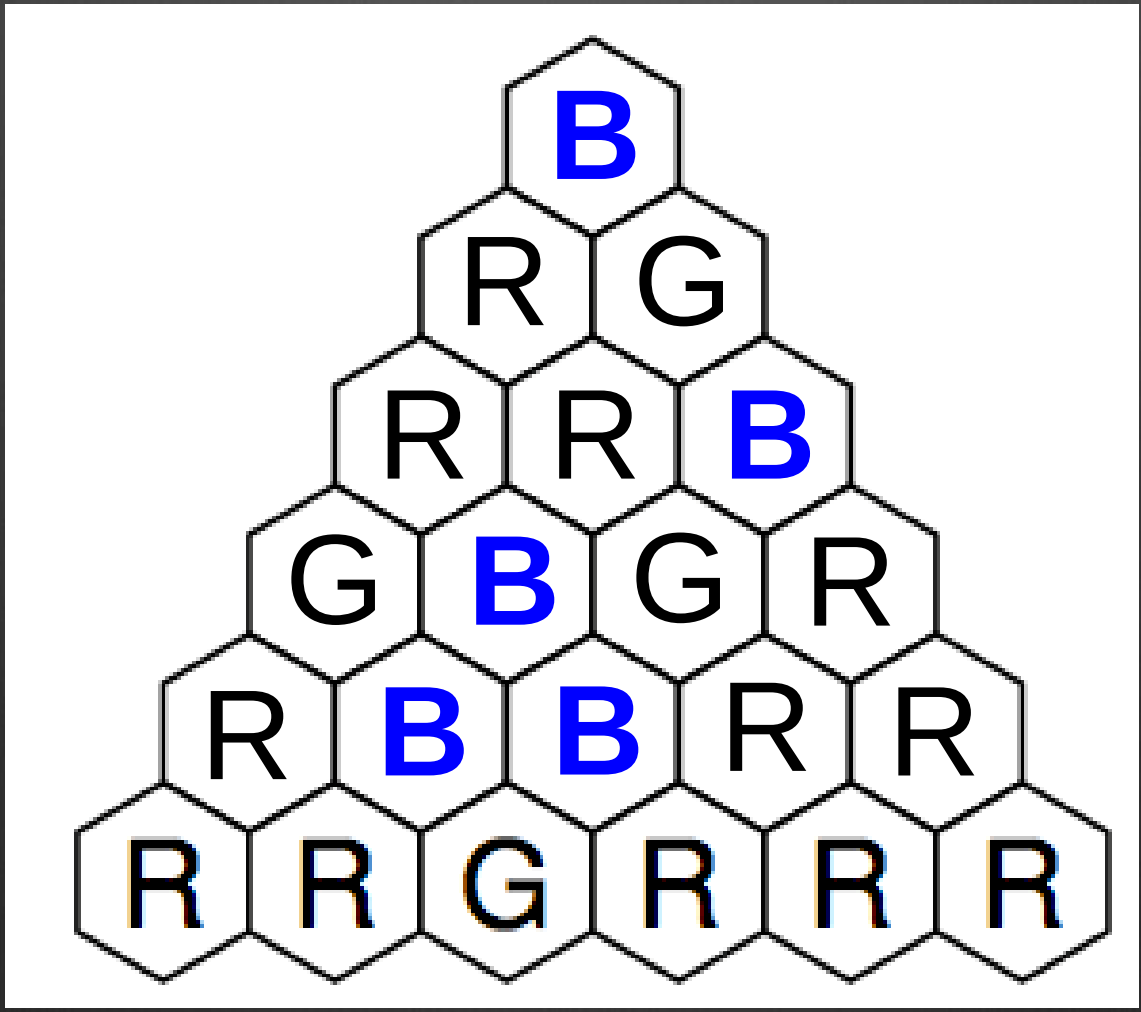

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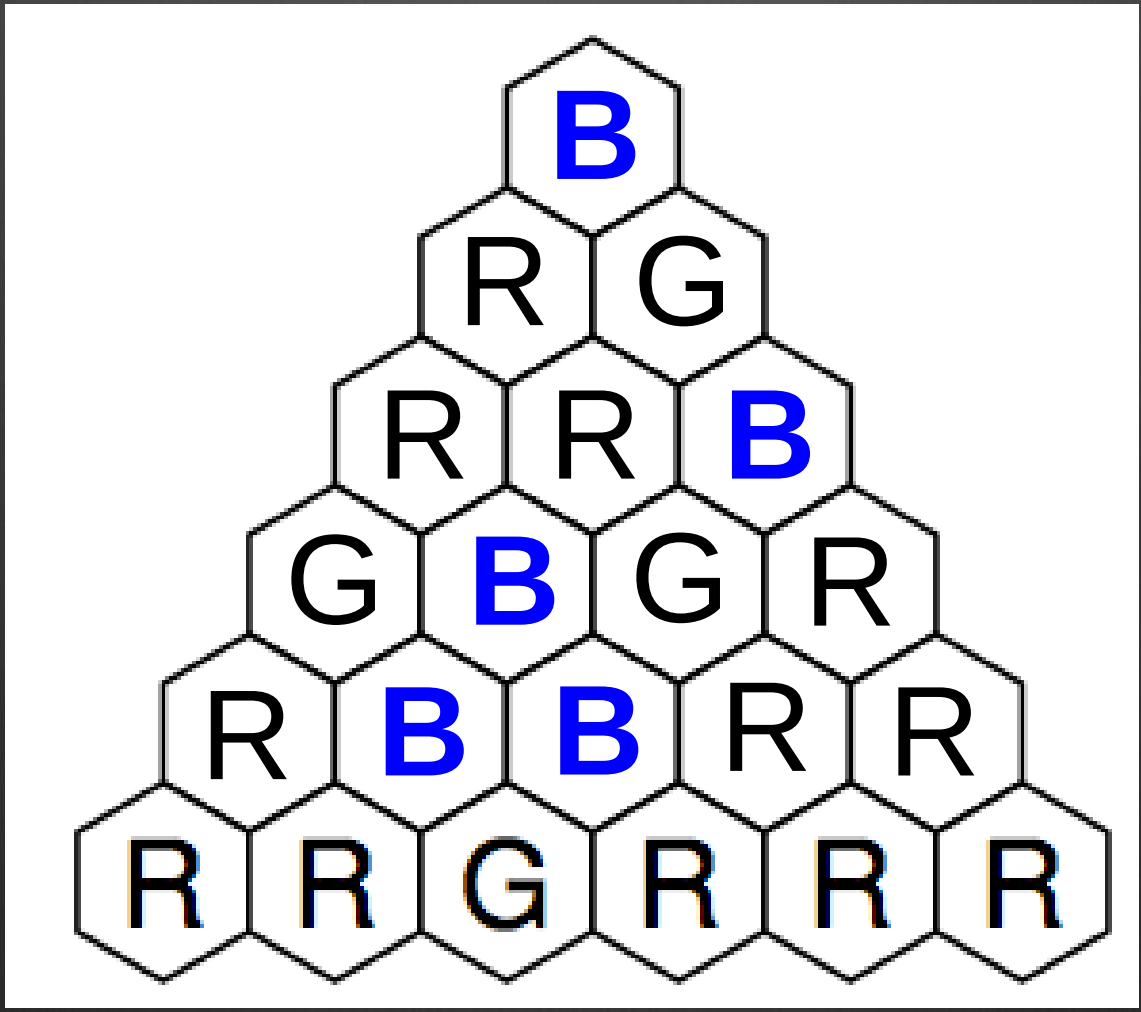
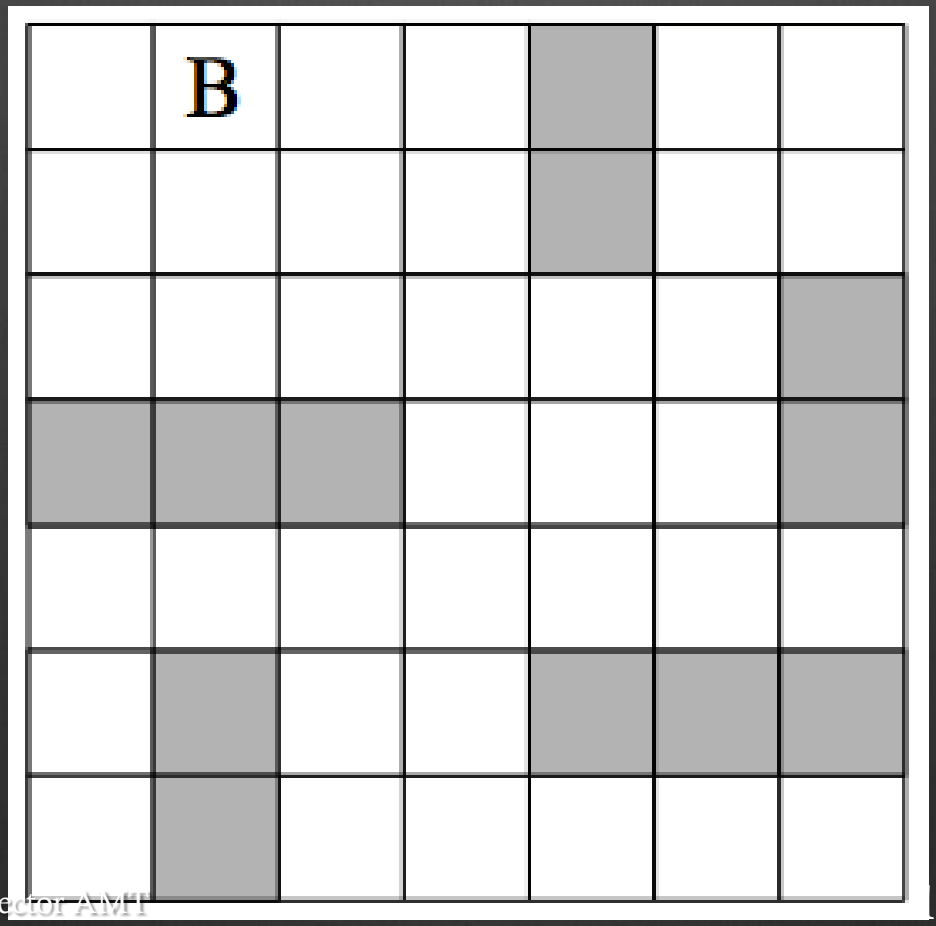
How many blue patches are there in the quilt below? **5**



# Junior 5. Emu

Emus will take at most 3 steps from the nesting burrow in search of food.
They refuse to walk through water or even over water.
B is the location of the burrow and the shaded cells are wet.
An emu can cover up to 4 cells in one step, and a step may not turn a corner.
In the diagram below, in how many dry cells will the emu NOT search for food?



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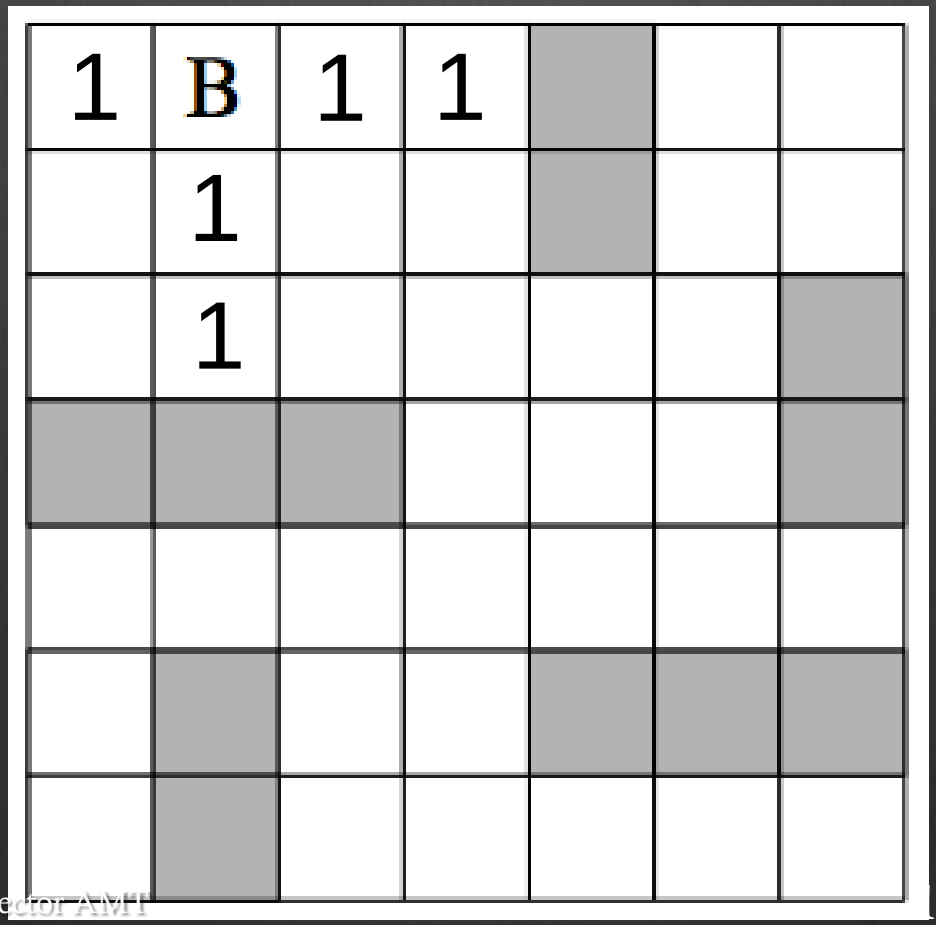

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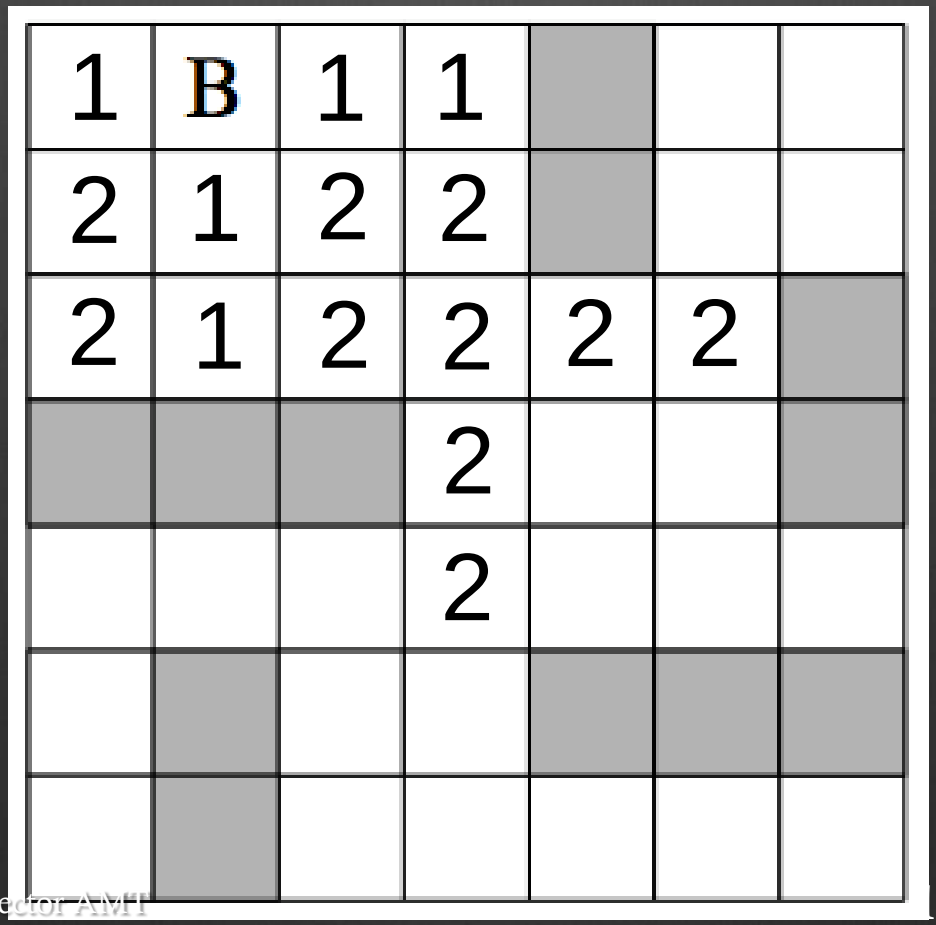

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In the diagram below, in how many dry cells will the emu NOT search for food?

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | B | 1 | 1 | | 3 | |
| 2 | 1 | 2 | 2 | | 3 | |
| 2 | 1 | 2 | 2 | 2 | 2 | |
| | | | 2 | 3 | 3 | |
| 3 | 3 | 3 | 2 | 3 | 3 | 3 |
| | | | 3 | | | |
| | | | 3 | | | |

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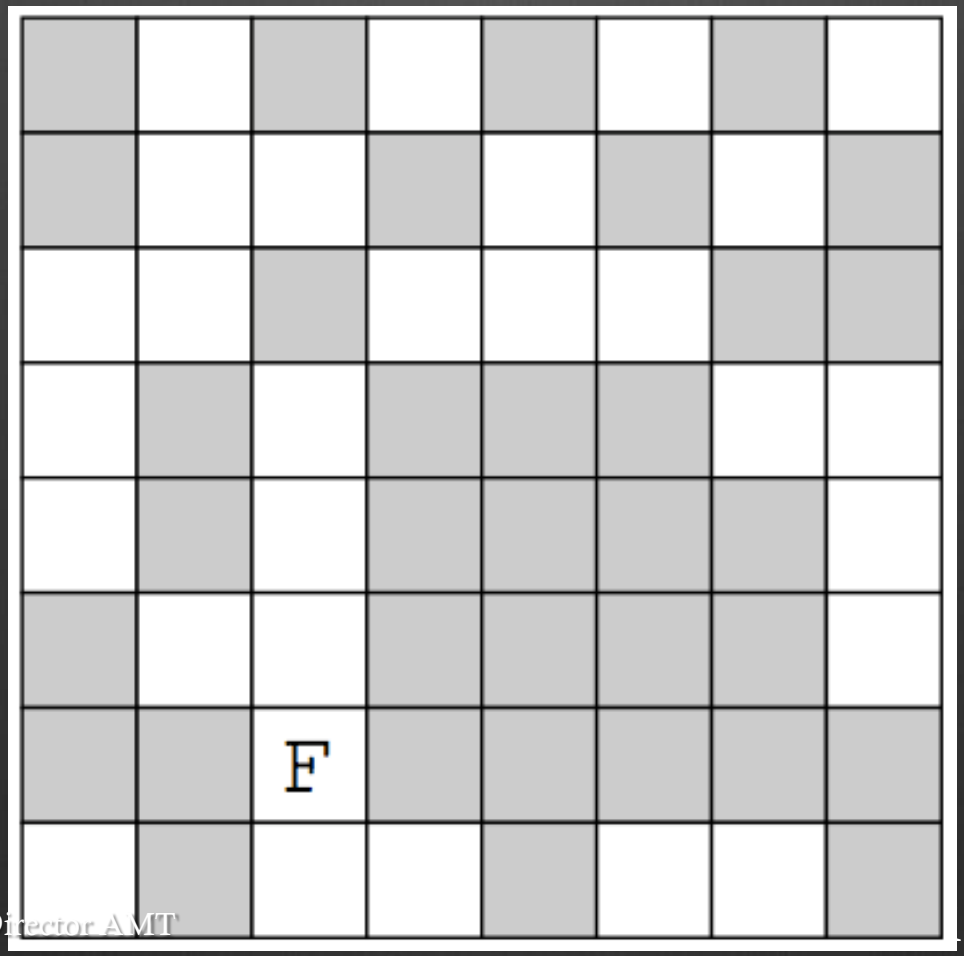
In the diagram below, in how many dry cells will the emu NOT search for food? **9**

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | B | 1 | 1 | | 3 | |
| 2 | 1 | 2 | 2 | | 3 | |
| 2 | 1 | 2 | 2 | 2 | 2 | |
| | | | 2 | 3 | 3 | |
| 3 | 3 | 3 | 2 | 3 | 3 | 3 |
| | | | 3 | | | |
| | | | 3 | | | |

# Senior 3. Frog

White squares represent lily pads and grey squares represent water.
A frog may jump from one lily pad to another, horizontally or vertically.
It can jump over large spans of water, but it may not jump over another lily pad.
Beginning on the lily pad marked 'F', what is the minimum number of jumps the frog needs in order to ensure that it can reach any other lily pad?



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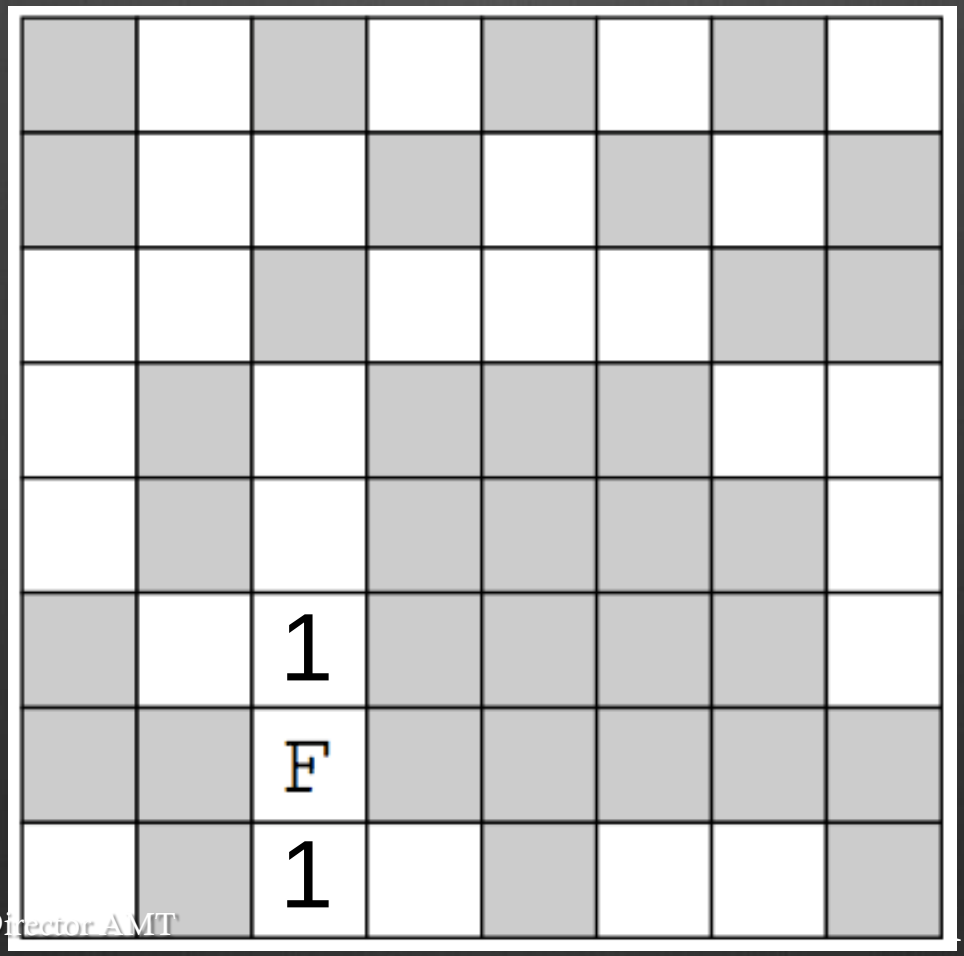

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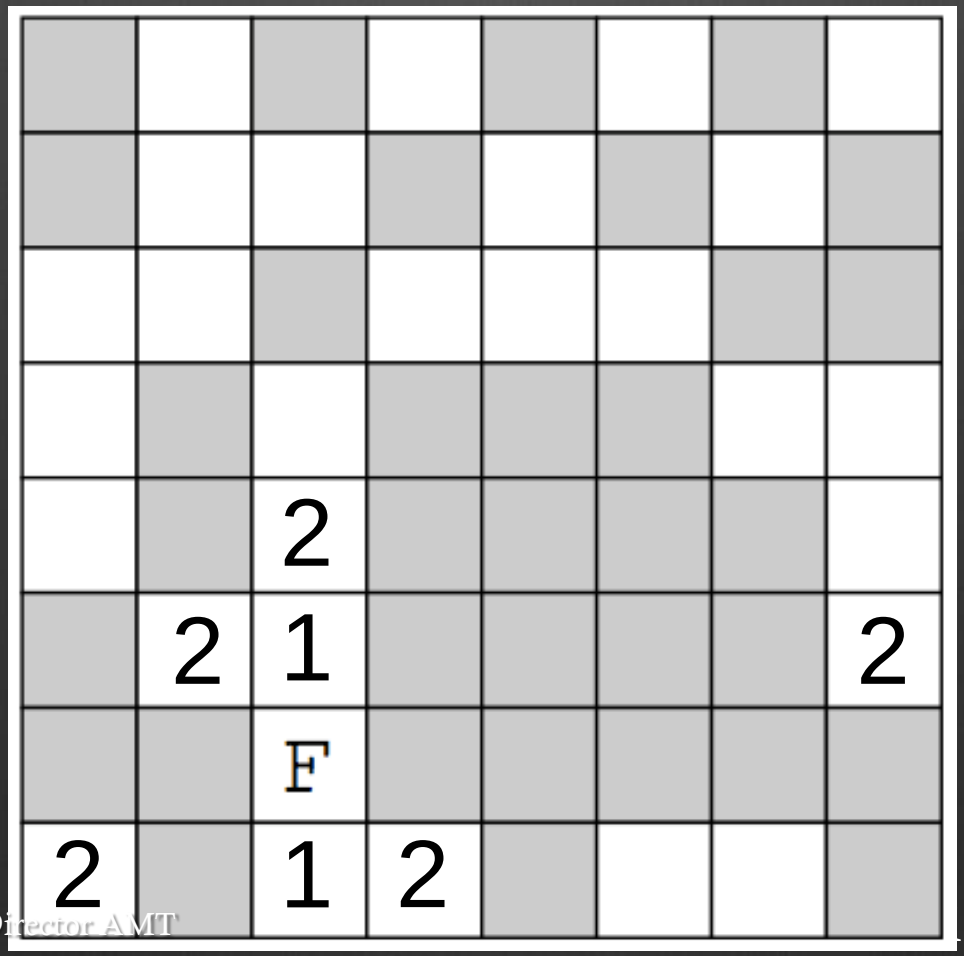

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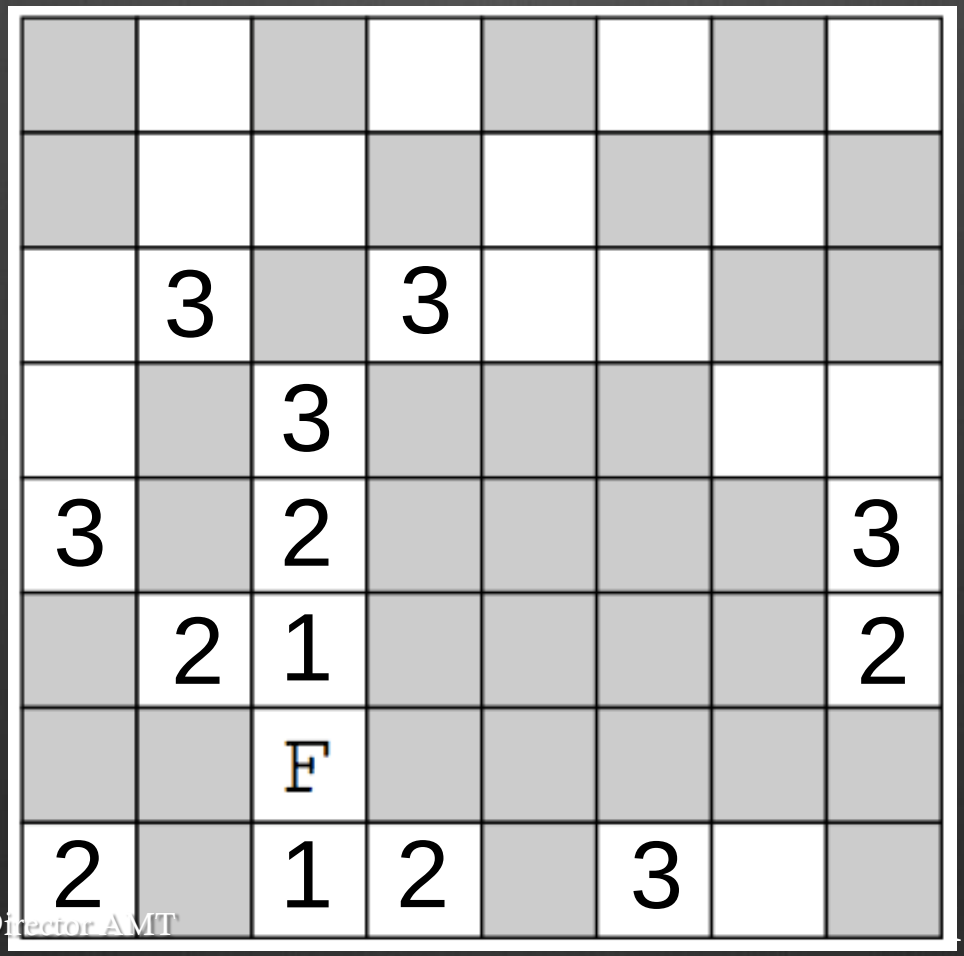

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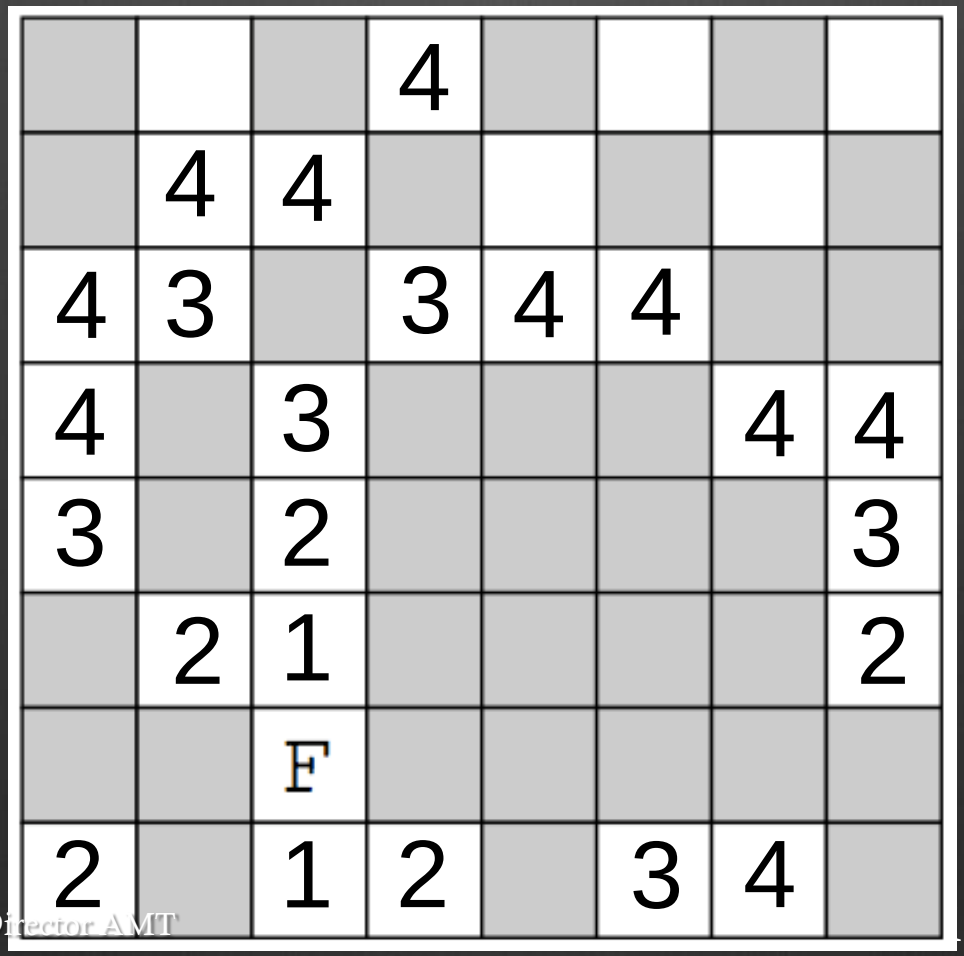

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| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 5 | | 4 | | 5 | | 5 |
| | 4 | 4 | | 5 | | 5 | |
| 4 | 3 | | 3 | 4 | 4 | | |
| 4 | | 3 | | | | 4 | 4 |
| 3 | | 2 | | | | | 3 |
| | 2 | 1 | | | | | 2 |
| | | F | | | | | |
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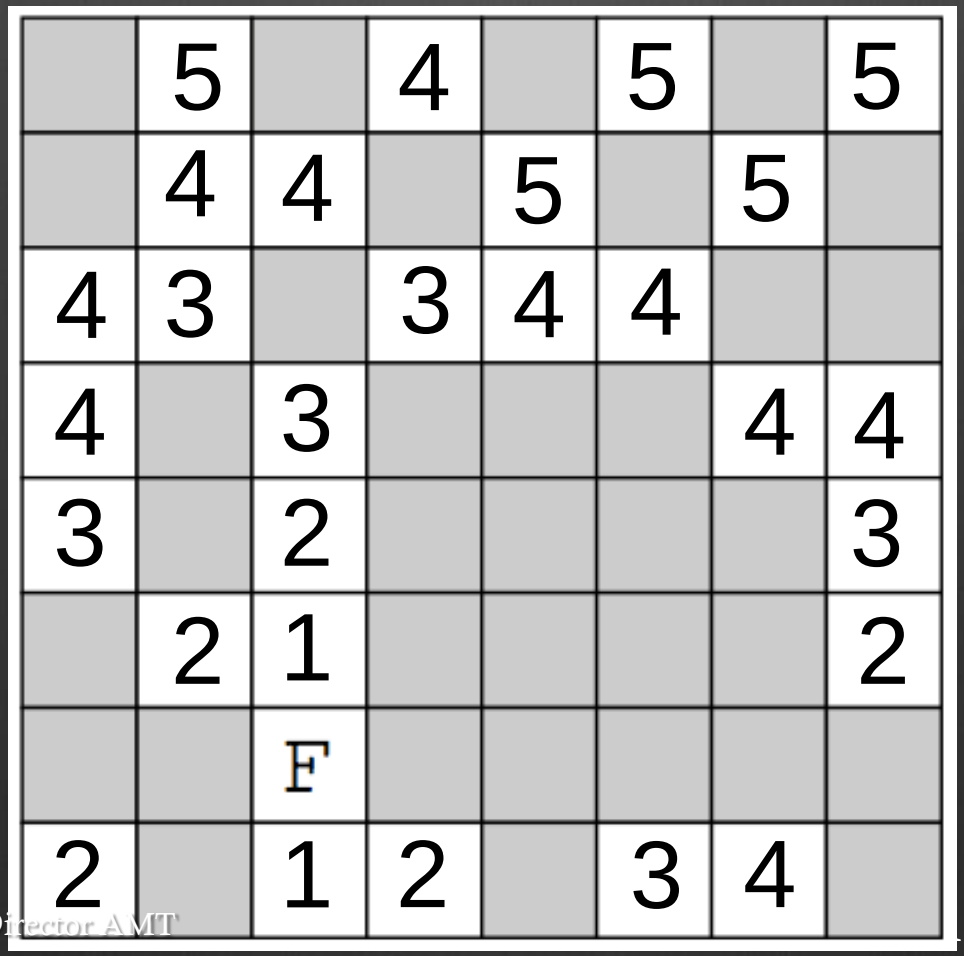
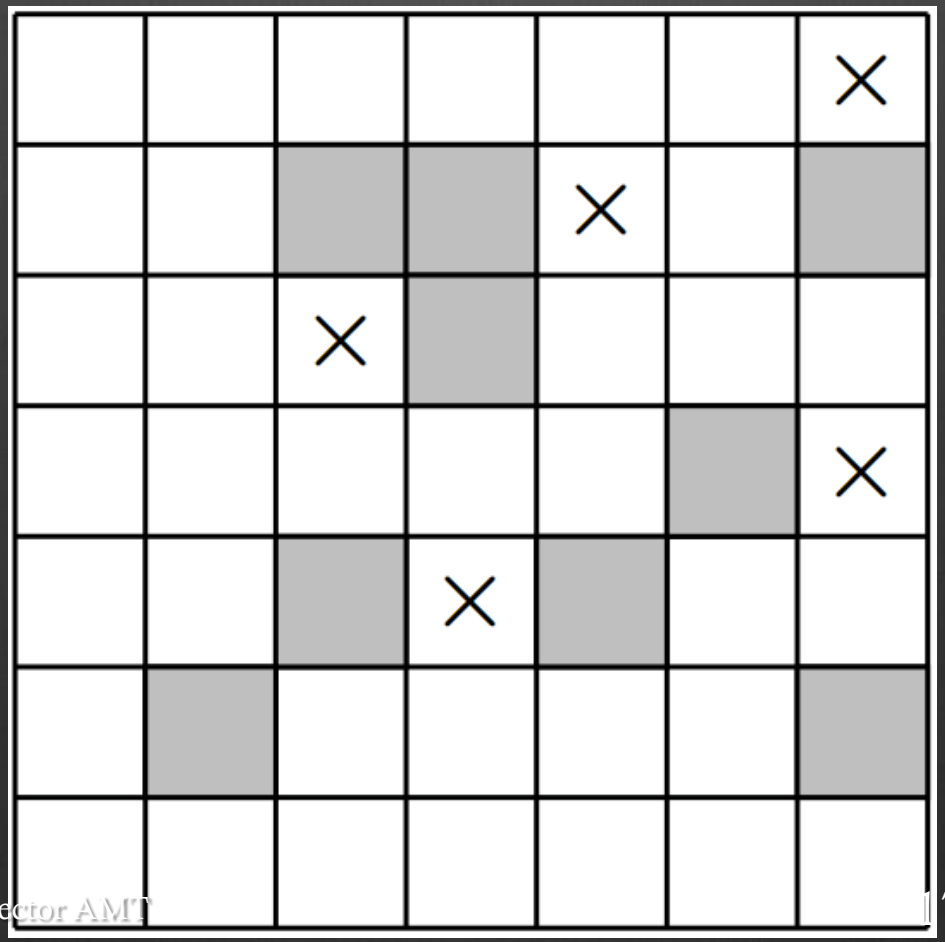


# Senior 4. Musca

Two players take turns to move a counter one or more cells to the left or down.
The counter may not be moved onto or across a shaded cell.
The winner is the player who moves the counter to the bottom left hand corner.
Some cells will be winning cells for the player who goes first, and some will be losing cells.
In the diagram below, how many of the cells marked × are winning cells?



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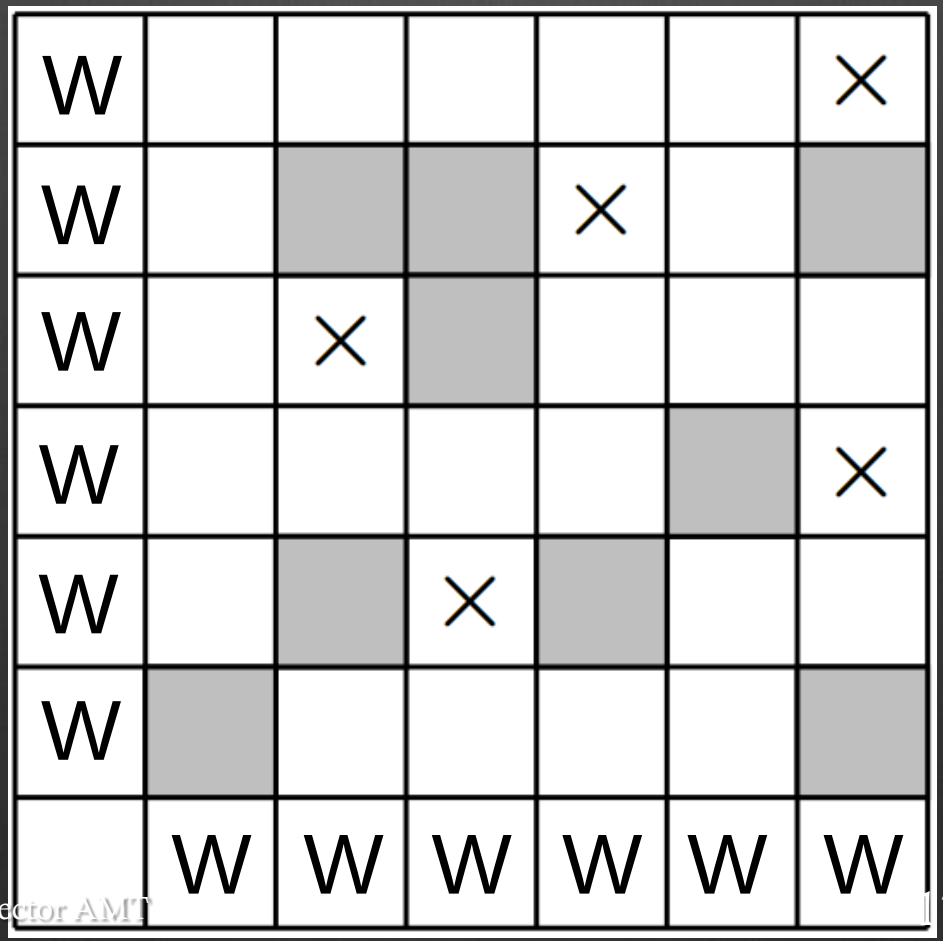

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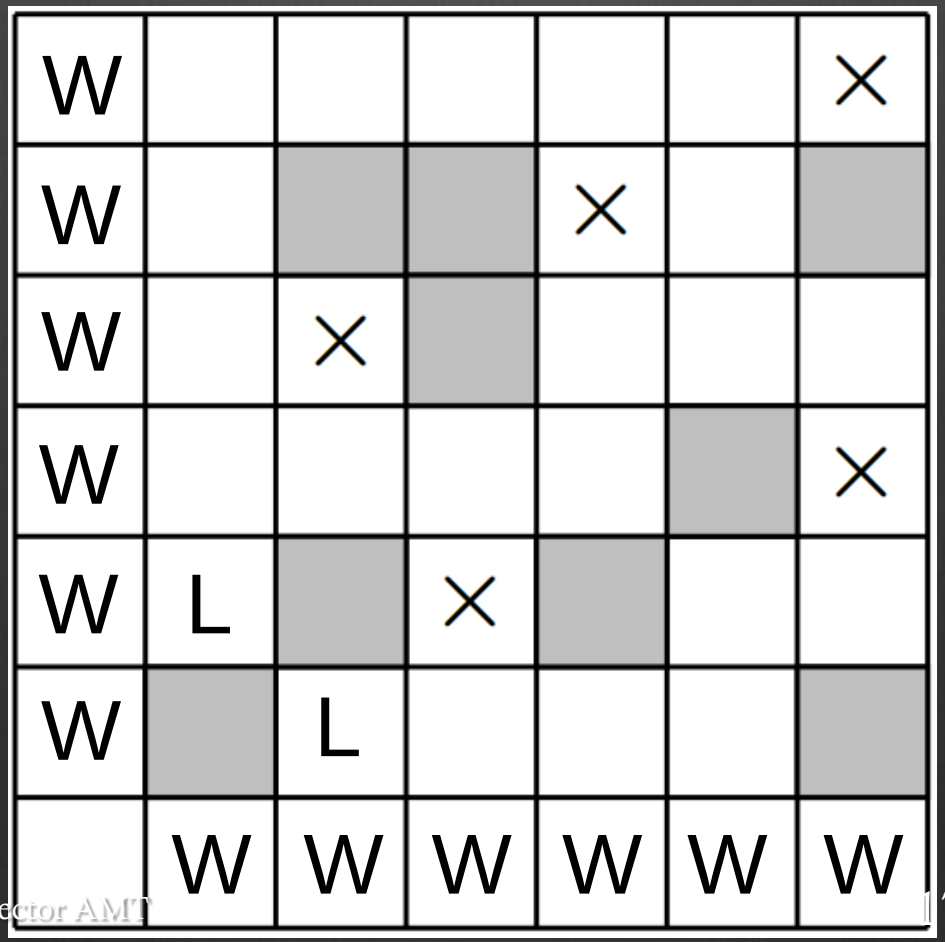

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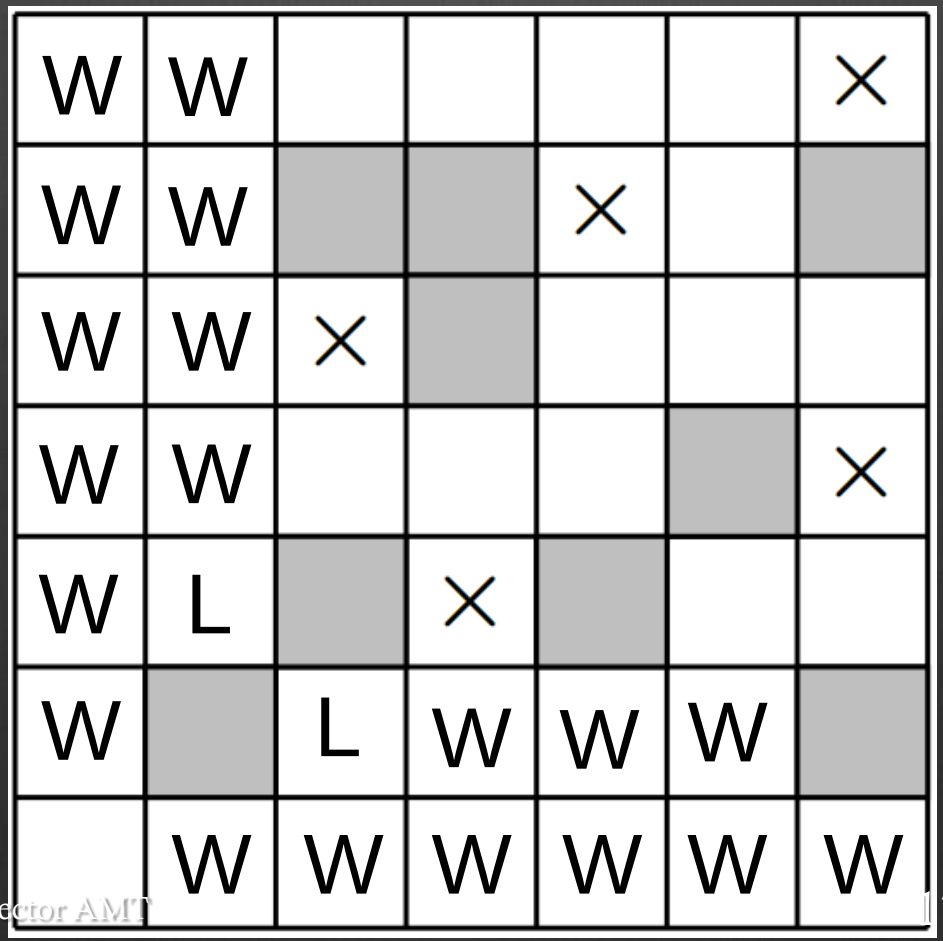

# Senior 4. Musca

Two players take turns to move a counter one or more cells to the left or down.
The counter may not be moved onto or across a shaded cell.
The winner is the player who moves the counter to the bottom left hand corner.
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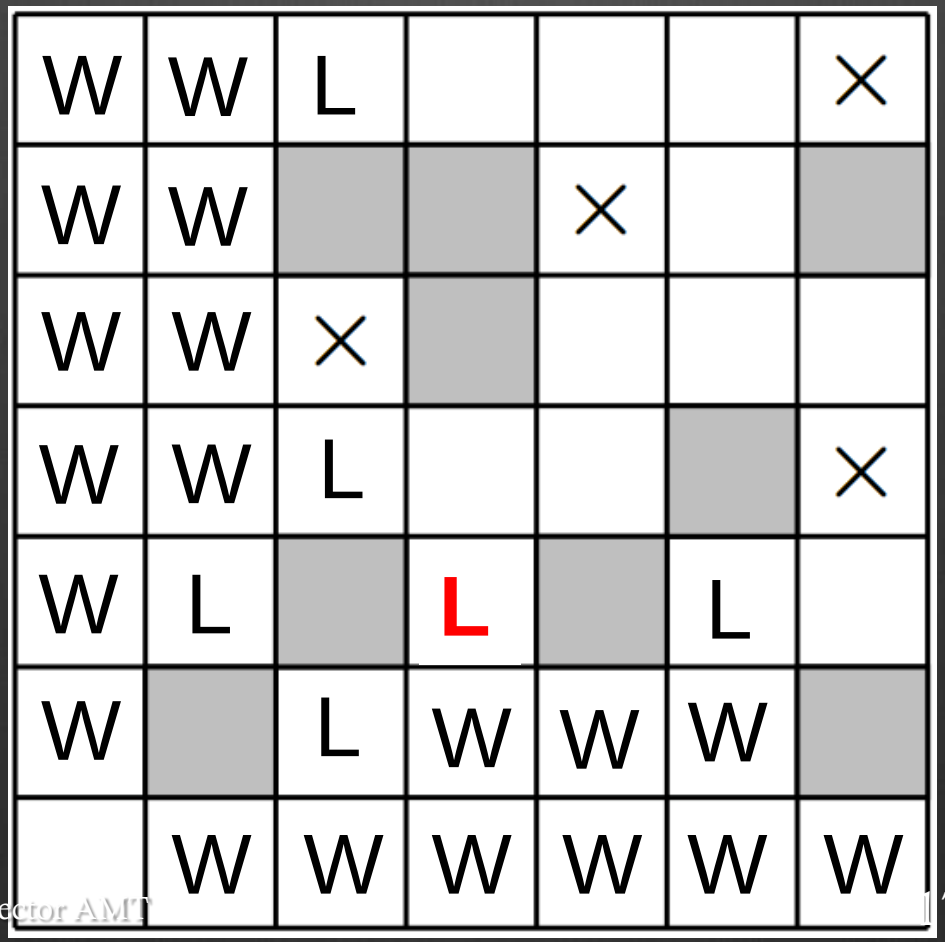

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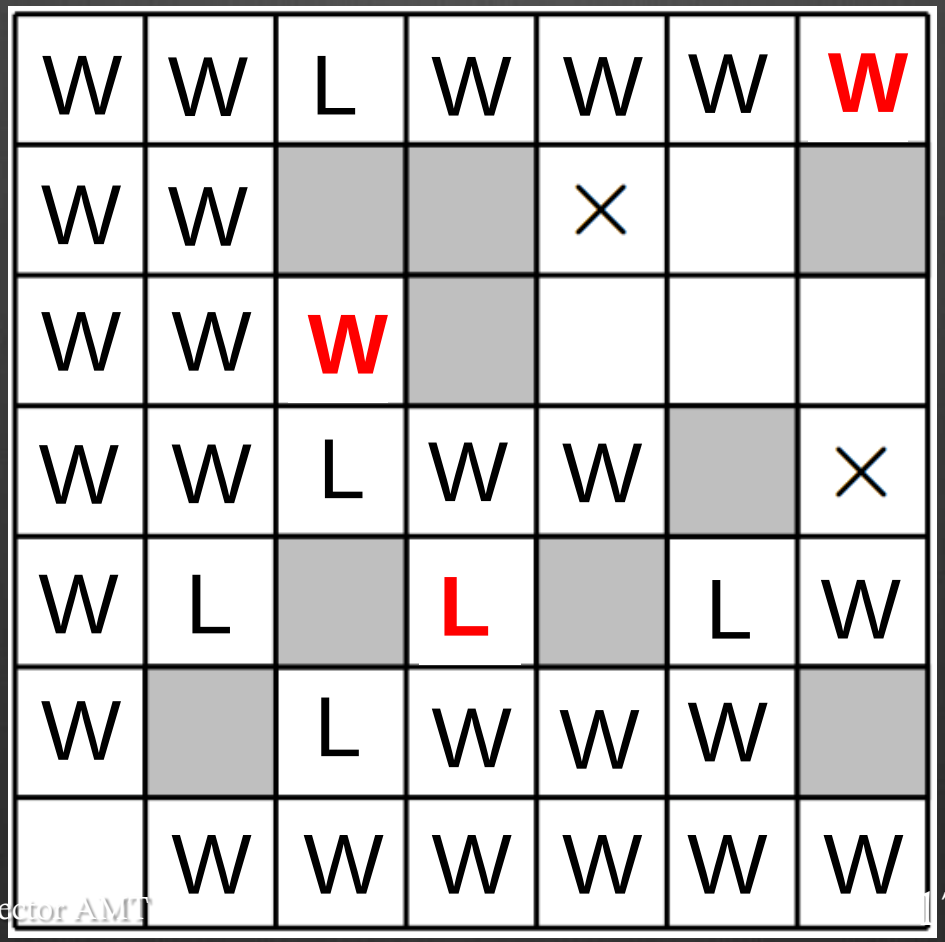

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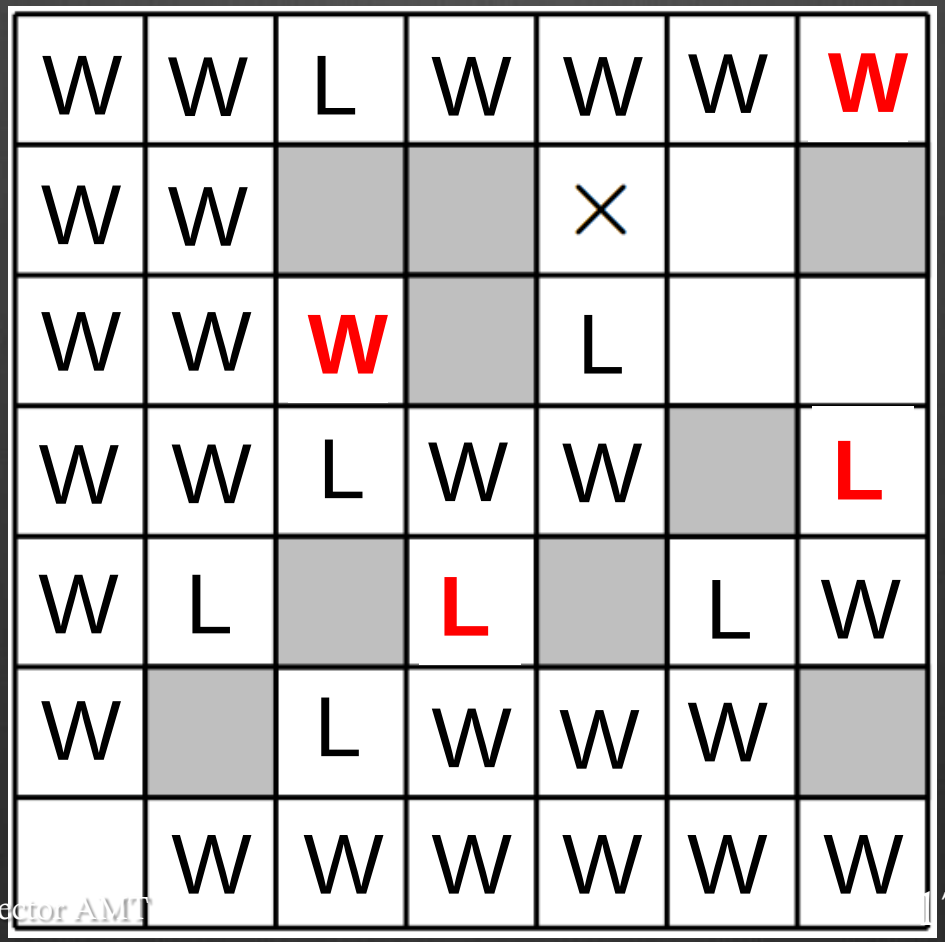

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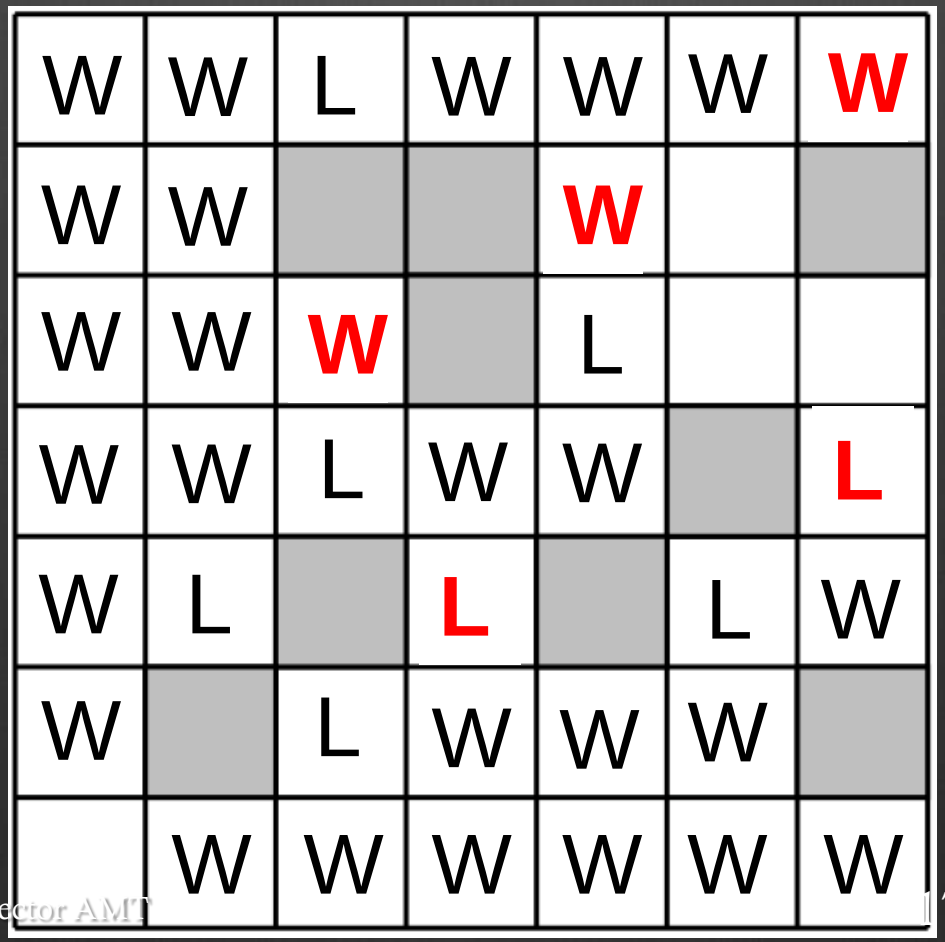

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In the diagram below, how many of the cells marked × are winning cells? **3**

| | | | | | | |
|---|---|---|---|---|---|---|
| W | W | L | W | W | W | **W** |
| W | W | | | **W** | | |
| W | W | **W** | | L | | |
| W | W | L | W | W | | **L** |
| W | L | | **L** | | L | W |
| W | | L | W | W | W | |
| | W | W | W | W | W | W |

# Resources for teaching algorithmic thinking

- Yourself
- Code.org: Anybody can learn
- Code Club Australia
- Australian Mathematics Trust: AMT
- Scratch - Imagine, Program, Share
- Codecademy
- mbots
- Bebras Australia | Digital Careers Computational Thinking